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PHYS-013 2nd Midterm Exam Solutions Instructor: T. Hübsch

DISCLAIMER: The completeness and detail presented herein were by no means expected in the Student's solutions for full credit. The additional information given here is solely for the Student's convenience and education. For each problem, the result is derived in the algebraic form first. Concrete values are then easy to substitute and compare with the choices offered; this is shown in the table at the end of each problem.

This time, there was only one group, so all numerical values are the same. Nevertheless, I first solve the problems algebraically and then substitute the numerical values. In all cases, the choice of the *closest* offered value should be straightforward; not one problem asked you to 'round up' or 'round down'!

1. A crew pushes a sofa of mass m=200 kg up the (tilted) ramp of length L=2.75 m (with a kinetic friction coefficient of μ_k=0.4) into the moving truck (h=1 m off the ground) without accelerating. Let α be the angle of the ramp; it may be calculated from $\sin\alpha = h/L$ and $\cos\alpha = \sqrt{L^2-h^2}/L$. In fact, we can just use these expressions, without ever evaluating the numerical value of α.

a. The crew is pushing with a force F_c (along the tilted ramp) that must balance the other two forces acting on the sofa: the component of the weight along the ramp, $F_{G\|}$, and the friction force, F_f. (If the forces along the ramp were not balanced, it would have to be accelerating—which it isn't.) Since the sofa is moving *upward* the ramp, F_f acts *downward* the ramp, and we have that $F_c = F_{G\|} + F_f$.

Now, notice that the angle between the direction of the weight (vertical) and the component of the weight perpendicular to the surface of the ramp, $F_{G\perp}$, is α. Then, the component of the weight along the ramp, $F_{G\|}$, equals $mg\sin\theta = mg(h/L)$. On the other hand, the friction force is proportional to the normal force, which is equal in magnitude (and opposite in direction) to the component of the weight perpendicular to the surface: $F_f = \mu_k F_{G\perp} = \mu_k mg\cos\alpha = \mu_k mg\sqrt{L^2-h^2}/L$.

Thus, $F_c = F_{G\|} + F_f = mg(h + \mu_k\sqrt{L^2-h^2})/L = 1.445\,\text{kN} \approx 1.5\,\text{kN}$. [=20pt]

b. The net work done (against gravity) by placing the sofa into the truck is simply the (gravitational) potential energy, $mgh = 200{\cdot}9.81{\cdot}1 = 1.962\,\text{kN} \approx 2\,\text{kN}$. [=5pt]

c. The total work done while moving the sofa may be calculated as the work done by the force exerted by the crew, F_c, over the length of the ramp, L. Thus: $W = \big(F_{G\|} + F_f\big)L = mgh + \mu_k mg\sqrt{L^2-h^2} = 3.972\,\text{kN} \approx 4\,\text{kN}$.

This final result is easily interpreted: the total work done by the crew on the sofa is the work against gravity (lifting it into the truck) *plus* the work done against friction, moving the sofa (horizontally) over the distance $\sqrt{L^2-h^2}$ against the friction force $\mu_k mg$ (since now all of the weight, mg, is perpendicular to the surface).

Since the work does not depend whether it is calculated (1) along the tilted ramp, and (2) horizontally to the truck and then vertically into it, it is tempting to conclude that it does not depend on the path taken. This however is not true: just consider pushing the sofa back and forth along the ramp. While the net (gravitational) potential energy in the end is the same, much more work will have been done against friction. Indeed, it is the dissipative effects such as friction which make work depend on the path in general.

