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PHYS-013 1st Midterm Exam Solutions—**Revised** Instructor: T. Hübsch

DISCLAIMER: The completeness and detail presented herein were by no means expected in the Student's solutions for full credit. The additional information given here is solely for the Student's convenience and education. For each problem, the result is derived in the algebraic form first. Concrete values are then easy to substitute and compare with the choices offered; this is shown in the table at the end of each problem.

1. A car accelerates, at a constant rate, from v_i to v_f in t seconds.

a. The average acceleration is $\bar{a} = \dfrac{v_f - v_i}{t}$, using the standard expression. [=10pt]

b. The distance traveled during this time is $d = v_i\,t + \frac{1}{2}\bar{a}\,t^2$, using that the initial position is 0 m, since we measure the distance from the initial position. With the previous result for $\bar{a}$, this becomes $d = v_i\,t + \frac{1}{2}\frac{v_f - v_i}{t}\,t^2 = v_i\,t + \frac{1}{2}(v_f - v_i)\,t$, or, $d = \frac{1}{2}(v_f + v_i)\,t$. Indeed, this is the distance covered during time t, travelling at the *average* speed, $\frac{1}{2}(v_f + v_i)$. [=10pt]

c. It is tempting (and indeed, I did so myself, in the original solution) to turn the previous result, $d = \frac{1}{2}(v_f + v_i)\,t$, around and solve for the time, $t = \frac{2d}{(v_f + v_i)}$. Given the distance and the initial and final speeds, we can now calculate the time required. Unfortunately, as given in the problem, the initial and final speeds do not refer to the 50 m trip, but to the trip made in t seconds (*credits to Prof. A. Batra for noticing this caveat*). So, while v_i *is* the initial speed at the beginning of the trip, v_f is the "final" speed at the end of the time period of t seconds,– not at the end of the distance d! Since we do not know what the speed at the end of the distance d was, we cannot use this simple formula, $d = \frac{1}{2}(v_f + v_i)\,t$.
Thus, we are forced to use the correct, albeit a bit more complicated formula $d = v_i\,t + \frac{1}{2}\bar{a}\,t^2$, in which we can insert the numerical value for $\bar{a}$ (either the exact result from part a, or the choice from the four offered values). Now you end up with having to solve this quadratic expression for t, and have $t_\pm = \frac{1}{\bar{a}}\big(-v_i \pm \sqrt{v_i^2 + 2\bar{a}d}\big)$ for the two solutions of the quadratic equation $\frac{1}{2}\bar{a}\,t^2 + v_i\,t - d = 0$. Clearly, only $t_+ = \frac{1}{\bar{a}}\big(\sqrt{v_i^2 + 2\bar{a}d} - v_i\big)$ is positive, and so t_+ is the solution we need. [=10pt]

v_i [m/s]	v_f [m/s]	t [s]	$\bar{a}$ *[m/s²]*	*d [m]*	d [m]	*t [s]* in part *c*.
10	20	5	*2 (2)*	*75 (75)*	50	*3.667 (3)*
10	15	3	*1.667 (1.5)*	*37.5 (35)*	50	*3.798 (4)*
5	20	8	*1.875 (2)*	*100 (95)*	50	*5.108 (4)*
12	16	2	*2 (2)*	*28 (30)*	50	*3.274 (4)*

Table 1: Data and results (*in italic*) for problem #1. Identifying the closest of the offered values (*shown in parentheses*) should not be too difficult now.

2. A fire hose held near the ground, on level ground, shoots water at a speed of v_i.

a. If aimed at θ from the ground, the vertical motion is governed by the vertical component of the initial velocity, $v_{iy} = v_i \sin\theta$, and the constant gravitational acceleration. The vertical component of the velocity then is $v_y(t) = v_{iy} - gt$. The water turns downward when $v_y(t) = 0$, *i.e.*, when $v_{iy} = gt$. The time that the water takes to hit ground is twice this much, so $t_f = 2\frac{v_{iy}}{g} = 2\frac{v_i \sin\theta}{g}$. [=10pt]

b. The horizontal motion of the water stream is unaffected by the (vertical) gravitational acceleration, so $d(t) = v_{ix}t = v_i \cos\theta\, t$. At the time when it hits the ground, this becomes $d(t_f) = (v_i\cos\theta)\big(2\frac{v_i\sin\theta}{g}\big) = \frac{v_i^2}{g}\sin(2\theta)$. To hit a spot a distance of d away, the nozzle must be aimed at $\theta = \frac{1}{2}\arcsin\big(\frac{g\,d}{v_i^2}\big)$ from the level ground. [=20pt]

v_i [m/s]	θ [°]	*t [s]*	d [m]	*θ [°]*
6	30	*0.616 (0.6)*	3	*27.428 (30)*
5	45	*0.721 (0.7)*	2	*25.851 (25)*
10	45	*1.442 (1.4)*	5	*14.687 (15)*
5	45	*0.721 (0.7)*	2	*25.851 (25)*

Table 2: Data and results (*in italic*) for problem #2. Identifying the closest of the offered values (*shown in parentheses*) should not be too difficult now.

