Quantum Mechanics II

The Quantum Nature of Nature

History, Motivation and Mental Modeling (Hmmm)

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Pink Floyd: "Another Brick in the Wall (Pt. II)"

QMI

The Quantum Nature of Nature

History & Motivation

- 1900, Max Planck
  - Rayleigh+Jeans, 1900
    spectral energy density $u(f,T)$ = energy per unit f, per unit volume
    $$u(f,T)\,\mathrm{d}f = \overline{E}\,n(f)\,\mathrm{d}f$$
    - Average energy:
    $$\overline{E} = \frac{\int_0^\infty \mathrm{d}E\,E\,e^{-E/k_BT}}{\int_0^\infty \mathrm{d}E\,e^{-E/k_BT}} = k_BT$$
    - Modes per unit frequency:
    $$A\sin(k_x x)\sin(k_y y)\sin(k_z z) \qquad x,y,z\in[0,L] \quad (k_x,k_y,k_z)=(n_x,n_y,n_z)\tfrac{\pi}{L}$$
    $$\mathrm{Vol}_{\vec{k}}(k) = \tfrac{4}{3}\pi k^3 \qquad \mathrm{Vol}_{\vec{k}}(k,k+\mathrm{d}k) = 4\pi k^2\,\mathrm{d}k$$
    $$N(k) = 2\,\frac{\frac{1}{8}\mathrm{Vol}_{\vec{k}}(k,k+\mathrm{d}k)}{(\frac{\pi}{L})^3} = \frac{L^3}{\pi}k^2\,\mathrm{d}k \qquad k = 2\pi f/c$$
    $$n(f)\mathrm{d}f = \frac{N(k)\mathrm{d}k}{L^3} = \frac{1}{\pi}\left(\tfrac{2\pi f}{c}\right)^2\mathrm{d}\left(\tfrac{2\pi f}{c}\right) = \frac{8\pi f^2}{c^3}\mathrm{d}f$$
  - So…
    $$u(f,T)\,\mathrm{d}f = \frac{8\pi f^2}{c^3}k_BT\,\mathrm{d}f$$

$\int u(f,T)\,\mathrm{d}f = \infty$

Ooops!!

"UV catastrophe"
—P. Ehrenfest, 1911

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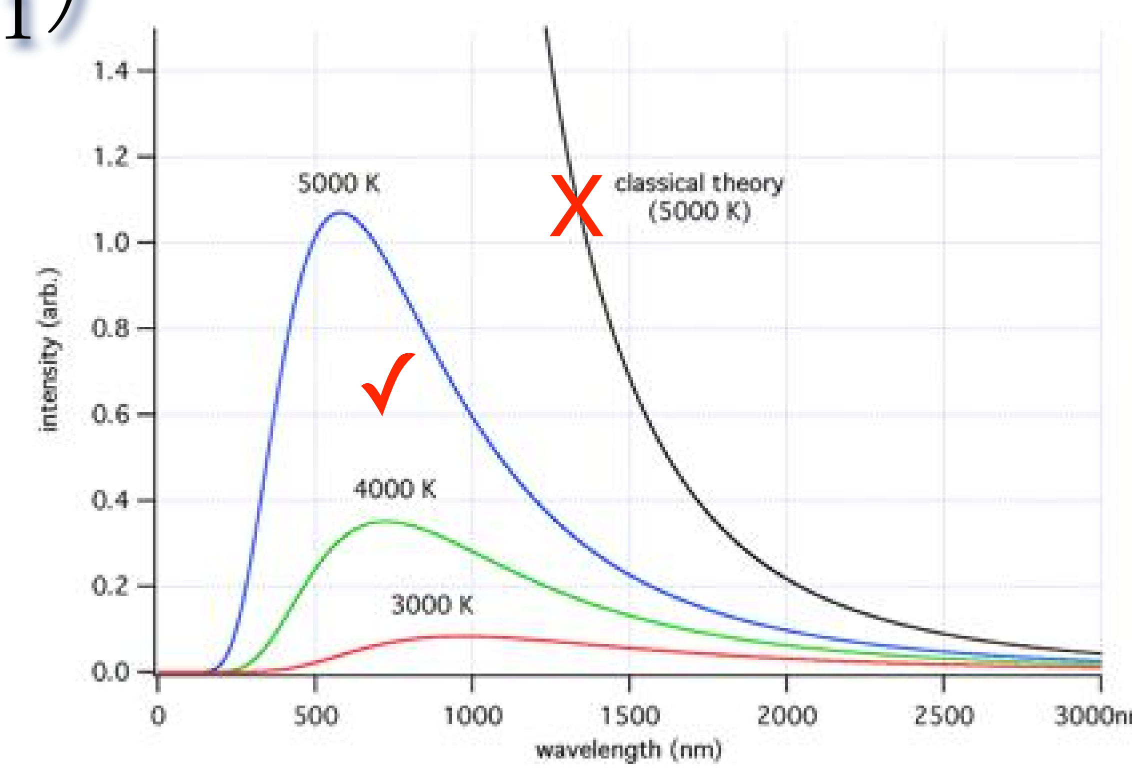
History & Motivation