Finally, while moving the sofa, the crew of course had also to haul their own weight, but that component of work cannot be estimated, as we do not know the number and the mass of the crew. [=10pt]

2. A pendulum of length L=2 m is released (from rest) at an angle θ_0=30°.

a. The speed of its bob as it swings through the equilibrium position $\theta = 0°$ is determined from conservation of energy: $K.E. + V = const.$ Initially, at θ_0, the pendulum was at rest and so had no kinetic energy; the potential energy on the other hand equals $V(\theta_0) = mg(L - L\cos\theta_0)$, where the quantity in the parentheses is the height measured upward, from the height of the lowest possible position (when $\theta = 0$). When passing through the equilibrium position, the potential energy vanishes, having been converted completely into the kinetic energy, $K.E. = \frac{1}{2}mv^2$. This gives us

$$mg(L - L\cos\theta_0) = \big[K.E. + V\big]_{\text{at }\theta=\theta_0} \stackrel{!}{=} \big[K.E. + V\big]_{\text{at }\theta=0} = \tfrac{1}{2}mv^2 ,$$

where "$\stackrel{!}{=}$" is the equality enforced by conservation of energy. Solving this for the speed, we have $v = \sqrt{2gL(1 - \cos\theta_0)} = 2.293\,\text{m/s} \approx 2.25\,\text{m/s}$. [=10pt]

b. The *total energy* of the bob of mass m=0.1 kg remains constant, by conservation of energy. Therefore, it is the same when swinging through any particular position, such as $\tau = \frac{1}{2}\tau_0$ or any other, and we are free to pick the one where the evaluation is easiest, such as the original position, at $\theta = \theta_0$. There $K.E. + V = 0 + mgL(1 - \cos\theta_0) = 0.263\,\text{J} \approx 0.26\,\text{J}$, which is determined in terms of the explicitly given quantities. Alternatively, at the equilibrium point, θ=0, $K.E. + V = \frac{1}{2}mv^2 + 0$, for which we'd need to use the value for v calculated in part a., and so introduce additional calculational error. To calculate the total energy as $E = \frac{1}{2}mv^2 + mg(1 - \cos\theta)$, you need to know both τ and v *at that particular position*, which (both clearly, and clearly unnecessarily) complicates the calculations even more. [=10pt]

3. A white marble (of mass m_w=0.01 kg) hits a red marble (of mass m_r=0.02 kg, originally at rest) at a speed v_{wi}=10 m/s and is (itself, the white marble) scattered at the angle θ=60° left of its original direction. This problem is very closely modeled on the worked Example 9-11, and we follow the same procedure, noting however that *the masses of the two objects are not the same* (see problem 53). Then conservation of energy, and momentum in the direction of the incoming white marble and perpendicular to it produces the system of three equations:

$$\tfrac{1}{2}m_w v_{wi}^2 = \tfrac{1}{2}m_w v_{wf}^2 + \tfrac{1}{2}m_r v_{rf}^2 , \qquad (1a)$$

$$m_w v_{wi} = m_w v_{wf}\cos\theta + m_r v_{rf}\cos\phi , \qquad (1b)$$

$$0 = m_w v_{wf}\sin\theta + m_r v_{rf}\sin\phi . \qquad (1c)$$

From Eq. (1c), we obtain

$$\sin\phi = -\frac{m_w}{m_r}\frac{v_{wf}}{v_{rf}}\sin\theta , \qquad (2)$$

and see that we need the ratio v_{wf}/v_{rf} to find the scattering angle of the red marble.

In Eqs. $(1b, c)$, we move the terms with v_{wf} to the left and square them:

$$m_w^2 v_{wi}^2 - 2m_w^2 v_{wi} v_{wf} \cos\theta + m_w^2 v_{wf}^2 \cos^2\theta = m_r^2 v_{rf}^2 \cos^2\phi \,, \tag{1b'}$$

$$m_w^2 v_{wf}^2 \sin^2\theta = m_r^2 v_{rf}^2 \sin^2\phi \,. \tag{1c'}$$

We sum these equations, use that $\sin^2\alpha + \cos^2\alpha = 1$ for any angle and that

$$v_{rf}^2 = \frac{m_w}{m_r}\left(v_{wi}^2 - v_{wf}^2\right) \,, \tag{3}$$

from Eq. $(1a)$, to obtain:

$$m_w^2 v_{wi}^2 - 2m_w^2 v_{wi} v_{wf} \cos\theta + m_w^2 v_{wf}^2 = m_r^2 \cdot \frac{m_w}{m_r}\left(v_{wi}^2 - v_{wf}^2\right) \,,$$

