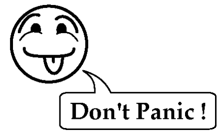
Howard University

WASHINGTON, D.C. 20059

DEPARTMENT OF PHYSICS AND ASTRONOMY
(202)-806-6245 (Main Office)
(202)-806-5830 (FAX)

2355 Sixth Str., NW, TKH Rm.215
thubsch@howard.edu
(202)-806-6257



Quantum Mechanics II
2nd Midterm Exam

30th March '98.

Instructor: T. Hübsch

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(Student name and ID)

This is an "open Textbook (Park), open lecture notes" two-part exam: problems #1–2 are due at the end of the Monday class, #3 and any additions/corrections to #1–2 are **due Friday, 5:00 p.m.** For full credit, show all your work. If you cannot complete one part of a calculation, a clear description of the procedure/method will still earn you partial credit. No collaboration is allowed; by handing in the exam you implicitly agree to abide by this.

1. Consider a modification of the Kronig-Penney model (p.454), where $c \to 0$, $V_0 \to \infty$ but $\Delta = cV_0$ remains constant; this is called the Dirac comb.

a. Determine the matching conditions across $V = \Delta\delta(x)$. [7*pt*]

b. Find the appropriate analogue of Eq. (15.11). [7*pt*]

c. Find the appropriate analogue of Eq. (15.12); explain the result. [7*pt*]

d. Describe the effect of $\Delta \to 0$ on the energy bands. [7*pt*]

e. Describe the effect of $\Delta \to \infty$ on the energy bands. [7*pt*]

(Hint: For part a., integrate the Schrödinger equation accross a barrier. Then, either carefully evaluate the Dirac comb limit of Eqs. (15.11) and (15.12), or use your result in part a.)

2. Consider the effects of a strong $\vec{B}$-field perturbation on the $|2,\ell,m,m_s\rangle$ states of the H-atom, where $\vec{B} = B_0\, z^2\, \hat{e}_z$, and b_0 is a suitable constant. Neglect the $\vec{B}^2$ term.

a. Carefully determine the operators with which this perturbation commutes. [5*pt*]

b. Specify which of the $|2,\ell,m,m_s\rangle$ will be mixed by the perturbation. [5*pt*]

c. Calculate the first order shift to the energy of the $|2,\ell,m,m_s\rangle$ states. [15*pt*]

3. Modify Bethe's model of α-decay, so that the effective nuclear potential is $V(r) = \hbar c r/R^2 - V_0$, for $r \le R$, and $V(r) = 2Ze'^2/r$, for $r \ge R$, and V_0 ensures that $V(r)$ is continuous.

a. Determine V_0, and the turning points r_1, r_2 for an α-particle of energy $E > 0$. [10*pt*]

b. Assuming spherical symmetry, find the differential equation for $u(r) = r\psi(r)$. [10*pt*]

c. Transform this differential equation into the Bessel equation, and determine which solutions are physical. [10*pt*]

d. Using regularity at $r = 0$, write down and sketch the final radial dependence of the ground state ψ_0; elliminate all but an overall constant by aplying the boundary matching conditions. [10*pt*]

e. Calculate $\frac{\mathrm{d}}{\mathrm{d}t}\int \mathrm{d}^3\vec{r}\,|\psi_0|^2$ within a sphere of radius r_2 using the equation of continuity, and specify its physical significance of this quantity. [10*pt*]

f. Evaluate the Gamow factor $e^{-2\sigma}$, $\sigma \stackrel{\text{def}}{=} \frac{1}{\hbar}\int_{r_1}^{r_2} \mathrm{d}r \sqrt{2m(E - V(r))}$ exactly. [10*pt*]