"UV catastrophe"
—P. Ehrenfest, 1911

- 1900, Max Planck:
  - Wien, 1893: $u(f,T) \propto f^3\, e^{-\beta f/T}$ fits very well qualitatively & for large f…
  - …but is a "little off" for small f.
  - In October 1900, Planck heard of the low-f discrepancy from Heinrich Rubens, and (having worked on deriving Wien's law for six years), had:

$$u(\lambda,T) = \frac{8\pi ch}{\lambda^5}\left(\frac{1}{e^{hc/\lambda k_B T}-1}\right)$$

  - then fit experimentally h
  - and verified that $k_B = N_A/R$
- …and justified h as the smallest packet ("quantum") of the EM radiation that a material object may emit



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History & Motivation

- 1905, Albert Einstein:
  - Heinrich Herz, 1887: UV-illuminated cathode sparks more easily
  - Philipp Lenard, 1902: (KE of the UV-released e^-) $\propto f$
    (the number of the UV-released e^-) $\propto$ Intensity

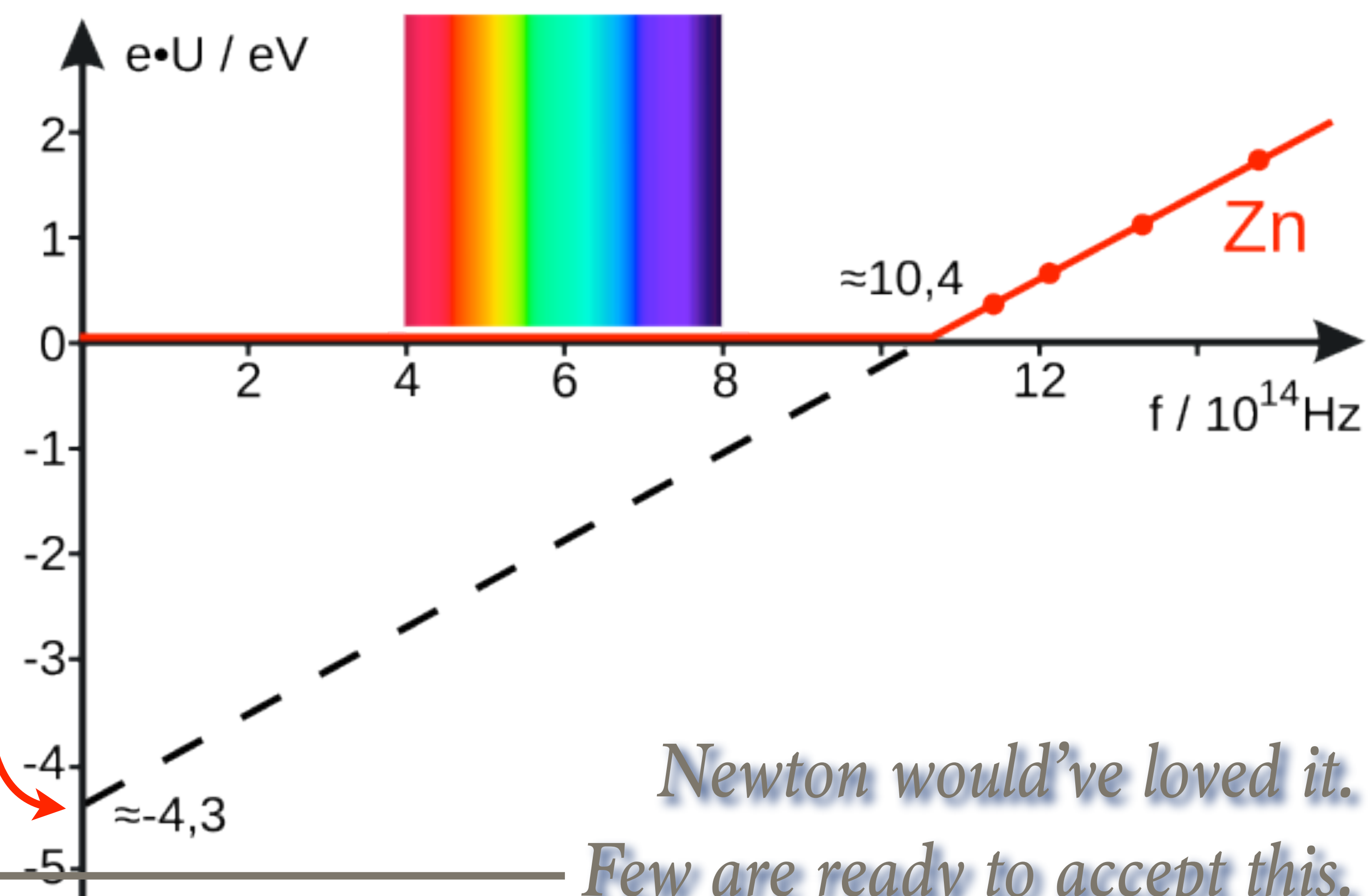
Contradicts classical EM

  - Conservation of energy

$$hf = W + KE$$

Planck (→ hf) liberation from metal (→ W)

  - EM radiation is absorbed also in "quanta"



Einstein concludes: EM radiation also <u>exists</u> only as quanta

Newton would've loved it.

Few are ready to accept this.

