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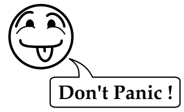

Mathematical Methods I
Midterm 1: 2010, Oct. 4.

Student: ____________________

This is an "open Textbook (Arfken & Webber), open lecture notes" in-class exam. For full credit, show all your work. The part of your solutions completed in class staple to the question sheet; then complete the *rest* of the Exam and hand it in **by Friday, 10/08/10, 5:00 pm, in the Main Office**, for 2/3 of the indicated credit. **Budget your time:** do first what you are sure you know how; use shortcuts (but be prepared to explain them afterwards).

1. Given a vector $\vec{A} = \sin(\varphi)\,\hat{e}_z$, so specified in cylindrical coordinates,

a. Calculate $\vec{\nabla}\cdot\vec{A}$. [=5*pt*]

b. Calculate $\vec{\nabla}\times\vec{A}$. [=5*pt*]

c. Calculate the three components of $\vec{\nabla}^2\vec{A}$. [=20*pt*]

2. Calculate $\oint_S \mathrm{d}\vec{\sigma} \times \big(\hat{z}\,(x^2+y^2)^n\big)$ for $n \in \mathbb{Z}$, where S is a pill-box of radius R and height H, body-centered at the origin:

a. by performing the surface integral directly; [=10*pt*]

b. upon applying an appropriate integration/derivative identity. [=10*pt*]

c. Is any value of n exceptional? Explain. [=5*pt*]

(Hint: changing into cylindrical coordinates first should simplify calculations significantly.)

3. Consider a (generalized) coordinate system (ξ, η, ζ) which is related to the Cartesian system (x, y, z) through the relations

$$x = \xi\eta, \quad y = \tfrac{1}{2}(\eta^2 - \xi^2), \quad z = \zeta\ .$$

a. Calculate the (inverse) transformation matrix $\mathbb{J} = \frac{\partial(x,y,z)}{\partial(\xi,\eta,\zeta)}$. [=10*pt*]

b. Calculate the metric, $g_{ij}(\xi, \eta, \zeta)$, for the (ξ, η, ζ) coordinate system. [=10*pt*]

c. Determine if the (ξ, η, ζ) system is orthogonal or not. Explain. [=10*pt*]

d. State the relationship between $\mathbb{J}$ and the matrix $[g_{ij}(\xi, \eta, \zeta)]$. [=5*pt*]

4. For $i, j = 1, 2, 3$, A_i, B^j are components of a covariant and a contravariant vector, and C^{kl} are the components of a type-$(2, 0)$ tensor.

a. Determine the (tensorial) transformation properties of $(A_i C^{ij} B^k)$. [=5*pt*]

b. Determine the (tensorial) transformation properties of $\varepsilon^{ijk} A_i \big(\frac{\partial B^m}{\partial x^j}\big)$ with respect to *general* coordinate changes. [=5*pt*]

c. Write down two *algebraically* independent general coordinate transformation scalars (invariants) constructed only from the components A_i, B^j and C^{kl}. [=10*pt*]

d. How many independent components does the set of quantities $\varepsilon_{ijk} B^i C^{j\ell} A_\ell$ represent? [=10*pt*]

(Note: the Einstein (implicit) summation convention is in effect: indices that are repeated once as a subscript and once as a superscript are being summed over.)