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PHYS-014 1st Midterm Exam Solutions

Don't Panic !

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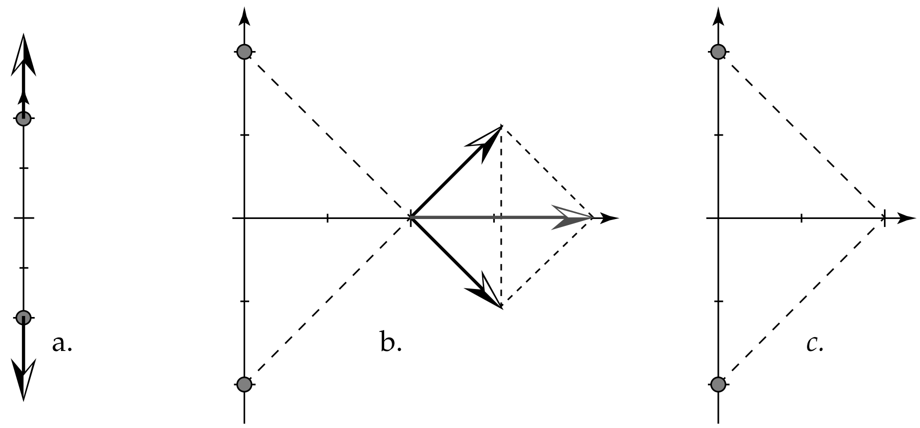
DISCLAIMER: The completeness and detail presented herein were by no means expected in the Student's solutions for full credit. The additional information given here is solely for the Student's convenience and education. For each problem, the result is derived only in the algebraic form. The Student should have no difficulty in substituting the concrete numerical values as appropriate.

1. Two charges, $Q_1 = q = Q_2$ are placed along the y-axis, a distance L on each side of the origin; see figure below:



a. The electrostatic force on each charge is calculated from the Coulomb law: [=10pt]

$$|\vec{F}| \;=\; \frac{1}{4\pi\epsilon_0}\,\frac{Q_1\,Q_2}{r^2} = \frac{1}{4\pi\epsilon_0}\,\frac{q^2}{(2\,L)^2}\ ,$$

since the charges are equal and the distance is $2L$. The direction of $\vec{F}$ is along the x axis, and is repulsive.

b. The electric field on the x-axis, a distance ℓ from the origin is calculated as the *vector* sum of the electric fields caused by each of the two charges: [=10pt]

$$\vec{E} \;=\; \frac{1}{4\pi\epsilon_0}\,\frac{Q_1}{r^2}\,\hat{e}_1 + \frac{1}{4\pi\epsilon_0}\,\frac{Q_2}{r^2}\,\hat{e}_2 = 2\,\frac{1}{4\pi\epsilon_0}\,\frac{q}{L^2+\ell^2}\,\cos\theta\,\hat{\imath}\ .$$

Here the unit vectors $\hat{e}_1$ and $\hat{e}_2$ specify the direction of the electric field caused by the first and second charge, respectively. Now, in all groups, $L = \ell$, so $\theta = 45°$, so $\hat{e}_i\cdot\hat{e}_x = \cos\theta = \frac{1}{\sqrt{2}}$ for both $i = 1, 2$. Their y-components precisely cancel, while their x-components, each of magnitude $\cos(45°)$, add up,– whence the overall pre-factor '2'.

c. The electric potential on the x-axis, a distance ℓ from the origin is calculated as the *scalar* sum of the electric potentials caused by each of the two charges: [=10pt]

$$V \;=\; \frac{1}{4\pi\epsilon_0}\,\frac{Q_1}{r} + \frac{1}{4\pi\epsilon_0}\,\frac{Q_2}{r} = 2\,\frac{1}{4\pi\epsilon_0}\,\frac{q}{\sqrt{L^2+\ell^2}}\ .$$

This –of course– is quite simpler than the expression for the electric field, as the potential is not a vector, has no direction, and so only its magnitude is being added.

