Quantum Mechanics I

Angular Momenta

Three-Dimensional Space:

Spin and Addition of Angular Momenta

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Pink Floyd: "Another Brick in the Wall (Pt. II)"

Angular Momenta

Adding Angular Momenta

- Recall:

$$e^{-i\theta\cdot J}\Psi(\vec{r},t) = \left\langle \vec{r} \middle| e^{-i\theta\cdot J} \middle| \Psi(t) \right\rangle = \left\langle \vec{r} \middle| e^{-i\theta\cdot(L+S)} \middle| \Psi(t) \right\rangle$$

  - That is, a single object has both a *position* and a *state*
  - Both can, independently, be transformed by rotations
  - so that object transforms in a “joint”/“composite” fashion
- Also, a system of two particles depends
  - on *two* independent positions of the two particles
  - and the *two*-particle state of the system
  - so that

$$e^{-i\theta\cdot J}\Psi(\vec{r}_1,\vec{r}_2,t) = \left\langle \vec{r}_1,\vec{r}_2 \middle| e^{-i\theta\cdot(L+S)} \middle| \Psi(t) \right\rangle$$

  - the two-particle system transforms “jointly”/“compositely”
  - Need to know how to add/compose angular momenta

Angular Momenta

Adding Angular Momenta (A Digression)

- There is no such thing as angularomomentumometer
- In macroscopic, classical physics
  - objects have distinguishing features (nose, face, …)
  - to follow as they change positions/orientations
- In microscopic, quantum physics
  - particles have electric and magnetic multipole moments
  - G.E. Uhlenbeck and S.A. Goudsmit reverse-engineered the e^{-} magnetic dipole moment as if created by its spinning charge
  - Magnetic dipole moments interact w/outer magnetic fields
  - Electric dipole moments interact w/outer electric fields
  - …and cause externally detectable/controllable behavior
  - …of the system as a whole, with “joint”/“composite” angular momentum (whether real or reverse-engineered)

Angular Momenta

Adding Angular Momenta

$$[J_j, J_k] = i\varepsilon_{jk}{}^{\ell} J_\ell$$
$$[L_j, L_k] = i\varepsilon_{jk}{}^{\ell} L_\ell$$
$$[S_j, S_k] = i\varepsilon_{jk}{}^{\ell} S_\ell$$

- Consider the generic setting (regardless of physics!)

$$\vec{J} = \vec{L} + \vec{S} \qquad \Rightarrow \qquad J_3 = L_3 + S_3$$

  - Each (size^2 & proj.) pair of operators has simult. eigenstates

$$\vec{L}^2\,|\ell, m_\ell\rangle = \ell(\ell+1)\,|\ell, m_\ell\rangle\,, \qquad L_3\,|\ell, m_\ell\rangle = m_\ell\,|\ell, m_\ell\rangle\,;$$
$$\vec{S}^2\,|s, m_s\rangle = s(s+1)\,|s, m_s\rangle\,, \qquad S_3\,|s, m_s\rangle = m_s\,|s, m_s\rangle\,;$$
$$\vec{J}^2\,|j, m_j\rangle = j(j+1)\,|j, m_j\rangle\,, \qquad J_3\,|j, m_j\rangle = m_j\,|j, m_j\rangle\,.$$

  - Only four operators mutually commute & are independent:

composite			
composite $\vec{J}^2 \quad J_3$	$\vec{L}^2 \quad \vec{S}^2$	$L_3 \quad S_3$ **product**
$|j, \ell, s; m_j\rangle$		$|\ell, s; m_\ell, m_s\rangle := |\ell, m_\ell\rangle \otimes |s, m_s\rangle$

  - There must exist two different ways of accounting
    - ...and a translation between the two

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$$[S_j, S_k] = i\varepsilon_{jk}{}^{\ell} S_{\ell}$$

- Both the electron and the proton in the Hydrogen atom
  - have spin, $s_e = s_p = ½$, and $m_{se} = m_{sp} = \pm½$, independently
  - The sum of their spin-projections is then $(\pm½ \pm½) = \{-1,0,+1\}$
  - Do this a bit more carefully $\vec{S} = \vec{S}_e + \vec{S}_p \quad \Rightarrow \quad S_3 = S_{3e} + S_{3p}$