The Quantum Nature of Nature

History & Motivation

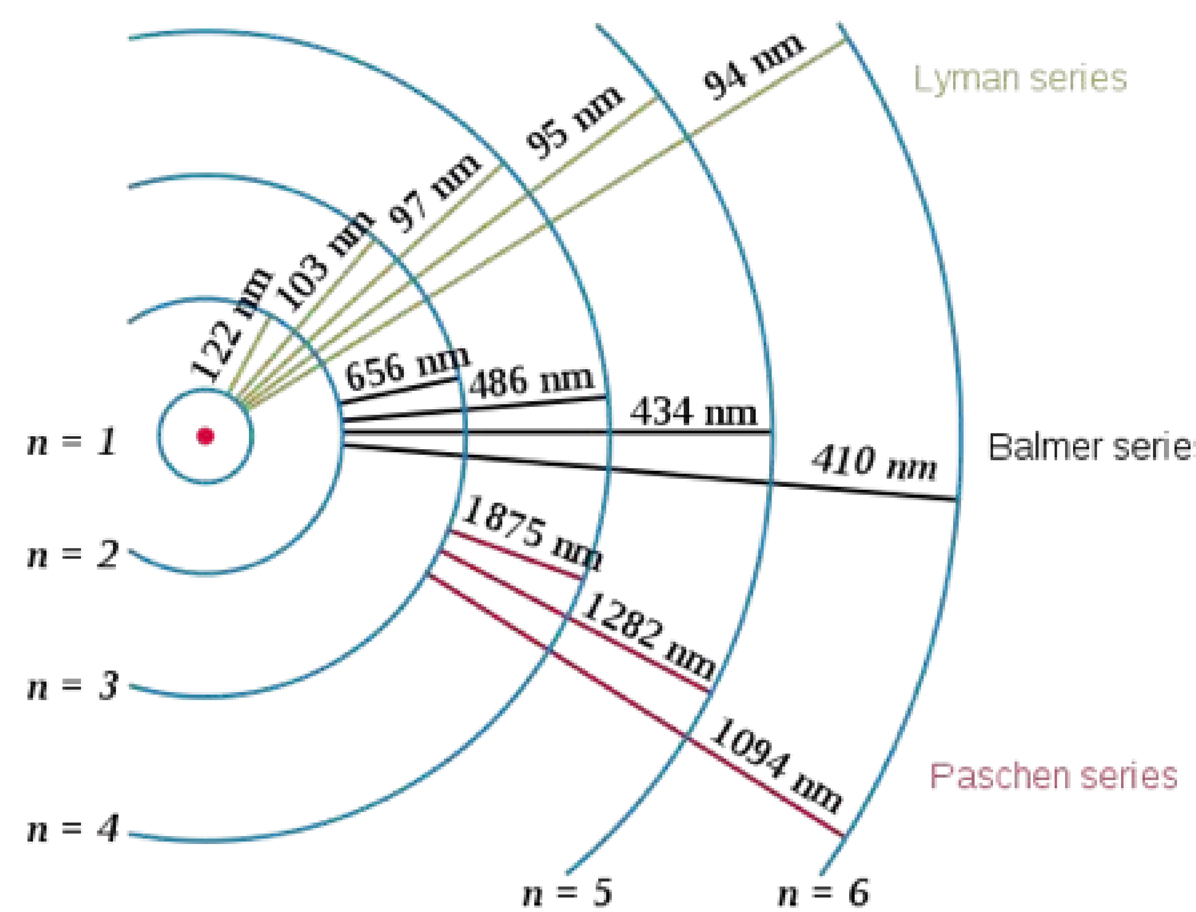
- 1910, Peter Debye: Planck's result follows from supposing that the Fourier modes of the EM field have $E_n = nhf$
  - Only in 1915: Born, Heisenberg & Jordan: (nhf)-mode = $n[(hf)$-modes]
- 1913, Niels Bohr:
  - Fits 1911 Rutherford's planetary model of the H-atom
  - …so it correctly reproduces the Rydberg line spectrum formulae

$$\frac{1}{\lambda_{n_i \to n_f}} = R\left(\frac{1}{{n_f}^2} - \frac{1}{{n_i}^2}\right)$$

$$= \left(\frac{2\pi^2 (k\,e^2)^2\, m_e}{h^3 c}\right)\left(\frac{1}{{n_f}^2} - \frac{1}{{n_i}^2}\right)$$

Planck

  - …by postulating $L = L_n = nh$



Q M I

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History & Motivation

- 1923, Louis de Broglie:
  universal wave-particle duality

$$\lambda_{\text{dB}} = \frac{h}{p}, \quad p = \frac{h}{\lambda}$$

- 1922-23, Arthur Compton (and Peter Debye):
  - X-ray scattering off of free electrons
  - Classical theory: e^- should be propelled in the direction of the EM wave
  - …and re-radiate EM waves of $f' \leq f_i$, with f' depending on intensity
  - Exp.: f' depends only on the scattering angle θ, and $\theta \neq 0$!!
- The EM wave = particles of energy ηf, linear momentum $\eta f/c$

$$\lambda' - \lambda_i = \frac{h}{m_e c}(1 - \cos\theta)$$

- which follows from:
  - analyzing the (X-ray)-e^- scattering as a particle collision
  - and using conservation of both energy and linear momentum
- 1926, Gilbert Lewis named the "photon"

