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Of Marginal Kinetic Terms and Anomalies

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ABSTRACT

Certain special kinetic terms in supersymmetric field theories lead to exactly marginal operators in the sense of renormalization flow. Such terms are shown to arise naturally in 2-dimensional σ-models describing 4-dimensional superstring compactification, but many other models can also contain them. They are closely related to chiral gauge and gravitational anomalies on one hand and Ricci-flatness on the other.

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Introduction. The interest in 2-dimensional mean-field theories with world sheet (2,2)-supersymmetry has been revived recently in connection to 4-dimensional string models [1]. It has been argued on general grounds, and subsequently proven [2], that exact quantum characteristics of the simplest such models can be extracted from a suitable classical action functional even though its (super)conformal invariance is spoiled by quantum corrections. In particular, the classical Wess-Zumino type action[♯2]

$$\mathcal{A}_n \quad \stackrel{\rm def}{=} \quad \int \mathrm{d}^2\sigma\, \mathrm{d}^2\varsigma\, \mathrm{d}^2\overline{\varsigma}\ K(\Phi^\dagger, \Phi) \quad + \quad \Big(\int \mathrm{d}^2\sigma\, \mathrm{d}^2\varsigma\ \Phi^{n+1} \quad + \quad \text{h.c.} \Big) \tag{1}$$

suffices to determine the central charge of the (super)conformally invariant model into which $\mathcal{A}_n$ flows under renormalization, the complete set of marginal operators and their operator product expansion coefficients. Moreover, this data is independent of the explicit form of the D-term $K(\Phi^\dagger, \Phi)$, many choices of which are in a wide universality class represented by the simplest such choice, $K_0(\Phi^\dagger, \Phi) = \Phi^\dagger\Phi$. The latter is argued to have positive anomalous dimension and thus irrelevant [1], whence all such choices of $K(\Phi^\dagger, \Phi)$ seem to yield irrelevant operators in the sense of renormalization flow.

The purpose of this short note is to point at a perhaps less generic class of D-terms which, in a wide variety of field theories, do produce exactly marginal operators; we refer to these as marginal kinetic terms. It is of course gratifying to realize that precisely such kinetic terms arise naturally in many 2-dimensional σ-models which describe superstring models with 4-dimensional Minkowski spacetime. Inclusion of such kinetic terms opens the possibility to describe Kähler variations in complement to the Landau-Ginzburg description of Calabi-Yau superstring vacuua [1], which was known to describe well the complex structure deformations. Perhaps not surprisingly, this will involve the "twisted chiral" superfields of Ref. [3]. This is in accord with the general (super)conformal field theory analysis : polynomials in the chiral superfields Φ^μ correspond to elements of the (c,c)-ring, while twisted chiral (composite) fields relate to the (a,c)-ring.

It will also transpire that the same marginal kinetic terms carry the information about the various anomalies in the models. In particular, in suitable 2-dimensional σ-models, the Ricci-flatness requirement in the Calabi-Yau compactification appears as a chiral gauge anomaly cancellation, while the central charge of the Virasoro algebra is of course the chiral gravitational anomaly. That the latter one cancels non-trivially between various fields and their ghost counterparts, rather than directly among (super)fields of a single sector, ensures the appearance of such marginal kinetic terms. In field theories where spacetime is more than 2-dimensional, the rôle of these marginal kinetic terms is less universal, but otherwise analogous to the 2-dimensional case.

[♯2]We use the superspace notation where $\sigma^{\pm\pm}$ are the world sheet light-cone coordinates and $\varsigma^\pm, \overline{\varsigma}^\pm$ are their (2,2)-superpartners; the $\pm$ sub- and superscripts are simply units of spin.