Angular Momenta

Adding Angular Momenta

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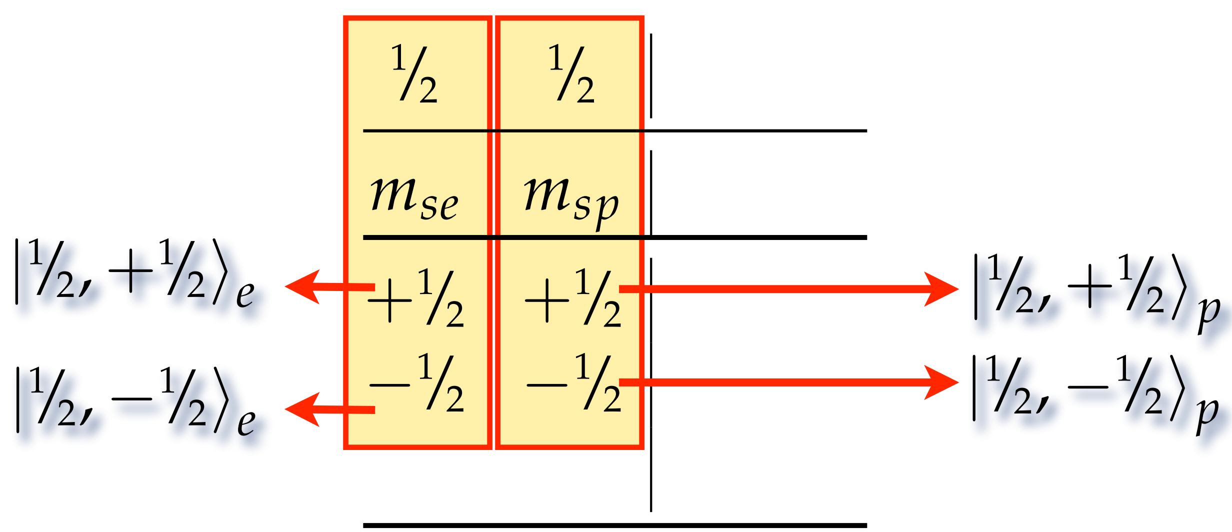
  - Do this a bit more carefully $\vec{S} = \vec{S}_e + \vec{S}_p \quad \Rightarrow \quad S_3 = S_{3e} + S_{3p}$
  - Plot:



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- Both the electron and the proton in the Hydrogen atom
  - have spin, $s_e = s_p = ½$, and $m_{se} = m_{sp} = ±½$, independently
  - The sum of their spin-projections is then $(±½ ±½) = \{–1,0,+1\}$
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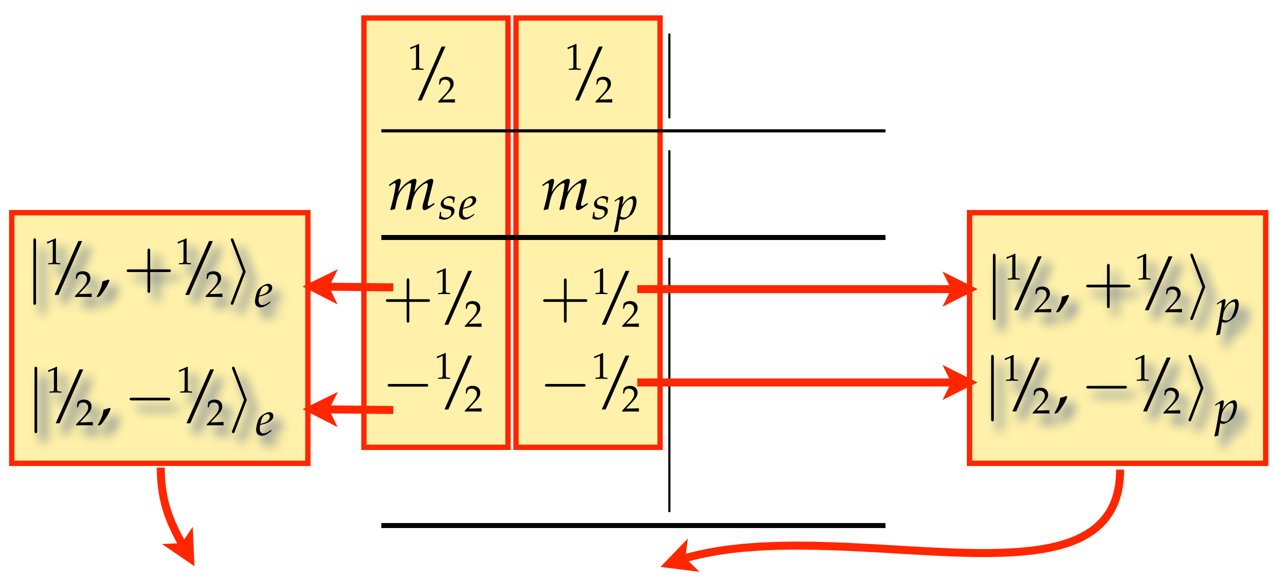