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History & Motivation

universal constant of Nature

- To summarize:
  - 1900, Max Plank: quantum of emitted EM radiation (hf)
  - 1905, Albert Einstein: quantum of absorbed EM radiation (hf)
  - 1910, Peter Debye: Fourier modes with $E_n = n\,hf$
  - 1913, Niels Bohr: H-atom & line spectra, $R = \left(\frac{2\pi^2 (k\,e^2)^2\, m_e}{h^3 c}\right)$
  - 1913, Louis de Broglie: universal wave-particle duality, $\lambda_{\rm dB} = h/p$
  - 1923, Arthur Compton: X-ray scattering, $\Delta\lambda = (h/m_e c)(1{-}\cos\theta)$
- It is not that classical physics never quantizes:
  - Waves in a cavity (on a string) have quantized wave-lengths & energy
    - — this phenomenon actually will (re)appear in very many places
  - Saturn's rings have quantized gaps
    - — this is nonlinear dynamics of a large ensemble
  - But, in the above cases, there is no classical reason for quantization

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History & Motivation

- One more thing…
- The Gibbs "paradox"
  - A container is partitioned, with two gases on each side.
  - The partition is removed and the gasses mix.
  - With distinguishable gases, entropy grows and work can be extracted.
  - But, then: individual molecules/atoms of any gas are indistinguishable.
  - Contriving delayed identification, ΔS appears to be subjective.
- **HW# 0**:
read [http://bayes.wustl.edu/etj/articles/gibbs.paradox.pdf]
  - see [http://en.wikipedia.org/wiki/Gibbs_paradox] & [your choice]
  - Write a (2–3)k-word & 5–10 equation explanation of the "paradox," its logical resolution, and relevance to the "classical *vs*. quantum" physics, in your own words. **Due 09/04/13**
    - Try to answer: Could QM really have been discovered in 1875?

The Quantum Nature of Nature

Mental (Caricatures and) Modeling

- Universal wave-particle duality (de Broglie, 1913)
- [particle: E & p] $\rightleftarrows$ [wave: ω & λ]
  - Relations: $E = \hbar\omega$ and $p = \hbar k \equiv \hbar/\lambda$ where $\hbar = h/2\pi = 1.055\times10^{-34}$ J s.
- How fine is the ballerina's angular momentum quantization?
  That is, what is the relative error of $\Delta n = 1$ in $L_{\text{balerina}} = n\hbar$?
- How fine is a pitched baseball energy quantization?
  That is, what is the relative error of $\Delta n = 1$ in $\int \mathrm{d}t\, L_{\text{b-ball}} = n\hbar$?

Hamilton's action

- Why does the instructor not diffract through the door?
  - With what speed should he walk through?
  - Can that speed be achieved? (Why or why not?)
  - When would the diffraction pattern appear in a 1-m distant screen?

HW#0 due 09/04/13

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Mental (Caricatures and) Modeling

- Spinning ballerina
  - Assume a solid-cylindrical ballerina
  - radius 0.20 m, height 1.70 m, mass 50 kg, period 0.333 s, $\omega = 2\pi(3\ \text{s}^{-1})$
  - $I = \frac{1}{2} m r^2 = 1.00\times10^{-2}$ kg m²
  - $L_{\text{ballerina}} = I\omega = 1.885\times10^{-1}$ J s
  - Relative error $(1\hbar / L_{\text{ballerina}}) = 5.597\times10^{-34}$ ← **Hilariously unmeasurable**
- Pitched baseball
  - Assume horizontal flight, no gravity
  - distance 18.39 m, speed 100 mph = 44.69 m/s, mass 0.142 kg
  - $\text{KE} = \frac{1}{2} m v^2 = 1.418\times10^{2}$ J
  - Time of flight $t_{\text{flight}} = d/v = 0.412$ s
  - Hamilton's action $A_{\text{b-ball}} = \int \text{d}t\, L_{\text{b-ball}} = \text{KE}\cdot t = 58.35$ J s
  - Relative error $(1\hbar / A_{\text{b-ball}}) = 1.808\times10^{-36}$ ← **Hilariously unmeasurable**
- Quantum billiards (à la Mr. Tomkins) anyone?

