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Quantum Mechanics I
Final Exam

23rd Nov. '98.

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(Student name and ID)

This is an "open Textbook (Park), open lecture notes" take-home exam, **due 12 noon of Wednesday, 2nd Dec. '98**. For full credit, show all your work. If you cannot complete one part of a calculation, a clear description of the procedure/method will still earn you partial credit. No collaboration or consultation is allowed, but you may quote (full reference, please) any published source for intermediate results that you may use.

1. Consider the potential: $V(x) = \lambda|x|$ for $|x| < a$, and $V(x) = a\lambda$, for $|x| > a$; $a, \lambda > 0$.

a. For energies $0 < E < V_\infty$, specify *all* boundary conditions. [10pt]

b. Use WKB approximation to find the energy quantization condition, and for $V_\infty = a\lambda$ estimate λ_0, the smallest value of λ, such that there be at least one bound state. [10pt]

c. Assume that λ is slowly changed from $\lambda > \lambda_0$ to $\lambda < \lambda_0$. Describe what happens to the bound state(s). [10pt]

d. Describe the Hilbert space of the system when $\lambda > \lambda_0$ and when $\lambda < \lambda_0$. [5pt]

2. For a spherical potential well, with $V = 0$ for $r < R$ and $V = \infty$ for $r > R$,

a. solve for the wave-functions in spherical coordinates [15pt]

b. Calculate the energy spectrum (= list of allowed values). [10pt]

c. Specify the degeneracy of the stationary states. [5pt]

d. Shift the potential by adding $\alpha \sin(\theta)$, where $\alpha > 0$ is constant, and calculate the lowest order non-zero perturbative correction to the energy. Is the degeneracy lifted? Fully or only partially? [10pt]

3. Consider the 2-dimensional harmonic oscillator, $\hat{H} = \hbar\omega\big[\hat{a}_1^\dagger\hat{a}_1 + \hat{a}_2^\dagger\hat{a}_2 + 1\big]$, with its Hilbert space of stationary states $\big\{\,|m,n\rangle\,,\ m,n{=}0,1\ldots\big\}$, and the four quadratic operators $\hat{Q}_{ij} \stackrel{\text{def}}{=} \hat{a}_i^\dagger\hat{a}_j$ $(i,j = 1,2)$.

a. Determine the action of each of the four $\hat{Q}_{ij}$ on the stationary states $|m,n\rangle$. [10pt]

b. Prove by explicit computation that all $\hat{Q}_{ij}$ commute with $\hat{H}$. [10pt]

c. Prove that all $\hat{Q}_{ij}$ commute with $\hat{H}$ by only using their action on the $|m,n\rangle$. [10pt]

Write $\hat{L}_0 \stackrel{\text{def}}{=} (\hat{Q}_{11} + \hat{Q}_{22})$, $\hat{L}_\pm \stackrel{\text{def}}{=} C_\pm(\hat{Q}_{12} \pm \beta_\pm\hat{Q}_{21})$, and $\hat{L}_3 \stackrel{\text{def}}{=} C_3(\hat{Q}_{11} - \hat{Q}_{22})$.

d. Specify the constants $C_\pm, \beta_\pm, C_3$ so that the operators $\hat{L}_\pm, \hat{L}_3$ would satisfy the angular momentum commutation relations. [10pt]

e. What possibly physical meaning can be given to $\hat{L}_0$? [5pt]