The $\mathbb{CP}^n$ model. We start with the well known supersymmetric $\mathbb{CP}^n$ model in 2-dimensional spacetime and (2,2)-supersymmetry (the analogous will be true of N=1 supersymmetry in 4-dimensional spacetime). The classical action is

$$\mathcal{A}[\mathbb{CP}^n] \stackrel{\rm def}{=} \int \mathrm{d}^2\sigma\ \mathrm{d}^4\mathrm{d}^2\varsigma\,\mathrm{d}^2\bar{\varsigma}\,\Big(\,\|\Phi\|^2\mathrm{e}^{-\boldsymbol{V}} + \tfrac{n+1}{2f}\boldsymbol{V}\Big)\,, \qquad \|\Phi\|^2 \stackrel{\rm def}{=} \sum_{\mu=0}^{n}\Phi^\dagger_\mu\,\Phi^\mu\ . \qquad (2)$$

It involves $n{+}1$ chiral superfields, Φ^μ, and a gauge superfield, $\boldsymbol{V}$ for which no kinetic term is introduced. $\boldsymbol{V}$ gauges a complexified and supersymmetric version of $U(1)$, which we denote by $\widetilde{U}(1)$ and which includes $U(1)$, dilation and their supersymmertic counterpart, a chiral symmetry acting on the fermion component fields in Φ^μ. Indeed, the body of the classical configuration space (spanned by the unconstrained bosonic fields) is $[\mathbb{C}^{n+1}{-}\{0\}]/\mathbb{C}U(1) = \mathbb{CP}^n$. The $\widetilde{U}(1)$ gauge transformation, also known as the Kähler symmetry, acts by

$$\Phi \ \to\ \Theta\,\Phi\ , \qquad \boldsymbol{V} \ \to\ \boldsymbol{V} + \ \log\Theta + \log\Theta^\dagger\ , \qquad (3)$$

where Θ is an arbitrary chiral superfield and $\Theta^\dagger$ its antichiral hermitian conjugate.

The equation of motion for $\boldsymbol{V}$ reads $\boldsymbol{V} = \log(\frac{2f}{n+1}\|\Phi\|^2)$. Using this to eliminate $\boldsymbol{V}$ is equivalent to path-integration over $\boldsymbol{V}$ and produces, up to an uninteresting additive constant,

$$\mathcal{A}'[\mathbb{CP}^n] = \int \mathrm{d}^2\sigma\,\mathrm{d}^2\varsigma\,\mathrm{d}^2\bar{\varsigma}\ K_{(FS)}(\Phi^\dagger,\Phi)\ , \qquad K_{(FS)}(\Phi^\dagger,\Phi) = \tfrac{n+1}{2f}\log\|\Phi\|^2\ , \qquad (4)$$

where $\|\Phi\|^2 = \sum_{\mu=0}^{n}\Phi^\dagger_\mu\,\Phi^\mu$ and $K_{(FS)}$ is the Fubini-Study Kähler potential. Since the action (2) is quadratic in Φ's, it is also possible to integrate out the Φ's, which results [4] in :

$$\mathcal{A}''[\mathbb{CP}^n] \stackrel{\rm def}{=} \frac{n{+}1}{4\pi}\int \mathrm{d}^2\sigma\Big(\int \mathrm{d}\varsigma^+\,\mathrm{d}\bar{\varsigma}^-\ \boldsymbol{S}[\log(\boldsymbol{S}/\mu)-1] + \text{h.c.}\Big) + \ \ldots \qquad (5)$$

where the higher terms all involve $\int \mathrm{d}^2\varsigma\,\mathrm{d}^2\bar{\varsigma}$ -integrals.

The superfields $\boldsymbol{S} \stackrel{\rm def}{=} D_+\bar{D}_-\boldsymbol{V}$ and its hermitean conjugate are both twisted chiral superfields [3]. Since the leading terms in $\mathcal{A}''[\mathbb{CP}^n]$ involve integration over only "half" of the superspace, these terms are protected by the usual non-renormalization theorems regarding (twisted) F-terms. So, while many D-terms supply only irrelevant operators [2], there do exist D-terms such as in Eq. (2) which have a marginal residue, represented in Eq. (5). That the kinetic terms considered here are not in the universality class represented by $K_0(\Phi^\dagger,\Phi) = \Phi^\dagger\Phi$ is most easily seen from the distinct behaviour of the partition functionals. When defined with the kinetic term (4), a partition function clearly has an essential singularity at $\Phi^\mu = 0$. Removing the origin from the field-space, however, changes its topology and is seen to distinguish the two universality classes.