After combining like terms and dividing through by v_{rf}^2, we obtain:

$$v_{wf}^2 - 2\frac{m_w v_{wi} \cos\theta}{m_w + m_r} v_{wf} + \frac{m_w - m_r}{m_w + m_r} v_{wi}^2 = 0 \,,$$

from which the speed of the white marble after the collision is

$$v_{wf} = \left[\frac{m_w \cos\theta}{m_w + m_r} \pm \sqrt{\frac{m_w^2 \cos^2\theta}{(m_w + m_r)^2} - \frac{m_w - m_r}{m_w + m_r}}\right] v_{wi} = 7.676\,\mathrm{m/s} \,.$$

On inserting the numerical values, we see that the '-' option would lead to nevative speeds and is therefore unphysical.

a. Inserting this in Eq. (3) (chosing the '+' sign), we obtain the speed of the red marble after the collision to be $v_{rf} = 4.532\,\mathrm{m/s} \approx 5\,\mathrm{m/s}$. [=20pt]

b. The scattering angle of the red marble is now obtained from Eq. (2), substituting the above results. We may, fo course, substitute the above analytic expression for v_{wf}, but this does not simplify at all; it is therefore better to substitute the numerical value of the quantities on the right hand side: $\phi = -47.17° \approx -60°$, *i.e.*, $60°$ to the right since $\theta = 60° > 0$ was given to "the left." [=15pt]

I admit that $47.17°$ is *almost* half-way between $30°$ and $60°$, but it is still closer to $60°$. Note however that simply makes no sense chosing ϕ to be to the left—how could the marbles scatter off of each other but still go to the same side?!? So, even if you were guessing with but half a brain, you had 50 % to guess correctly.

4. a. To tighten bolts with a torque τ=200 m·N, a mechanic who can push at most F_m=1,000 N must use a wrench of a length at least $L = \tau/F_m = 0.2\,\mathrm{m}$. [=10pt]

b. The force at the edge of a bolt (with *diameter* D=10 mm) is $\tau/(D/2) = 2\tau/D = 40$ kN, since the distance from the pivotal point to the edge is $\frac{1}{2}D$, the *radius* of the bolt. [=10pt]

5. A m=70 Tg meteorite hits the Earth Eastward, tangentially and in the equatorial plane at a speed of v=10 km/s, and remains stuck. Since the meteorite is now stuck with the Earth, the angular momentum of the Earth after the collision equals the sum of angular momenta of the Earth before the collision and the meteorite (see Example 11-7 and problem 37). Equating then the angular momentum just before the collision with the one just after the collision, we have

$$I_E\omega_i + Rmv = L_i \stackrel{!}{=} L_f = (I_E + mR^2)\omega_f = \frac{I_E + mR^2}{T+\delta T} ,$$

where $I_E = \frac{2}{5}MR^2$ is the moment of inertia of the Earth (assuming that it is a uniform sphere rotating about an axis passing through its center), Rmv is the angular momentum of the meteorite at the moment of striking Earth. The Earth's rotational frequency before the collision is $\omega_i = 1/T$ and equals 1 revolution per day, so that T is the duration of a day before the collision: $T = 86,400\,\mathrm{s}$. *After* the collision, the frequency becomes $\omega_f = 1/(T{+}\delta T)$, where δT is the inflicted change in Earth's day. Solving for δT, we have that the Earth day changes by [=10pt]

$$\delta T = \frac{I_E + mR^2}{I_E/T + mRv} - T = \left[\frac{I_E + mR^2}{I_E + mRvT} - 1\right]T . \tag{4}$$

