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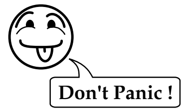

Mathematical Methods II

Exam Midterm 1; 2010, Feb. 22. Student: ______________________

This is an "open Textbook (Arfken), open lecture notes" exam. For full credit, show all your work. The part of your solutions completed in class staple to the question sheet; then complete the *rest* of the Exam and hand it in **by Wednesday, 02/24/10, 5:00 pm**, for 2/3 of the indicated credit. **Budget your time:** first do what you are sure you know how; use short-cuts whenever possible (but be prepared to explain them afterwards, if necessary).

1. Solve the differential equation [=15*pt*]

$$\frac{\mathrm{d}x}{\mathrm{d}t} + e^{t/t_0 - x/x_0} = \frac{x_0}{t_0}\;, \qquad x_0, t_0 \neq 0\;.$$

b. In your solution, take carefully the limit $x_0 \to 0$, and verify that this solves the $x_0 \to 0$ limit of the equation itself. [=10*pt*]

Hint: *Try the "separation" method, or a judicious change of variables x, t. You may benefit from using L'Hospital's rule. I haven't collected the material in "Know Thy Math" in vain, I hope.*

2. Solve the differential equation [=15*pt*]

$$(\dot{x})^2 + 2g_0\Big(t\,\big[\dot{x} + v_0\big(e^{\dot{x}/v_0} + 1\big)\big] - x(t)\Big) = 0\;, \qquad g_0, v_0 \neq 0\;.$$

b. In your solution, take carefully the limit $v_0 \to 0$, and verify that this solves the $v_0 \to 0$ limit of the equation itself. [=10*pt*]

Hint: *See the hint to the previous problem. Also, $Li_{s+1}(z) := \int_0^z \frac{\mathrm{d}t}{t} Li_s(z)$ is a recursive definition of a class of special functions, where $Li_0(z) = \frac{z}{1-z}$ and $Li_1(z) = -\ln(1-z)$, but where $Li_s(z)$ for $s > 1$ cannot be expressed in terms of elementary functions without integration.*

3. Consider solving the differential equation $x^2y'' + (\alpha x - \beta)y' - \gamma y = 0$, in the series form, $y = \sum_{k=0}^{\infty} c_k x^{k+s}$.

a. Calculate the possible value(s) of s; [=5*pt*]

b. Calculate the recursion relation for the c_k's. [=10*pt*]

c. Determine if the resulting series converges. Explain why (not). [=5*pt*]

d. Determine a α, β, γ for which this solution (nevertheless) does make sense. Explain. [=10*pt*]

4. Given that $y(x) = \sqrt{x}$ solves $2x^2y'' + 5xy' - 2y = 0$, find and verify the second solution. [=10*pt*]

5. Consider the 2nd order ordinary differential equation $f''_\alpha - \sqrt{x+1}\, f'_\alpha + (x+\alpha)f_\alpha = 0$.

a. Modify it, so as to become self-adjoint; cf. p. 623 and Eq. (10.7). [=10*pt*]

b. Once so rewritten, compare with Eq. (10.8) and identify α as the eigenvalue; read off $\mathcal{L}$, the weight-function $w(x)$, and check if the interval $x \in [-1, 1]$ is compatible with Eq. (10.25) and self-adjointness, *i.e.*, Hermiticity. See § 10.2. [=10*pt*]

c. Write down the orthogonality relation for the $f_\alpha(x)$, in the manner of Eq. (10.36). [=5*pt*]

Hint: *The last two parts are bonus—for reading the text beyond what was covered in class.*