A few remarks are in order before carrying on to related models. Firstly, the continuous gauge symmetry (3) is anomalous; while the classical actions (2) and (4) are invariant, the action (5) is not, except for the leading term $\boldsymbol{S}$ and its conjugate [4]. The "leading log" term

$\boldsymbol{S}\log(\boldsymbol{S}/\mu)$ and its conjugate carry the $\widetilde{U}(1)$ gauge-anomaly of the $\mathbb{CP}^n$ model; the numerical value is $(n{+}1)$ and equals the first Chern class of $\mathbb{CP}^n$; the $1/4\pi$ factor turns out to be merely a suitable normalization. Because of the $(n{+}1)$ prefactor, however, a $\mathbb{Z}_{n+1}$ subgroup of (3) does remain a symmetry and will have to be included in the GSO projection in Landau-Ginzburg models.

Secondly, note that the equation of motion for $\boldsymbol{V}$ and the definition of the superfield $\boldsymbol{S}$ lets us re-interpret Eq. (5) in terms of the Φ's. To that end we record

$$\boldsymbol{S} \;\sim\; (D_+\Phi^\mu)\, G^{(FS)}_{\mu\bar\nu}\, (\bar D_-\Phi^{\dagger\bar\nu})\;, \qquad G^{(FS)}_{\mu\bar\nu} \;\stackrel{\text{def}}{=}\; \partial_\mu\,\partial_{\bar\nu}\, K_{(FS)}(\Phi,\Phi^\dagger)\;. \tag{6}$$

where $G^{(FS)}_{\mu\bar\nu}$ is the Fubini-Study metric. Clearly, the (anticommuting) fermionic derivatives act as the field theory generalization of the dz's and d$\bar z$'s[♯3]. Hence, the quantity

$$(\boldsymbol{S}\,“{+}”\,\boldsymbol{S}^\dagger) \;\;\stackrel{\text{def}}{=}\;\; (\int \mathrm{d}\varsigma^+\,\mathrm{d}\bar\varsigma^-\;\boldsymbol{S} \;+\; \int \mathrm{d}\varsigma^-\,\mathrm{d}\bar\varsigma^+\;\boldsymbol{S}^\dagger) \tag{7}$$

has a natural interpretation as the (super)field theory generalization of the Fubini-Study Kähler (1,1)-form; this supports the identification of the $\widetilde{U}(1)$-anomaly with the first Chern class. Note that the term (7), with the interpretation (6), also appears when the "twisted half" of the fermionic integral is explicitly performed in Eq. (4).

Finally, the coupling constant f from $\mathcal{A}[\mathbb{CP}^n]$ has undergone a "dynamical transmutation" : in Eq. (5), it occurs through the renormalization 'mass' scale[♯4] $\mu{=}\mathit{const.}\,\varepsilon\,\mathrm{e}^{2\pi/f}$, where ε is the dimensional regularization parameter. When describing a σ-model with $\mathbb{CP}^{n_1}\times\cdots\times\mathbb{CP}^{n_m}$ target space, the effective action (5) is replaced by a linear combination of such actions, one for each $\mathbb{CP}^{n_i}$ factor, which contains the linear combination $\sum_{i=1}^m(\mathit{const.}{+}(2\pi/f_i))(\boldsymbol{S}_i\,“{+}”\,\boldsymbol{S}_i^\dagger)$. Indeed, the Kähler class of a product of $\mathbb{CP}^n$'s is a linear superposition of the individual Kähler classes.