$$|½, ½; +½, +½\rangle$$
$$|½, ½; +½, -½\rangle$$
$$|½, ½; -½, +½\rangle$$
$$|½, ½; -½, -½\rangle$$

Angular Momenta

Adding Angular Momenta

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  - Plot:

$^1\!/_2$	$^1\!/_2$
m_{se}	m_{sp}
$+^1\!/_2$	$+^1\!/_2$
$-^1\!/_2$	$-^1\!/_2$

product spin factors

- $|^1\!/_2, ^1\!/_2; +^1\!/_2, +^1\!/_2\rangle$
- $|^1\!/_2, ^1\!/_2; +^1\!/_2, -^1\!/_2\rangle$
- $|^1\!/_2, ^1\!/_2; -^1\!/_2, +^1\!/_2\rangle$
- $|^1\!/_2, ^1\!/_2; -^1\!/_2, -^1\!/_2\rangle$

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  - Plot:

$^1/_2$	$^1/_2$	
m_{se}	m_{sp}	
$+^1/_2$	$+^1/_2$	$+1$
$-^1/_2$	$-^1/_2$	0
		-1

product spin factors

$|^1/_2, ^1/_2; +^1/_2, +^1/_2\rangle$
$|^1/_2, ^1/_2; +^1/_2, -^1/_2\rangle$
$|^1/_2, ^1/_2; -^1/_2, +^1/_2\rangle$
$|^1/_2, ^1/_2; -^1/_2, -^1/_2\rangle$

QMI

Angular Momenta

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  - Plot:

$^1/_2$	$^1/_2$		
m_{se}	m_{sp}		
$+^1/_2$	$+^1/_2$	$+1$	
$-^1/_2$	$-^1/_2$	0	0
		-1	

product spin factors

$|^1/_2, ^1/_2; +^1/_2, +^1/_2\rangle$

$|^1/_2, ^1/_2; +^1/_2, -^1/_2\rangle$

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  - Plot:

$^1\!/_2$	$^1\!/_2$	1	0
m_{se}	m_{sp}	m	m
$+^1\!/_2$	$+^1\!/_2$	$+1$	
$-^1\!/_2$	$-^1\!/_2$	0	0
		-1	

product spin factors

$|^1\!/_2, ^1\!/_2; +^1\!/_2, +^1\!/_2\rangle$
$|^1\!/_2, ^1\!/_2; +^1\!/_2, -^1\!/_2\rangle$
$|^1\!/_2, ^1\!/_2; -^1\!/_2, +^1\!/_2\rangle$
$|^1\!/_2, ^1\!/_2; -^1\!/_2, -^1\!/_2\rangle$

$|1, ^1\!/_2, ^1\!/_2; +1\rangle$
$|1, ^1\!/_2, ^1\!/_2; 0\rangle \qquad |0, ^1\!/_2, ^1\!/_2; 0\rangle$
$|1, ^1\!/_2, ^1\!/_2; -1\rangle$

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  - Plot:

$\frac{1}{2}$	$\frac{1}{2}$	1	0
m_{se}	m_{sp}	m	m
$+\frac{1}{2}$	$+\frac{1}{2}$	$+1$	
$-\frac{1}{2}$	$-\frac{1}{2}$	0	0
		-1	

product spin factors

$|\frac{1}{2},\frac{1}{2};+\frac{1}{2},+\frac{1}{2}\rangle$
$|\frac{1}{2},\frac{1}{2};+\frac{1}{2},-\frac{1}{2}\rangle$
$|\frac{1}{2},\frac{1}{2};-\frac{1}{2},+\frac{1}{2}\rangle$
$|\frac{1}{2},\frac{1}{2};-\frac{1}{2},-\frac{1}{2}\rangle$