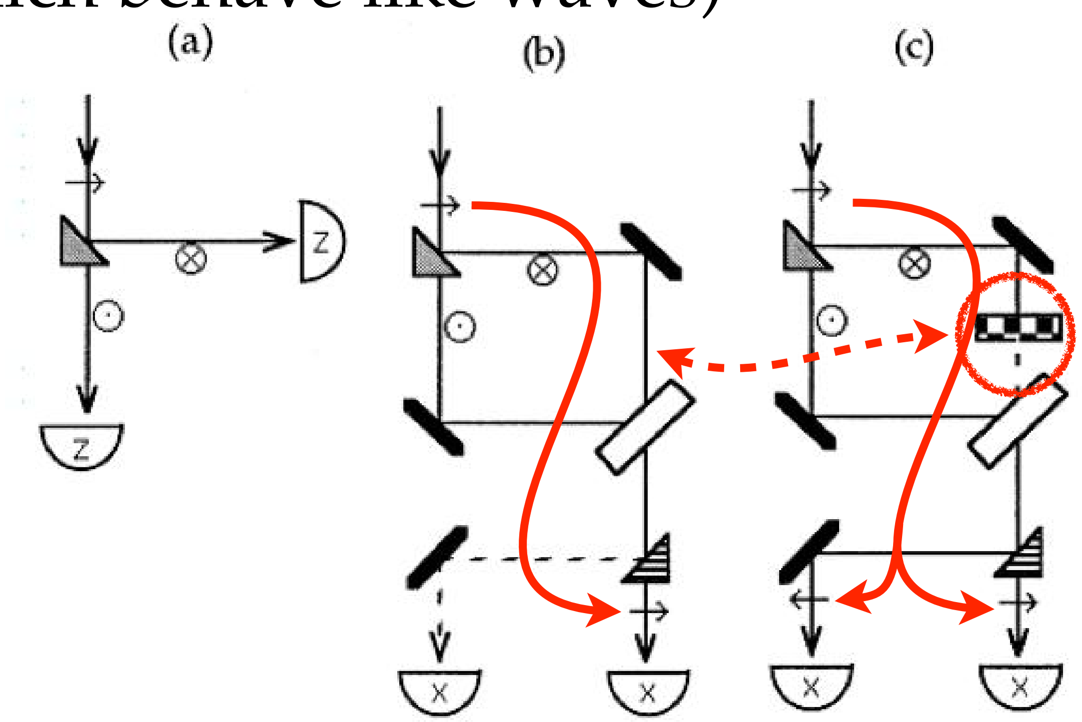
Stick to classical physics

The Quantum Nature of Nature

Mental (Caricatures and) Modeling

- **Discreteness**
  - Not everything (by far) is discrete, but some quantities are
  - Energies, (lin. & ang.) momenta — owing to boundary conditions
  - Involves $n\hbar$ — and so is either commensurate ($L = n\hbar$) or proportional ($E = n\hbar\omega$)
- **Diffraction**
  - Hallmark characteristic of wave-like phenomena
  - True of everything in Nature (particles = wave-packets)
  - …even single particles (ensembles of which behave like waves)
- **Coherence** (& **Superposition**)
  - A.k.a. "phase stability" → interference
  - Universal to everything in Nature
  - May link "distant" "objects"
  - Profoundly "non-classical"

    …and bewilderingly confusing!



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Now, go forth and calculate!!

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