Constrained Models. Many Kähler manifolds are constructed by embedding in $\mathbb{CP}^n$'s and rather important examples are obtained as the (sub)space of common solutions to a system of constraints in a product of complex projective spaces. Most naturally, one imposes the constraints $P^a(\Phi){=}0$ by means of Lagrange multiplier fields on a system of $\mathbb{CP}^n$ models [6,7]. The corresponding action is $\mathcal{A}_{\text{kin.}} + \mathcal{A}_{\text{con.}}$, where

$$\mathcal{A}_{\text{kin.}} \;=\; \sum_{i=1}^m \int \mathrm{d}^2\sigma\,\mathrm{d}^2\varsigma\,\mathrm{d}^2\bar\varsigma\,\Big(\,\|\Phi_i\|^2\mathrm{e}^{-\boldsymbol{V}_i} \;+\; \frac{n_i{+}1}{2f_i}\boldsymbol{V}_i\,\Big)\;, \tag{8}$$

$$\mathcal{A}_{\text{con.}} \;=\; \sum_{a=1}^K \int \mathrm{d}^2\sigma\,\mathrm{d}^2\varsigma\,\Big(\,\boldsymbol{\Lambda}_a\,P^a(\Phi) \;\;+\;\; \text{h.c.}\,\Big) \tag{9}$$

♯3 Indeed, the lowest component of the superfield $D_\pm\Phi^\mu$ is $\psi^\mu_\pm$ and has been identified as a formal analogue of dz ever since the early work on σ-models [5].

♯4 In d-dimensional spacetime, μ has dimension $(d{-}2)/2$ and is actually dimensionless in $d=2$.

and where $\Phi_i^{\mu_i}$ are coordinate superfields on $\mathbb{CP}^{n_i}$. While this provides the exact field theory parallel of the construction of such Kähler spaces, its use in practical computations is limited, because of the non-linear couplings in (8) and since the Lagrange multiplier fields effectively introduce infinitely strong coupling (see however Ref. [8,9] for some applications).

Since the terms in $\mathcal{A}_{\rm con.}$ are generally not quadratic in the Φ's, we cannot in general perform the $\int D[\Phi]$ path-integration and obtain the constrained space analogues of Eqs. (5) and (6). On general grounds, we know that analogous relations must exist although an explicit evaluation eludes us. Clearly, the analogue of $\boldsymbol{S}$ should then again be of the form (6), featuring, however the physical metric on the constrained space $\mathcal{M}$,

$$\boldsymbol{S}_{\mathcal{M}} \;\sim\; (D_+\Phi^{\mu})\, G^{(\mathcal{M})}_{\mu\overline{\nu}}\, (\bar{D}_-\Phi^{\dagger\overline{\nu}}) \;, \tag{10}$$

where the indices μ and $\overline{\nu}$ have to be restricted so as to label directions locally (co)tangent to $\mathcal{M} \subset \mathbb{CP}^{n_1}\times\cdots\times\mathbb{CP}^{n_m}$. This is most easily performed by inserting appropriate local projection operators of the form $\mathbb{1}-[\partial P]$, where the matrix of gradients $\partial_M P^a(\Phi)$ and its conjugate serve as projections (locally on $\mathcal{M}$) from $\mathbb{CP}^{n_1}\times\cdots\times\mathbb{CP}^{n_m}$ to the directions transversal to $\mathcal{M}$.

Having identified the $\widetilde{U}(1)$-anomaly $(n{+}1)\,\boldsymbol{S}\log(\boldsymbol{S}/\mu)$ with the first Chern class of $\mathbb{CP}^n$, one expects that integration over the Φ's, for a degree-q polynomial constraint would reduce this anomaly to $(n{+}1-q)\,\boldsymbol{S}\log(\boldsymbol{S}/\mu)$. For example, if the constraint polynomial were linear, it would simply require a linear combination of Φ^{μ}'s to vanish. Integrating the remaining n Φ^{μ}'s would yield the decreased anomaly $n\,\boldsymbol{S}\log(\boldsymbol{S}/\mu)$, as expected.