$|1,\frac{1}{2},\frac{1}{2};+1\rangle$
$|1,\frac{1}{2},\frac{1}{2};0\rangle$ $\quad |0,\frac{1}{2},\frac{1}{2};0\rangle$
$|1,\frac{1}{2},\frac{1}{2};-1\rangle$

composite spin factors

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  - Do this a bit more carefully $\vec{S} = \vec{S}_e + \vec{S}_p \quad \Rightarrow \quad S_3 = S_{3e} + S_{3p}$
  - Plot:

$\frac{1}{2}$	$\frac{1}{2}$	1	0
m_{se}	m_{sp}	m	m
$+\frac{1}{2}$	$+\frac{1}{2}$	$+1$	
$-\frac{1}{2}$	$-\frac{1}{2}$	0	0
		-1	

symmetric!

$$|1,\tfrac{1}{2},\tfrac{1}{2};0\rangle = \tfrac{1}{\sqrt{2}}\,|\tfrac{1}{2},\tfrac{1}{2};+\tfrac{1}{2},-\tfrac{1}{2}\rangle + \tfrac{1}{\sqrt{2}}\,|\tfrac{1}{2},\tfrac{1}{2};-\tfrac{1}{2},+\tfrac{1}{2}\rangle$$

product spin factors	composite spin factors	
$|\tfrac{1}{2},\tfrac{1}{2};+\tfrac{1}{2},+\tfrac{1}{2}\rangle$	$|1,\tfrac{1}{2},\tfrac{1}{2};+1\rangle$	
$|\tfrac{1}{2},\tfrac{1}{2};+\tfrac{1}{2},-\tfrac{1}{2}\rangle$	$|1,\tfrac{1}{2},\tfrac{1}{2};0\rangle$	$|0,\tfrac{1}{2},\tfrac{1}{2};0\rangle$
$|\tfrac{1}{2},\tfrac{1}{2};-\tfrac{1}{2},+\tfrac{1}{2}\rangle$	$|1,\tfrac{1}{2},\tfrac{1}{2};-1\rangle$	
$|\tfrac{1}{2},\tfrac{1}{2};-\tfrac{1}{2},-\tfrac{1}{2}\rangle$		

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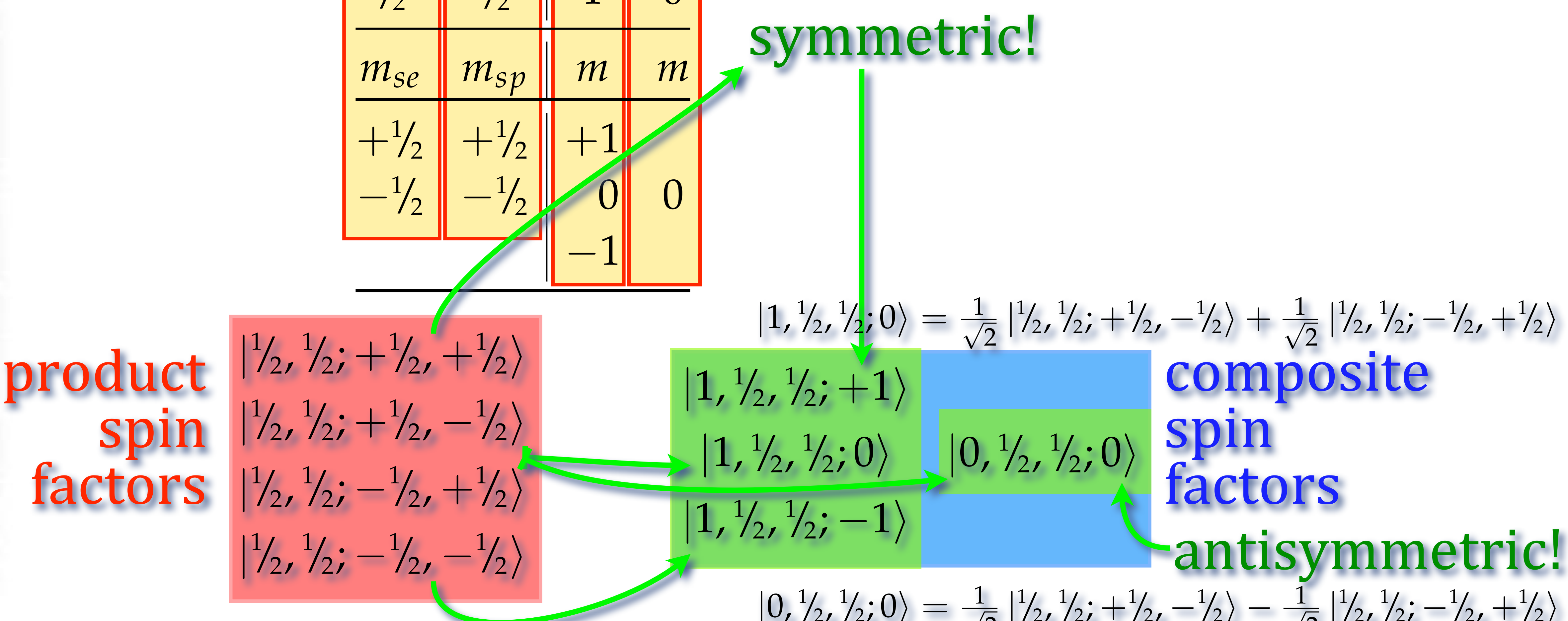
  - Plot:

$^1\!/_2$	$^1\!/_2$	1	0
m_{se}	m_{sp}	m	m
$+^1\!/_2$	$+^1\!/_2$	$+1$	
$-^1\!/_2$	$-^1\!/_2$	0	0
		-1	

$$|1,½,½;0\rangle = \tfrac{1}{\sqrt{2}}\,|½,½;+½,-½\rangle + \tfrac{1}{\sqrt{2}}\,|½,½;-½,+½\rangle$$



$$|0,½,½;0\rangle = \tfrac{1}{\sqrt{2}}\,|½,½;+½,-½\rangle - \tfrac{1}{\sqrt{2}}\,|½,½;-½,+½\rangle$$

QMI

Angular Momenta

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$$[S_j, S_k] = i\varepsilon_{jk}{}^{\ell} S_\ell$$

- So (while the memory is fresh):

$$|1,\tfrac12,\tfrac12;+1\rangle = 1|\tfrac12,\tfrac12;+\tfrac12,+\tfrac12\rangle$$
$$|1,\tfrac12,\tfrac12;0\rangle = \frac{1}{\sqrt2}|\tfrac12,\tfrac12;+\tfrac12,-\tfrac12\rangle + \frac{1}{\sqrt2}|\tfrac12,\tfrac12;-\tfrac12,+\tfrac12\rangle$$
$$|1,\tfrac12,\tfrac12;+1\rangle = 1|\tfrac12,\tfrac12;+\tfrac12,+\tfrac12\rangle$$

composite spin = 1

  - are symmetric w.r.t. exchange of the two spins

$$|0,\tfrac12,\tfrac12;0\rangle = \frac{1}{\sqrt2}|\tfrac12,\tfrac12;+\tfrac12,-\tfrac12\rangle - \frac{1}{\sqrt2}|\tfrac12,\tfrac12;-\tfrac12,+\tfrac12\rangle$$

composite spin = 0

  - is antisymmetric.

- This elementary basis construction in fact determines the Clebsh-Gordan coefficients
- Also

$$|\tfrac12,\tfrac12;+\tfrac12,-\tfrac12\rangle = \frac{1}{\sqrt2}|1,\tfrac12,\tfrac12;0\rangle + \frac{1}{\sqrt2}|0,\tfrac12,\tfrac12;0\rangle$$
$$|\tfrac12,\tfrac12;-\tfrac12,+\tfrac12\rangle = \frac{1}{\sqrt2}|1,\tfrac12,\tfrac12;0\rangle - \frac{1}{\sqrt2}|0,\tfrac12,\tfrac12;0\rangle$$

Angular Momenta

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$$[S_j, S_k] = i\varepsilon_{jk}{}^{\ell} S_\ell$$

Adding Angular Momenta

- Let's try:

$\ell = 1$	$s = ½$	
m_ℓ	m_s	
$+1$	$+½$	
0	$-½$	
-1		

Angular Momenta

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$$[S_j, S_k] = i\varepsilon_{jk}{}^{\ell} S_\ell$$

Adding Angular Momenta

- Let's try:

$\ell = 1$	$s = \frac{1}{2}$		
m_ℓ	m_s		
$+1$	$+\frac{1}{2}$	$+\frac{3}{2}$	
0	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$
-1		$-\frac{1}{2}$	$-\frac{1}{2}$
		$-\frac{3}{2}$	

