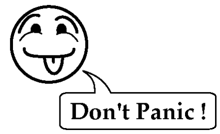
HOWARD UNIVERSITY
WASHINGTON, D.C. 20059

DEPARTMENT OF PHYSICS AND ASTRONOMY
(202)-806-6245 (Main Office)
(202)-806-5830 (FAX)

2355 Sixth Str., NW, TKH Rm.215
thubsch@howard.edu
(202)-806-6257



Quantum Mechanics I 9th Oct. '98.

1st Midterm Exam

Instructor: T. Hübsch ______________ (Student name and ID)

This is an "open Textbook (Park), open class-notes" exam. For full credit, show all your work. If you cannot complete one part of a calculation, a clear description of the procedure/method will still earn you partial credit. **Budget your time**: first do what you are sure you know how; use short-cuts whenever possible (but be prepared to explain them afterwards, if necessary).

1. Consider a particle in an rectangular potential well: $V(x) = +\infty$ for $x < 0$, $V(x) = 0$ for $0 < x < a$, and $V(x) = +V_0$ for $x > 0$, where V_0 and a are positive constants. For $-V_0 < E < 0$:

a. sketch potential and the wave-function, on the same plot, and specify the (boundary) matching conditions for the wave-function; [=5pt]

b. find the condition on E (the bound state energy levels); [=10pt]

c. sketch the potential and the wave-function when $V_0 \to \infty$; [=5pt]

d. find the allowed values of E when $V_0 \to \infty$. [=10pt]

Hint: d. can be obtained both as a (careful!) limit of part b., and also by an independent calculation.

2. Consider a (1-dimensional) particle moving in the potential $V(x) = 0$ for $|x| > a$ and $V(x) = \lambda(x^2 - a^2)$ for $|x| < a$, where $a, \lambda > 0$.

a. Determine the wave-function, in the WKB approximation (state all the conditions that determine the constants), for allowed negative energies. [=15pt]

b. Determine the WKB estimate of the allowed negative energy levels. [=15pt]

3. Find the asymptotic form of the stationary wave-function, $\psi_\infty(x)$ as $x \to \infty$, for a single particle in the potential $V(x) = \lambda x^8$. [=15pt]

Hint: This can be solved both by a WKB *estimate*, or by *approximately* solving the Schrödinger equation for very large x (keeping only highest order terms throughout the calculation).

4. Sketch the potential, sketch a sample bound-state wave-function (if any), and write down the energy quantization condition ($V_0, a > 0$) for each of:

a. $V(x) = V_0 \log|x + 1|$; [=10pt]

b. $V(x) = V_0/(1 + x^2)$; [=10pt]

c. $V(x) = -V_0/(1 + x^2)$. [=10pt]

d. $V(x) = 0$ for $x < 0$ and $x > 2a$, but $V(x) = V_0 \sin(x\pi/a)$. [=10pt]