2. A negligibly thin line with a uniform charge per unit length, $+Q/L$, is surrounded by a negligibly thin cylinder of radius R and opposite charge per unit length $(-Q/L)$; both are much longer than R. Let r denote the perpendicular distance from the central line.

a. The electric field inside the cylinder (for $r < R$) then is, using Gauss's law: [=10pt]

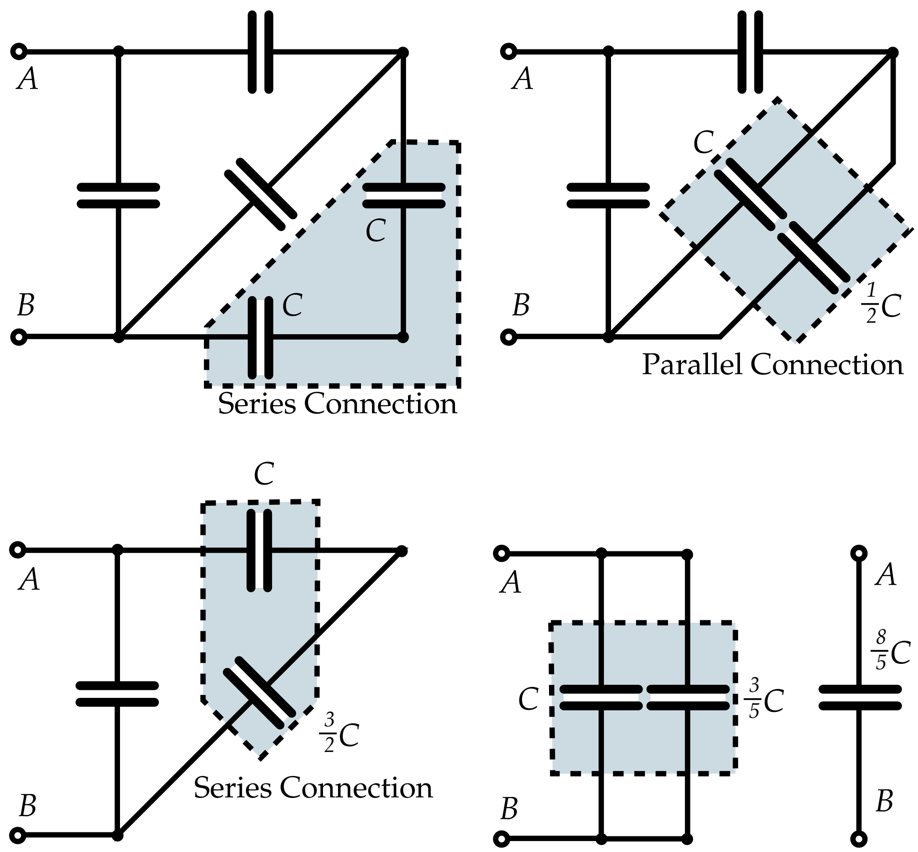
$$\vec{E} \;=\; \frac{1}{2\pi\epsilon_0}\frac{Q/L}{r}\,\hat{r}\;, \quad \text{since} \quad \oint_S \mathrm{d}\vec{A}\cdot\vec{E} = \frac{Q}{\epsilon_0}\;.$$

To see this, note that $\vec{E}$ is directed radially outward, and is constant on the Gaussian, concentrically cylindric surface S of radius $r<R$ and length L. Thus:

$$\oint_S \mathrm{d}\vec{A}\cdot\vec{E} = |\vec{E}|\oint_S |\mathrm{d}\vec{A}| = |\vec{E}|\,2\pi r\,L\;, \quad \text{so} \quad |\vec{E}|\,2\pi r\,L = \frac{Q}{\epsilon_0}\;.$$

b. To obtain the electric field outside the cylinder, we wrap the arrangement into a Gaussian, concentrically cylindirc surface of a radius $r > R$. Since the net charge contained inside this Gaussian cylinder is $Q/L + (-Q/L) = 0$, the electric field on this cylinder, and so for all $r > R$ must vanish. [=10pt]

