HOWARD UNIVERSITY
WASHINGTON, D.C. 20059

DEPARTMENT OF PHYSICS AND ASTRONOMY
(202)-806-6245 (Main Office)
(202)-806-5830 (FAX)

2355 Sixth Str., NW, TKH Rm.215
thubsch@howard.edu
(202)-806-6257

Quantum Mechanics II 20th Feb. '98.
1st Midterm Exam

Instructor: T. Hübsch (Student name and ID)

This is an "open Textbook (Park), open class-notes" exam. For full credit, show all your work. **Budget your time**: first do what you are sure you know how; use short-cuts whenever possible (but be prepared to explain them afterwards, if needed). Hand in the part done in class at the end of the class, with a copy of this question sheet stapled to the top. Then, take another question sheet and complete the rest of the problems and hand those in by Wed., 2/25/98, 5:00 pm.

1. The two two-component wave-functions for a spin-$\frac{1}{2}$ electron in a central potential are

$$\psi_{\pm}(r,\theta,\phi) \;=\; \frac{F_l(r)}{\sqrt{2l+1}} \begin{pmatrix} \sqrt{l+\frac{1}{2}\pm m_j}\; Y_l^{\,m_j\mp 1/2}(\theta,\phi) \\ \mp\sqrt{l+\frac{1}{2}\mp m_j}\; Y_l^{\,m_j\pm 1/2}(\theta,\phi) \end{pmatrix} ,$$

where $F_l(r)$ is the properly normalized radial function, $j = l \pm \frac{1}{2}$ for the two wave-functions and m_j is the z-projection of $\vec{J} = \vec{L} + \vec{S}$. Let $\langle r^n \rangle \stackrel{\text{def}}{=} \int_0^\infty \mathrm{d}r\; r^{n+2}\, |F_l(r)|^2$.

a. Calculate $\langle z \rangle$ in the one-electron S-state ($l = 0$). [=15pt]

b. Calculate $\langle z^2 \rangle$ in the one-electron P-state ($l = 1$). [=20pt]

(Note: when $l = 0$, $\psi_- \equiv 0$, since $j \geq 0$. Some spherical harmonics are listed on p.572.)

2. An electron is trapped in an impenetrable cube of side L, wherein it moves freely.

a. Write down the complete set of wave-functions for this electron. [=5pt]

b. Calculate the lifetime of a first excited state (which decays into the ground state through dipole emission). [=15pt]

c. What is the polarisation of the photon emitted in a $|2,1,1\rangle \to |1,1,1\rangle$ decay? [=10pt]

(Note: $|n_x, n_y, n_z\rangle$ is the wave-function in a Cartesian basis.)

3. A Lithium atom has three electrons. Assume that the wave-function of the electrons can be constructed from one-particle wave-functions for each electron, $\psi_{n,l,m,m_s}(\vec{r}_i)$, where n, l, m and m_s denote a principal (radial) quantum number, the orbital angular momentum, and the projections of angular momentum and spin.

a. Construct the 3-particle ground state wave-function(s) for the 3-electron system. [=15pt]

b. What is the degeneracy of the ground state? [=10pt]

4. Consider the system of N atoms of mass m, with one electron each, forming a long molecule approximated by a line of length L (wherein the electrons move freely).

a. Find $\rho(E) = \mathrm{d}N/\mathrm{d}E$ as a function of only energy E, and perhaps m and L. [=10pt]

b. Calculate the Fermi energy, E_F, as a function of the number density $\nu = N/L$. [=10pt]

c. Calculate the average energy in terms of E_F. [=5pt]

d. Calculate the pressure (careful!) in terms of previously determined quantities. [=5pt]

(Note: your results may of course involve constants of Nature.)