QMI

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$$[S_j, S_k] = i\varepsilon_{jk}{}^{\ell} S_\ell$$

- Let's try:

$\ell = 1$	$s = 1/2$	$3/2$	$1/2$
m_ℓ	m_s	m	m
$+1$	$+1/2$	$+3/2$	
0	$-1/2$	$+1/2$	$+1/2$
-1		$-1/2$	$-1/2$
		$-3/2$	

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m_ℓ	m_s	m	m
$+1$	$+1/2$	$+3/2$	
0	$-1/2$	$+1/2$	$+1/2$
-1		$-1/2$	$-1/2$
		$-3/2$	

$\ell = 1$	$S = 1$	
m_ℓ	m_s	
$+1$	$+1$	
0	0	
-1	-1	

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m_ℓ	m_s	m	m
$+1$	$+1/2$	$+3/2$	
0	$-1/2$	$+1/2$	$+1/2$
-1		$-1/2$	$-1/2$
		$-3/2$	

$\ell = 1$	$S = 1$			
m_ℓ	m_s			
$+1$	$+1$	$+2$		
0	0	$+1$	$+1$	
-1	-1	0	0	0
		-1	-1	
		-2		

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m_ℓ	m_s	m	m
$+1$	$+1/2$	$+3/2$	
0	$-1/2$	$+1/2$	$+1/2$
-1		$-1/2$	$-1/2$
		$-3/2$	

$\ell = 1$	$S = 1$	2	1	0
m_ℓ	m_s	m	m	m
$+1$	$+1$	$+2$		
0	0	$+1$	$+1$	
-1	-1	0	0	0
		-1	-1	
		-2		

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m_ℓ	m_s	m	m
$+1$	$+1/2$	$+3/2$	
0	$-1/2$	$+1/2$	$+1/2$
-1		$-1/2$	$-1/2$
		$-3/2$	

$\ell = 1$	$S = 1$	2	1	0
m_ℓ	m_s	m	m	m
$+1$	$+1$	$+2$		
0	0	$+1$	$+1$	
-1	-1	0	0	0
		-1	-1	
		-2		

$$J_3 = L_3 + S_3$$

Find all values of m_J, given all choices of m_L and m_s

QMI

Angular Momenta

Clebsch-Gordan Coefficients

http://pdg.lbl.gov/2002/clebrpp.pdf

$1\times1/2$	3/2 +3/2	3/2 +1/2	1/2 +1/2	3/2 −1/2	1/2 −1/2
+1 +1/2	1				
+1 −1/2		1/3	2/3		
0 +1/2		2/3	−1/3		
0 −1/2				2/3	1/3
−1 +1/2				1/3	−2/3

$$|{}^1\!/\!_2, 1, {}^1\!/\!_2; +{}^1\!/\!_2\rangle = \sqrt{\tfrac{2}{3}}\,|1, {}^1\!/\!_2; +1, -{}^1\!/\!_2\rangle - \sqrt{\tfrac{1}{3}}\,|1, {}^1\!/\!_2; 0, +{}^1\!/\!_2\rangle$$

$$|{}^3\!/\!_2, 1, {}^1\!/\!_2; +{}^1\!/\!_2\rangle = \sqrt{\tfrac{1}{3}}\,|1, {}^1\!/\!_2; +1, -{}^1\!/\!_2\rangle + \sqrt{\tfrac{2}{3}}\,|1, {}^1\!/\!_2; 0, +{}^1\!/\!_2\rangle$$

$$|1, {}^1\!/\!_2; -1, +{}^1\!/\!_2\rangle = \sqrt{\tfrac{1}{3}}\,|{}^3\!/\!_2, 1, {}^1\!/\!_2; -{}^1\!/\!_2\rangle - \sqrt{\tfrac{2}{3}}\,|{}^1\!/\!_2, 1, {}^1\!/\!_2; -{}^1\!/\!_2\rangle$$

$$|1, {}^1\!/\!_2; 0, -{}^1\!/\!_2\rangle = \sqrt{\tfrac{2}{3}}\,|{}^3\!/\!_2, 1, {}^1\!/\!_2; -{}^1\!/\!_2\rangle + \sqrt{\tfrac{1}{3}}\,|{}^1\!/\!_2, 1, {}^1\!/\!_2; -{}^1\!/\!_2\rangle$$

and so on…

Quantum Mechanics I

Now, go forth and calculate!!

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