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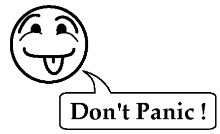

Quantum Mechanics II 17th April '98.

The Final Exam

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Instructor: T. Hübsch (Student name and ID)

This is an "open Textbook (Park), open lecture notes/handouts" take-home exam, due by 5 p.m. of Monday, 27th April '98. For full credit, show all your work. If you cannot complete one part of a calculation, a clear description of the procedure/method will still earn you partial credit. No collaboration or consultation is allowed, **and you may quote only the textbook, lecture notes and class-handouts**; all other results must be derived explicitly.

1. For $\hat{H} = \frac{1}{2m}\vec{p}^2 - \frac{1}{2}m\omega^2 r^2$, the Hamiltonian for the 3-dimensional harmonic oscillator:

a. Prove by explicit calculation of the relevant commutators that all 9 components of the matrix operator $\hat{T}_{ij} \stackrel{\text{def}}{=} \hat{a}_i^\dagger \hat{a}_j$ commute with $\hat{H}$. [10pt]

b. Prove that $(\hat{T}_{ij} + \hat{T}_{ji})$ and $i(\hat{T}_{ij} - \hat{T}_{ji})$ are hermitian. [5pt]

c. Prove the statement of part a. by careful examination of the action of $\hat{T}_{ij}$ on the eigenstates of $\hat{H}$. Express $\hat{H}$ in terms of $\hat{T}_{ij}$. [10pt]

d. Prove that the symmetries generated by $\hat{T}_{ij}$ account completely for the degeneracy of the 3-dimensional harmonic oscillator. [10pt]

e. In $\hat{L}_k \stackrel{\text{def}}{=} \alpha \sum_{ij} \epsilon_{ijk} \hat{T}_{ij}$, determine α so that the $\hat{L}_k$ satisfy the $SO(3) \approx SU(2)$ (angular momentum, *i.e.*, spin) algebra: $\big[\hat{L}_j, \hat{L}_k\big] = i\epsilon_{jkl}\hat{L}_l$, for all $j, k, l = 1, 2, 3$. [5pt]

f. Define $\hat{K}_k \stackrel{\text{def}}{=} \alpha \sum_{ij} |\epsilon_{ijk}| \hat{T}_{ij}$, $\hat{h}_1 = [\hat{T}_{11} - \hat{T}_{22}]$ and $\hat{h}_2 = [\hat{T}_{11} + \hat{T}_{22} - 2\hat{T}_{33}]$. Calculate the commutators $\big[\hat{L}_j, \hat{K}_k\big]$, $\big[\hat{h}_i, \hat{L}_k\big]$, $\big[\hat{h}_i, \hat{K}_k\big]$, and prove that $\big\{\hat{L}_k, \hat{K}_k, \hat{h}_1, \hat{h}_2\big\}$ generate a group. [30pt]

2. Compare the μ-onic Hydrogen atom (simply replace $e^- \to \mu^-$) with the standard one.

a. Write down the energy spectrum in the non-relativistic, spinless approximation. [5pt]

b. Determine the 1st order relativistic and the spin-orbit correction to energy, to lowest order in perturbation theory. [10pt]

c. Calculate the 2nd order relativistic correction to energy in lowest order perturbation theory. [10pt]

d. Estimate the 1st order relativistic and the spin-orbit correction to energy, to next-to-lowest order in perturbation theory. [10pt]

(Hint: Examine carefully the calculations for the Hydrogen atom, then identify and make the necessary changes. A μ^- is identical to e^-, except that its mass is ≈ 206 times bigger.)

3. Consider a 3-dimensional crystal where the electrons move in the periodic potential $V(\vec{r}) = \Omega \sum_{i=1}^{3} \delta\big(x_i - n_i L\big)$, for $n_i = 0, \pm1, \pm2, \ldots$.

a. Determine the complete translation and rotational symmetry. [5+5pt]

c. Write down the Bloch wave function, analogous to Park's Eqs. (15.3–6). [5pt]

d. Separate variables in Cartesian coordinates and determine the energy bands for each of the three 1-dimensional components. [5pt]

e. Determine the energy bands for the 3-dimensional crystal. [5pt]