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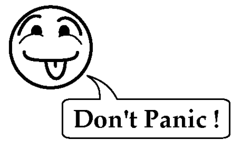

Quantum Mechanics II 16th Nov. '98.

2nd Midterm Exam

Instructor: T. Hübsch ______________ (Student name and ID)

This is an "open Textbook (Park), open class-notes" exam. For full credit, show all your work. If you cannot complete one part of a calculation, a clear description of the procedure/method will still earn you partial credit. **Budget your time**: first do what you are sure you know how; use short-cuts whenever possible (but be prepared to explain them afterwards, if necessary), Appendix 1 lists a good number of useful results. The take-home part (at 2/3 of credit and **due 20th Nov. '98.**), supersedes your in-class work: re-working of any one part of a problem anulls your in-class work on that part.

1. Let $\hat{R}_4$ be the counterclockwise 90°-rotation operator and $\hat{Z}_2$ be the $x \leftrightarrow y$ reflection operator in the (x, y)-plane. They act on the wavefunctions of the particle of mass M in the 2-dimensional square well of §6.1 in the obvious fashion.

a. List the possible eigenvalues of $\hat{Z}_2$. [=5pt]

b. From the stationary states $|m, n\rangle = \frac{2}{L}\sin(m\frac{x\pi}{L})\sin(n\frac{y\pi}{L})$ with $E = 65\frac{\hbar^2}{2M}$, construct the simultaneous eigenfunctions of $\hat{H}$ and $\hat{Z}_2$. [=5pt]

c. Write down the action of $\hat{R}_4$ on (x, y), and list the possible eigenvalues of $\hat{R}_4$. [=10pt]

d. From the stationary states $|m, n\rangle = \frac{2}{L}\sin(m\frac{x\pi}{L})\sin(n\frac{y\pi}{L})$ with $E = 65\frac{\hbar^2}{2M}$, construct the simultaneous eigenfunctions of $\hat{H}$ and $\hat{R}_4$; $\beta = \sqrt{\frac{m\omega}{\hbar}}$. [=10pt]

(Hint: First list all the states with $E = 65\frac{\hbar^2}{2M}$; don't panic: $n, m \le 8$.)

2. Consider a Hydrogen-like atom with the central potential $V(r) = -\frac{e'^2}{r}e^{-r/a_0}$, where a_0 is Bohr's radius and e' is the system-independent electric charge charge.

a. Treat this as a perturbated Coulomb potential; specify the perturbation $\hat{H}^{(1)}$ and give a quantitative argument for $\hat{H}^{(1)}$ to be regarded as a small perturbation. [=10pt]

b. Will this perturbation $\hat{H}^{(1)}$ mix any of the Hydrogen states? Which ones? Why? [=10pt]

c. Can one use (trust) 'non-degenerate perturbation theory' of §7.2? Why (not)? [=5pt]

d. Calculate the lowest nonzero correction to the ground state energy. [=15pt]

e. Calculate the lowest nonzero correction to the $|2, 0, 0\rangle$-state energy. [=15pt]

f. Will the energy of $|171, 0, 0\rangle$ be shifted more or less than these two? Why? [=5pt]

(Hint: See Table 6.1 (p.190) for the lowest five wavefunctions for Hydrogen-like atoms.)

3. The "free periodic electron" of §7.5 is being perturbed by a weak alternating electric field which induces $\hat{H}^{(1)} = -e\mathcal{E}_0\cos(\pi x/L)$.

a. Will this perturbation lift the degeneracy between $|k, +\rangle$ and $|k, -\rangle$? Why (not)? [=10pt]

b. Calculate the shift in the energy levels. [=20pt]

(Hint: Consider only states of the same but arbitrary momentum.)