HOWARD UNIVERSITY
WASHINGTON, D.C. 20059

DEPARTMENT OF PHYSICS AND ASTRONOMY
(202)-806-6245 (Main Office)
(202)-806-5830 (FAX)

2355 Sixth Str., NW, TKH Rm.215
thubsch@howard.edu
(202)-806-6257

Physics for Scientist and Engineers 29. March '02.

PHYS-014 2nd Midterm Exam Solutions Instructor: T. Hübsch

DISCLAIMER: The completeness and detail presented herein were by no means expected in the Student's solutions for full credit. The additional information given here is solely for the Student's convenience and education. For each problem, the result is derived in the algebraic form. The substitution of concrete values and comparison with the choices offered should present no difficulty.

In lieu of full credit for the correct answer given, partial/conditional credit for your work shown in the test itself must be claimed with explanation. In particular: if using your incorrect result from one part you answered *–using a correct procedure–* the subsequent parts and of course obtained incorrect (GIGO) answers, you should still be able to **claim** partial/conditional credit.

1. A coil of diameter D is made of n turns of diameter-d copper wire, of $\rho = 1.68\times10^{-8}\,\Omega\text{m}$ resistivity. That is, the wire of length $n\,(D\pi)$ forms a helix (corkscrew spiral).

a. The resistance of this piece of wire is then $\rho\times$(length)/(cross-section area). Since wires are circular in cross-section, the cross section area is $\pi(d/2)^2$. Putting this together, we have: [=10pt]

$$R = \rho\,\frac{n\,D\pi}{\pi(d/2)^2} = \frac{4nD\rho}{d^2}\ . \tag{1}$$

b. The current induced in the coil is "driven" by the emf, $\mathcal{E}$, created by the increase in the homogeneous magnetic field and is inversely proportional to the resistance, $I = \mathcal{E}/R$. Now: [=10pt]

$$I = \mathcal{E}/R = \Big(-\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}\Big)\frac{1}{R} = -\frac{\mathrm{d}(nAB)}{\mathrm{d}t}\,\frac{1}{R} = -\frac{n[\pi(D/2)^2]}{R}\,\frac{\mathrm{d}B}{\mathrm{d}t} = -\frac{\pi D d^2}{16\rho}\,\frac{\mathrm{d}B}{\mathrm{d}t}\ , \tag{2}$$

where $A = \pi(D/2)^2$ is the constant cross-section area of the coil. Notice also that since the magnetic field is homogeneous, the magnetic flux is particularly easy:

$$\Phi_B = n\int_A \mathrm{d}\vec{A}\cdot\vec{B} = n\int_0^\pi \mathrm{d}\theta\int_0^{D/2}\mathrm{d}r\ |\vec{B}| = n\pi\big(\tfrac{D}{2}\big)^2 B\ . \tag{3}$$

c. The rate at which the coil produces thermal energy is calculated from the fact that the wire presents a thermal ("ohmic") resistance, dissipating the power of the current as heat: [=10pt]

$$P = I^2 R = \Big(\frac{n[\pi(D/2)^2]}{R}\,\frac{\mathrm{d}B}{\mathrm{d}t}\Big)^2 R = \frac{n\pi^2 D^3 d^2}{64\rho}\Big(\frac{\mathrm{d}B}{\mathrm{d}t}\Big)^2\ . \tag{4}$$

2. A voltage-V (DC) battery, an inductance-L inductor and a resistance-R resistor are all connected in series, closing a loop.

a. The time constant of this LR circuit equals $\tau = L/R$. [=10pt]

b. The energy stored in the inductive coil after time t equals:

$$U_L = \tfrac{1}{2}LI^2 = \tfrac{1}{2}LI_{\max}^2(1-e^{-t/\tau})^2 = \tfrac{1}{2}\tfrac{LV^2}{R^2}(1-e^{-Rt/L})^2\ . \tag{5}$$

For all groups, the time given, t, was at least $\approx$606 times larger than τ, so that $1-e^{-t/\tau}\approx 1$; that is, the error committed by dropping the exponential is at most, roughly, $e^{-606}=10^{-606\times\log_{10}e}\approx 10^{-263}$. This, of course, is ridiculously small, and the $(1-e^{-t/\tau})^2$ factor can safely be neglected (approximated by 1). Thus, the correct answer to the question in the problem is $U=\frac{1}{2}\frac{LV^2}{R^2}$. [=10pt]

Alas! The offered solutions had a typo: for all the groups, the stored energy ranges 9.90 – 110 mJ, as given, but with units mistyped as mW. Hence, you may have been mislead to calculate the *rate* at which this energy is being stored:

$$\begin{aligned}P_L&=\frac{\mathrm{d}U}{\mathrm{d}t}=\frac{\mathrm{d}}{\mathrm{d}t}\Big(\tfrac{1}{2}\frac{LV^2}{R^2}(1-e^{-Rt/L})^2\Big)=\tfrac{1}{2}\frac{LV^2}{R^2}\Big(\tfrac{1}{2}(1-e^{-Rt/L})(+\frac{R}{L})\Big)\\&=\tfrac{1}{4}\frac{V^2}{R}\frac{\mathrm{d}}{\mathrm{d}t}(1-e^{-Rt/L})\approx\tfrac{1}{4}\frac{V^2}{R}\ ,\end{aligned}\tag{6}$$

which, curiously, is only a quarter of the "ohmic" power, $P=V^2/R$. Now, for all the groups, the *rate* of energy storing in the coil ranges 3.2 – 20 W, the smallest of which is still a bit over 29 times larger than the largest offered answer. Thus, for those of you who have *actually calculated the rate of energy storing*, the correct answer for all groups would have been either 110 mW or "none of these."