In the general case, the explicit path-integration eludes us, but the anomaly contribution can be discerned by considering the transformation of the path-integral measure [10]. Alternatively, in particle physics parlance, integrating out the superfields $\Phi_i^{\mu_i}$ allows us to interpret the $\boldsymbol{\Lambda}_a$ as bound states of the charge-conjugates of those Φ's which appear in $P^a(\Phi)$ (see Ref. [11] for a situation where such an interpretation is experimentally verified), so that the so "dressed" $\boldsymbol{\Lambda}_a$'s will have charge[♯5] $-q_a^i$ with respect to $\boldsymbol{V}_i$, where q_a^i is the degree of $P^a(\Phi)$ with respect to the coordinate superfields of $\mathbb{CP}^{n_i}$. There will also appear effective propagators for $\boldsymbol{\Lambda}_a$'s and $\boldsymbol{V}_i$'s ($\boldsymbol{S}_i$'s), whence the anomaly 1-loop diagrams yield the coefficients $(n_i{+}1-\sum_{a=1}^{K} q_a^i)$ in place of $(n_i{+}1)$ in Eq. (5). This is in perfect agreement with our identification of the chiral gauge symmetry anomaly with the first Chern class of the constrained subspace $\mathcal{M}$.

[♯5] Note that, before integrating the Φ^{μ} out, the Lagrange superfields $\boldsymbol{\Lambda}_a$ in $\mathcal{A}_{\rm con.}$ must have no charge with respect to the gauge fields $\boldsymbol{V}_i$; for, if they had, they would have to interact with the $\boldsymbol{V}_i$. In supersymmetric theories this would be possible only through kinetic terms for $\boldsymbol{\Lambda}_a$'s which would contradict their rôle as Lagrange multiplier superfields.

The Calabi-Yau case. For $\mathcal{M}$ to be a Calabi-Yau space, the first Chern class of $\mathcal{M}$ has to vanish, whence [12]

$$\sum_{a=1}^{K} \deg_{\mathbb{CP}^{n_i}}\Big(P^a(\Phi)\Big) \quad = \quad n_i{+}1\ , \qquad i = 1,\ldots,m\ . \tag{11}$$

Therefore, the the chiral gauge anomaly of each $\boldsymbol{V}_i$ is completely cancelled. However, the numerical pre-factor in the effective action analogous to $\mathcal{A}''[\mathbb{CP}^n]$, Eq. (5), appears also to have been annihilated. Can it be that the anomaly cancellation efforts have been counter-productive, in that Eq. (11) not only ensures cancellation of the chiral gauge anomaly, but also kills the marginal terms in an effective action which represent the Kähler variations?

The answer clearly ought to be negative. In fact, it is not hard to see how that comes about. In any realistic situation, the couplings to (super)gravity should be included. When considering 2-dimensional field theories to describe Calabi-Yau compactification of heterotic strings, for example, the constrained (product of) $\mathbb{CP}^n$ model(s), Eqs. (8) and (9), should be coupled to (1,0)-supergravity [7]. Indeed, the (super)gravitational anomaly [13] is nothing but the well-known trace-anomaly, i.e., the central charge of the Virasoro algebra. But then, the locally (1,0)-supersymmetric version of $\mathcal{A}_{\rm kin.}{+}\mathcal{A}_{\rm con.}$ in fact must be anomalous and precisely so as to cancel the contributions of a flat σ-model representing the 4-dimensional spacetime and of the super-reparametrization ghosts. In 2-dimensional field theories, the chiral gauge-, spin-3/2 and gravitational anomalies are all carried by the same operator [13]. It follows that the effective action for Calabi-Yau compactification of heterotic strings is bound to contain the marginal terms

$$\frac{c}{4\pi}\int\!{\rm d}^2\sigma\,\Big(\int\!{\rm d}\varsigma^+\,{\rm d}\bar\varsigma^-\ \boldsymbol{S}_{CY}[\log(\boldsymbol{S}_{CY}/\mu)-1]\ +\ {\rm h.c.}\Big)\ , \tag{12}$$

where c is the central charge, $c = 9$ for (2,2)-supersymmetric σ-models with a complex 3-dimensional target space. The $\boldsymbol{S}_{CY}$ may be thought of, in analogy to Eq. (10), as

$$\boldsymbol{S}_{CY}\ \sim\ (D_+\Phi^\mu)\,G^{(CY)}_{\mu\bar\nu}\,(\bar D_-\Phi^{\dagger\bar\nu})\ , \tag{13}$$