3. A rocket of mass m lifts off by expelling gases with a(n upward) force $F_L = |\vec{F}_L|$. The total sum of forces acting on the rocket then is $F_T = |\vec{F}_T| = |\vec{F}_L| - mg$, since the gravitational attraction acts downward.

a. The initial acceleration of the rocket is then $a = \frac{F_L - mg}{m} = \frac{F_L}{m} - g$. [=10pt]

b. The rocket's speed after a time t is then $v(t) = a\,t = \frac{F_L - mg}{m}\,t$, assuming constant acceleration. [=10pt]

m [kg]	F_L [N]	*a [m/s²]*	t [s]	*v [m/s]*
3.0×10^6	35×10^6	*1.857 (10)*	8	*14.853 (25)*
3.5×10^6	30×10^6	*−1.239:no liftoff (4)*	8	*0 (32→20*, 20†)*
3.5×10^6	30×10^6	*−1.239:no liftoff (5)*	8	*0 (40→50*, 20†)*
3.5×10^6	40×10^6	*1.619 (2)*	8	*12.949 (20)*

Table 3: Data and results (*in italic*) for problem #3. Identifying the closest of the offered values (*shown in parentheses*) should not be too difficult now, for the first and fourth group. The other two groups, obtaining a net downward acceleration (and so no liftoff), should have choosen the lowes offered value. Writing in the correct value, 0 or 'no liftoff,' instead warrants full credit. Similarly, in part b., (for the second two groups) should have obtained a speed of 0—the rocket isn't lifing. However, using the chosen value from part a. for this calculation (indicated by an asterisk), or choosing the lowest offered value (indicated by a dagger) similarly warrant full credit.

4. For Arlene ($m = 55$ kg) to walk accross a horizontal, $L = 10.0$ m long "high wire," with a $\theta = 10°$ sag when she is mid-way, her weight of $mg = (55{\cdot}9,81 = 539.55)$N must be cancelled by sum of the upward components of the tension forces in the two parts of the rope: $mg = 2F_{Ty} = 2F_T \sin\theta$. From this, it follows that $F_T = \frac{mg}{2\sin\theta}$.

m [kg]	θ [°]	F_T *[N]*
55	10	*1553 (1500, 5000*)*
45	15	*852 (850, 1500*)*
45	12	*1061 (1200)*
50	15	*948 (1000)*

Table 4: Data and results (*in italic*) for problem #4. Identifying the *closest* of the offered values (*shown in parentheses*) should not be too difficult now. Of course, if the *closest* value is *less* than the exact result, you may well opt for the next larger value (indicated by an asterisk), to save Arlene's life. Needless to say, that should warrant full credit.

Comment: So, why were you given the span of the wire when it does not feature in this calculation? And, why is the only question listed as part a.? Good questions. The answer is not 'so as to confuse you,' but rather because there were originally two more parts, to **b.** calculate the (horizontal) force with which the wire tugs on the anchors in the buildings (important, don't you think?) and to **c.** calculate the amount of sag in the wire when Arlene is in the middle. But then, I though that that would have been too much to ask...

5. For you to remain standing in a train that accelerates at a, you should not be moving (and so also not accelerating) with respect to the train. As the train accelerates horizontally (as trains do), this means that this acceleration force should be cancelled by the friction force. So, $F_{\rm fr} = \mu_S F_N = \mu_S mg$ is required to equal $F_{\rm tr} = ma$. This yields $\mu_S mg = ma$, or, $\mu_S = a/g$. Since the acceleration of the train is expressed as a multiple of the gravitational acceleration, $a = \alpha g$, the *minimal* numerical value of μ_S equals this coefficient of proportionality (the closest choice, amongst those offered, is given in parentheses, in *italic*): [=10pt]

$$\mu_S = \;0.2\;\textit{(0.2)},\qquad 0.4\;\textit{(0.5)},\qquad 0.6\;\textit{(1.0)},\qquad 0.4\;\textit{(0.5)}.$$

6. The gravitational acceleration on the surface of a planet is $g_X = G_N M_X/R_X^2$. So, on a planet that is α times heaveir than Earth (so $M_X = \alpha M_E$) but of the same size as Earth (so $R_X = R_E$), we have:

$$g_X \;=\; G_N \frac{M_X}{R_X^2} \;=\; G_N \frac{\alpha M_E}{R_E^2} \;=\; \alpha\, g_E\;.$$

The numerical results then are as follows (the closest choice, amongst those offered, is again given in parentheses, in *italics*): [=10pt]

$$g_X = \;3.0\,g_E\;\textit{(2g)},\qquad 1.2\,g_E\;\textit{(1g)},\qquad 4.1\,g_E\;\textit{(5g)},\qquad 1.2\,g_E\;\textit{(1g)}.$$