Before we substitute any numerical value, it is worthwhile noting that there are two *competing* effects: (1) the increase of the moment of inertia from I_E to $I_E{+}mR^2$ on account of the meteorite getting stuck in Earth's mantle, and (2) the addition of the meterorite's angular momentum, mRv, to that of Earth. While the former increases the length of the day, the latter decreases it. This becomes clearer upon bringing the quantity in square brackets in Eq. (4)to a common denominator:

$$\delta T = \frac{mR^2 - mRvT}{I_E + mRvT}T . \tag{4a}$$

As to the *amount* of change, δT, that these two effects impart, note that (in $\mathrm{kg\,m^2}$)

$$mR^2 = 2.849{\times}10^{24} , \qquad mRvT = 3.859{\times}10^{26} , \qquad I_E = 9.720{\times}10^{37} .$$

That is, the first (slowing down) effect is more than a hundred times smaller than the second (speeding up), but even this is eleven orders of magnitude smaller than Earth's original moment of inertia, I_E. We can therefore immediately conclude that the change in Earth's day should be about eleven orders of magnitude smaller than the day itself, *i.e.*, around $10^{-7}\,\mathrm{s}$. Indeed, inserting the numerical values in Eq. (4,) we find that $\delta T = -3.941{\times}10^{-12}T = -0.34\,\mu\mathrm{s}$, and so the closest value among those give (albeit still five orders of magnitude wrong) is $\delta T \approx 0.1\,\mathrm{s}$.

6. A plank (of mass $m{=}200\,\mathrm{kg}$ and length $L{=}2\,\mathrm{m}$) hangs on a cable attached a distance $d{=}0.1\,\mathrm{m}$ from each end. [=10pt]

a. The torque about each attachment point must be zero, for the plank is stationary. Thus the torque produced by the weight of the plank (at the center of mass) and by the tension in each cable balance:

$$mg(\frac{L}{2}{-}d) \;=\; F_t(L{-}2d) , \quad \text{so} \quad F_T(\text{empty}) = \tfrac{1}{2}mg = 981\,\mathrm{N} \approx 1\,\mathrm{kN} ,$$

as one would expect by noting that the weight of the plank is symmetrically supported by the two cables. [=10pt]

b. With the window-washer of mass M=65 kg at the very edge of the plank, the balance of torques through the attachment point further from her (which involves the tension force in the cable nearest to her) becomes

$$mg(\frac{L}{2}-d) + Mg(L-d) \;=\; F_T(L-2d) \;,$$

from which [=10pt]

$$F_T(\text{with washer}) = \tfrac{1}{2}mg + Mg\frac{L-d}{L-2d} = 1621.87\,\text{N} \approx 1.6\,\text{kN} \;.$$

Finally, two remarks:

1. Throughout the test, the number of significant figures given does not have any particular significance, and is done merely so as to (a) allow the student to check their own (*highly recommended!*) calculations, and to (b) offer enough precision so as to be able to choose among the offered answers. This practice makes sense in view of the fact that we have no information on the precision of any of the given data, and hence cannot even guess as to the actual (experimental) error in the calculations.

2. Several of you have displayed extreme lack of thinking in the test, by giving mutually exclusive answers to two parts in a given problem. For example, one student chose the answers in the last problem to be a.: 10 kN, and b.: 1.3 kN!

How can anyone, remotely awake, claim that the tension in a cable supporting the empty plank is almost seven times that of the tension in the cable supporting the same plank with the window washer on top of it!?! Such students should have been penalized, perhaps by *subtracting* the points for such hideously wrong answers!

Thus emerges a following grading scheme proposal:

a. full points for the correct answer;

b. no points for a wrong answer which does not contradict another given answer;

c. negative full points for wrong answers that contradict another answer;

d. an added option of “I don’t know,” to carry, say $\frac{1}{4}$ of points, so as to reward honesty.

Everyone who got this far in reading this posting, please send your opinions about this grading scheme proposal to `thubsch@howard.edu`.