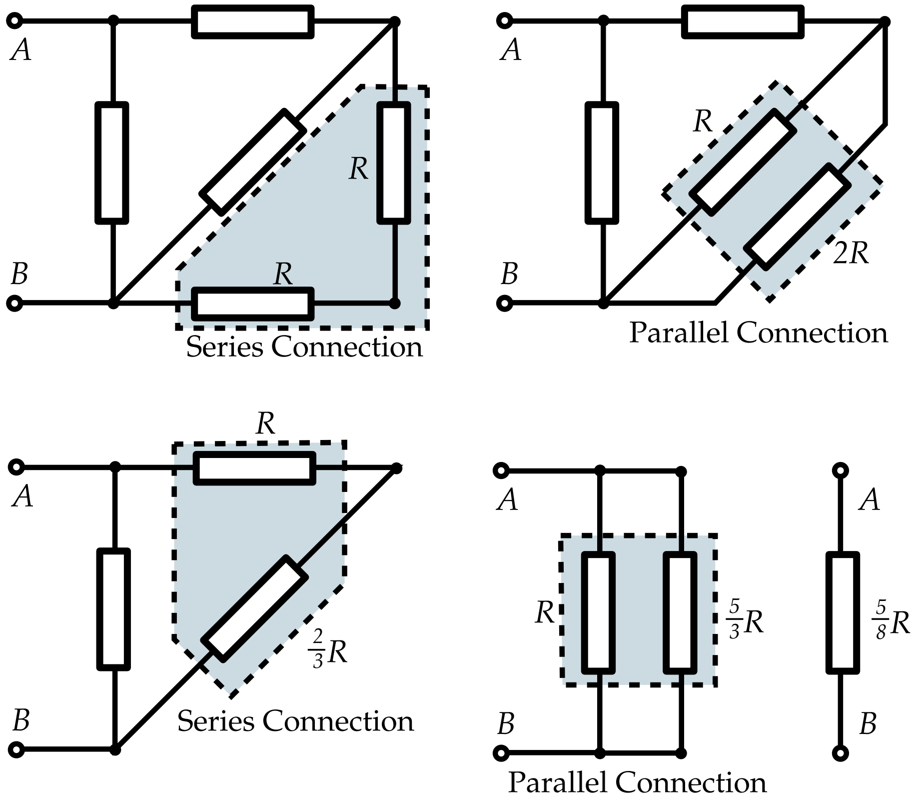
3. The circuit with all capacitors of equal capacitance, C, simplified:



a.
Note that this is the *only* way to (iteratively) simplify this circuit! [=10pt]

b. The capacitance of a capacitor equals $K\epsilon_0 A/d$, where K is the dielectric constant of the material placed between the capacitor plates of area A, separating them to a distance d. [=10pt]

4. The circuit with all resistors of equal resistance, R, simplified:



a. Note that this is the *only* way to (iteratively) simplify this circuit! [=10pt]

b. The resistance of a cylindrical resistor of length L and radius r equals $\rho L/A = \rho d/(\pi r^2)$, where ρ is the resistivity of the material. [=10pt]

5. Through n light-bulbs of power P, connected in parallel to the $V = 110\,\mathrm{V}$ wall outlet, there flows a current $I = P/V$.

a. The connection being parallel, the current drawn from the wall socket equals the sum of the currents running through the individual light-bulbs, so $I_{\mathrm{tot}} = nP/V$. [=10pt]

b. The resistance of each light-bulb is $R = V/I = V^2/P$. [=10pt]

6. The cyclotron orbit radius, for a particle of charge q entering a magnetic field of strength B with a perpendicular momentum mv is $R_c = \frac{mv}{qB}$. For an electron ($q = e$), the required magnetic field may be calculated from the formula $B = \frac{m_e v}{e\,R_c}$. [=10pt]