where the same remark applies as for Eq. (10) and where $G^{(CY)}_{\mu\bar\nu}$ is the "repaired" combination of Fubini-Study metrics : From the linear combination $\sum_{i=1}^{m}(const.{+}(2\pi/f_i))(\boldsymbol{S}_i\,\text{``}{+}\text{''}\,\boldsymbol{S}_i^\dagger)$, one constructs first the Ricci-flat metric following Yau [14], and then corrects this along the lines in Ref. [15] to restore (super)conformal invariance of the complete model order by order. Thus, $\boldsymbol{S}_{CY}$ is the correspondingly "repaired" linear combination of the $\boldsymbol{S}_i$ from the $\mathbb{CP}^n$ models. This again perfectly agrees with the identification of $(\boldsymbol{S}_{CY}\,\text{``}{+}\text{''}\,\boldsymbol{S}_{CY}^\dagger)$ with the Kähler class on the Calabi-Yau space and may be thought of as the result of integrating the true, superconformal D-term $\int{\rm d}^2\varsigma\,{\rm d}^2\bar\varsigma\,K_{CY}$ over the "twisted half" of the fermionic space. Note that even after dropping the anomalous $\boldsymbol{S}\log\boldsymbol{S}$ terms (after all, the anomaly eventually cancels out), the dependence on the f_i's remains through the $\log\mu$ factor; the f_i's in fact control the dependence of $\boldsymbol{S}_{CY}$ on the $\boldsymbol{S}_i$.

That the trace-anomaly had to appear through a marginal operator should have been obvious since it is a physical observable and can not be phased away through renormalization. That the gravitational and the gauge anomalies appear through the same marginal operator is a special feature of 2-dimensional spacetime; Eq. (12) then appears merely as a "twisted chiral" re-write of the more standard expressions.

Now, by choosing the 'physical' gauge in which $\boldsymbol{V}_i = 0$ (gauge fields in 2-dimensions are unphysical) and also $\boldsymbol{\Lambda}_a = \lambda_a$ (*const.*), in favourable circumstances, we obtain the Landau-Ginzburg models which were so successfully analyzed in Ref. [1,2]. Notably, however, these models generally lack a description of the Kähler moduli fields and also the related matter fields. Recall that the Kähler (1,1)-forms of the $\mathbb{CP}^n$'s span (at least a part, but often all of) the Kähler variations of the Calabi-Yau space. The lack of Kähler variations is therefore seen as the artefact of the $\boldsymbol{V}_i = 0$, i.e., $\boldsymbol{S}_i = 0$ gauge[♯6]. Indeed, the kinetic terms are now just the standard Wess-Zumino type, which are in the generic universality class and irrelevant. Renormalization flow will therefore fix them and this is explicitly seen through the correspondences with the exactly soluble models [1,2,6] which are known to have the Kähler moduli (i.e., relative and overall "sizes") fixed.

Finally, note that world-sheet instantons are expected to contribute to the terms (12). Determining these corrections appears to be a rather interesting problem for future study, because of the following intriguing possibility : Assume that the fully corrected terms will, instead of just $\boldsymbol{S}_{CY}$, contain a polynomial $Q(\boldsymbol{S})$ and that we may ignore the anomalous "$\boldsymbol{S}\log\boldsymbol{S}$" terms. The resulting action would be strikingly similar to a Landau-Ginzburg model in which the variations of $Q(\boldsymbol{S})$ are the marginal (twisted) F-terms describing the Kähler variations of our Calabi-Yau σ-model with target space $\mathcal{M}$. Alternatively, it could be interpreted as the Landau-Ginzburg model for describing the complex structure variations of the *mirror Calabi-Yau σ-model*, one which has target space $\mathcal{W}$, such that

$$H^q(\mathcal{M},\mathcal{T}_{\mathcal{M}}) \quad\cong\quad H^q(\mathcal{W},\mathcal{T}_{\mathcal{W}}{}^*) \tag{14}$$

is an equivalence of ring structures (Yukawa and all higher couplings), not only the equality of the dimensions of the respective spaces.

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[♯6] Note that it would have made no sense to fix a gauge if the respective gauge symmetry were anomalous, i.e., if the cancellation condition (11) were not enforced. This cancellation condition, in turn, selects Calabi-Yau target spaces.

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