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Mathematical Methods II 14th April '10.

The Final Exam

Instructor: T. Hübsch ______________________ (Student name and ID)

This is an "open Textbook (Arfken), open lecture notes and handouts" take-home exam, due by **12:00 noon of Wednesday, 04/21/10**. For full credit, **show all your work**. If you cannot complete one part of a calculation, a clear description of the procedure/method will still earn you partial credit. No collaboration or consultation is allowed, and you may rely only on results from the textbook, lecture notes and class-handouts.

1. For the *inhomogeneous* differential equation $xy'' - 2y' - \alpha xy = x$:

a. Find each singular point and whether it is essential or nonessential (regular). [5+5pt]

b. Find a power-series solution $y_{\alpha,h}(x)$ of the *homogeneous* equation and specify the possible choices of α for which this solution makes sense. [5+5pt]

c. Find the associated self-adjoint homogeneous equation and a domain $x \in [x_1, x_2]$ within which the associated differential operator is self-adjoint, and specify the orthogonality relation for the solutions $y_\alpha(x)$. [3×5pt]

d. Specify the singularity structure expected of the "second" homogeneous solution, as defined in Arfken's general formula (9.128). [5pt]

e. Find a low integral value of α for which the particular solution of the above *inhomogeneous* equation is a low-degree polynomial; find this particular solution. [5+5pt]

(Recall: if y_p is a particular solution and $y_{h,1}, y_{h,2}$ two independent solutions of the homogeneous part, the general solution of the inhomogeneous equation is $y_p + C_1 y_{h,1} + C_2 y_{h,2}$.)

2. Solve the diffusion equation $\frac{\partial}{\partial t}T = k\vec{\nabla}^2 T$ by (a) separating variables, (b) using standard solutions and (c) fitting the boundary conditions. $T(\vec{r}, t)$ is confined within a quarter-spherical cavity and vanishes on: the flat vertical walls at (1) $\varphi = 0$ and (2) $\varphi = 180°$ and (3) the flat horizontal floor at $\theta = 90°$, and (4) the quarter-spherical dome at $r = a$, respectively. Alternatively [10pt, *bonus*], solve this using the Fourier transform. [4×5pt]

3. Consider the function

$$f(x) = \begin{cases} x^2 - 1\ , & \text{for } -1 \le x < 1, \\ 4x - x^2 - 3\ , & \text{for } 1 < x \le 3, \\ 0\ , & \text{otherwise, within } x \in (-\infty, +\infty). \end{cases}$$

a. Find a Fourer *series* representation for $f(x)$. [5pt]

b. Find a Fourer *integral* representation for $f(x)$. [5pt]

(Sketch the function first; specify any additional conditions as may be necessary to solve either of the two tasks, and explain why.)

4. Using Laplace transformation, solve the system of differential equations

$$\dot{C}_1(t) = -C_1(t) + C_2(t)\ , \qquad C_1(0) = 1\ ,$$

$$\dot{C}_2(t) = -C_1(t) + C_2(t) - C_3(t)\ , \qquad C_2(0) = 0\ ,$$

$$\dot{C}_3(t) = +C_2(t) - C_3(t)\ , \qquad C_3(0) = 0\ ,$$

subject to the specified initial conditions, and where the over-dot denotes a derivative by time, t. [4×5pt]