DEPARTMENT OF PHYSICS AND ASTRONOMY (202)-806-6245 (Main Office) (202)-806-5830 (FAX)

Quantum Mechanics II

1st Midterm Exam

Instructor: T. Hübsch

This is an "open Textbook (Park), open class-notes" exam. For full credit, show all your work. **Budget your time**: first do what you are sure you know how; use short-cuts whenever possible (but be prepared to explain them afterwards, if needed). Hand in the part done in class at the end of the class, with a copy of this question sheet stapled to the top. Then, take another question sheet and complete the rest of the problems and hand those in by Wed., 2/25/98, 5:00 pm.

1. The two two-component wave-functions for a spin- $\frac{1}{2}$ electron in a central potential are

$$\psi_{\pm}(r,\theta,\phi) = \frac{F_l(r)}{\sqrt{2l+1}} \begin{pmatrix} \sqrt{l+\frac{1}{2}\pm m_j} & Y_l^{m_j\pm 1/2}(\theta,\phi) \\ \mp \sqrt{l+\frac{1}{2}\mp m_j} & Y_l^{m_j\pm 1/2}(\theta,\phi) \end{pmatrix}$$

where $F_l(r)$ is the properly normalized radial function, $j = l \pm \frac{1}{2}$ for the two wave-functions and m_j is the z-projection of $\vec{J} = \vec{L} + \vec{S}$. Let $\langle r^n \rangle \stackrel{\text{def}}{=} \int_0^\infty \mathrm{d}r \ r^{n+2} |F_l(r)|^2$.

- a. Calculate $\langle z \rangle$ in the one-electron S-state (l = 0).
- b. Calculate $\langle z^2 \rangle$ in the one-electron *P*-state (l = 1).

(Note: when l = 0, $\psi_{-} \equiv 0$, since $j \ge 0$. Some spherical harmonics are listed on p.572.)

- **2.** An electron is trapped in an impenetrable cube of side L, wherein it moves freely.
 - a. Write down the complete set of wave-functions for this electron.
 - b. Calculate the lifetime of a first excited state (which decays into the ground state through dipole emission). [=15pt]
 - c. What is the polarisation of the photon emitted in a $|2, 1, 1\rangle \rightarrow |1, 1, 1\rangle$ decay? [=10pt]

(Note: $|n_x, n_y, n_z\rangle$ is the wave-function in a Cartesian basis.)

3. A Lithium atom has three electrons. Assume that the wave-function of the electrons can be constructed from one-particle wave-functions for each electron, $\psi_{n,l,m,m_s}(\vec{r_i})$, where n, l, m and m_s denote a principal (radial) quantum number, the orbital angular momentum, and the projections of angular momentum and spin.

- a. Construct the 3-particle ground state wave-function(s) for the 3-electron system. [=15pt]
- b. What is the degeneracy of the ground state?

4. Consider the system of N atoms of mass m, with one electron each, forming a long molecule approximated by a line of length L (wherein the electrons move freely).

- a. Find $\rho(E) = dN/dE$ as a function of only energy E, and perhaps m and L. [=10pt]
- b. Calculate the Fermi energy, E_F , as a function of the number density $\nu = N/L$. [=10pt]
- c. Calculate the average energy in terms of E_F .
- d. Calculate the pressure (careful!) in terms of previously determined quantities. [=5pt]

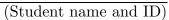
(Note: your results may of course involve constants of Nature.)



Don't Panic !

2355 Sixth Str., NW, TKH Rm.215 thubsch@howard.edu (202)-806-6257

20th Feb. '98.



[=15pt]

[=20pt]

[=5pt]

[=5pt]

[=10pt]