3. An inductance-L inductor with resistance R is connected in series to a capacitance-C capacitor and a frequency-f (AC) source of voltage V $(=V_{\rm rms})$.

a. The impedance of this LRC circuit is equals: [=10pt]

$$Z=\sqrt{R^2+\Big(2\pi fL-\frac{1}{2\pi fC}\Big)^2}\ .\tag{7}$$

b. The rms value of the current in this circuit equals: [=10pt]

$$I_{\rm rms}=\frac{V_{\rm rms}}{Z}=\frac{V_{\rm rms}}{\sqrt{R^2+\Big(2\pi fL-\frac{1}{2\pi fC}\Big)^2}}\ .\tag{8}$$

c. The phase angle, between the voltage and the current in this circuit is: [=10pt]

$$|\phi|=\cos^{-1}\Big(\frac{R}{Z}\Big)=\cos^{-1}\left(\frac{R}{\sqrt{R^2+\Big(2\pi fL-\frac{1}{2\pi fC}\Big)^2}}\right)\ .\tag{9}$$

Since $X_L=2\pi fL<(2\pi fC)^{-1}=X_C$, the voltage lags behind the current, so $\phi<0$. Sorry, there are no particularly nice simplifications of these analytic expressions.

4. An FM radio tuning circuit has a fixed capacitance-C capacitor and a variable inductor. Since $\omega_0=2\pi f_0=1/\sqrt{LC}$, to tune in to a frequency-f broadcast, the inductor must be set to an inductance of: $L=\big((2\pi f)^2C\big)^{-1}$ [=10pt]

5. A h_o-size object is placed a distance d_o in front of a convex (bulging) spherical mirror.

a. For an image of size h_i to form, its distance (d_i) from the mirror has to satisfy $d_o/d_i = -h_o/h_i$, so that: [=10pt]

$$d_i = -d_o \frac{h_i}{h_o} . \tag{10}$$

Note that, since the image forms behind the mirror, in the physically inaccessible region, d_i *must be negative.* Furthermore, by drawing a couple of cases, convince yourself that the image in a convex (bulging) mirror is *always* upright and *always* smaller than the object. This ensures that the above formula, with the explicit negative sign, always gives $d_i < 0$ and $|d_i| < d_o$.

b. The focal length of the mirror is calculated as: [=10pt]

$$\frac{1}{d_o} - \frac{1}{d_i} = -\frac{1}{f} , \quad \Rightarrow \quad f = \Big(\frac{1}{d_i} - \frac{1}{d_o}\Big)^{-1} = \frac{d_i d_o}{d_o - d_i} = -\frac{|d_i|\, d_o}{d_o + |d_i|} . \tag{11}$$

Substituting the above result we can also get:

$$f = \frac{(-d_o \frac{h_i}{h_o})d_o}{d_o - (-d_o \frac{h_i}{h_o})} = -\frac{h_i\, d_o}{h_o + h_i} . \tag{12}$$

c. The radius of the curvature of the mirror equals: [=10pt]

$$R = 2|f| = \frac{2\,|d_i|\, d_o}{d_o + |d_i|} = -\frac{2\, h_i\, d_o}{h_o + h_i} . \tag{13}$$

6. A person struggles to read by holding a book at a distance L away. The power of adequate prescription glasses (*lenses!*), placed ℓ in front of her eyes and for her to read at the "normal" distance of L_0, is then calculated as follows. When a book is placed at a distance L_0 in front of her eyes, and $d_o = L_0 - \ell$ in front of her glasses, it should produce a (virtual and upright) image of the text at the distance L from her eyes and $L - \ell$ from her glasses, where she can read. Thus, $d_i = -(L - \ell)$, and we have:

$$\begin{aligned} D = \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} &= \frac{1}{L_0 - \ell} + \frac{1}{-(L - \ell)} = \frac{1}{L_0 - \ell} - \frac{1}{L - \ell} \\ &= \frac{L - L_0}{(L_0 - \ell)(L - \ell)} . \end{aligned} \tag{14}$$

Note that this is a positive number, *i.e.*, the lenses must be converging. This follows since here $L > L_0$, which is true of farsighted people. Nearsighted folks hold their reading closer than "normal," $L < L_0$, and their corrective lenses must be diverging (with $D < 0$). [=10pt]