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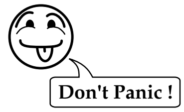

Mathematical Methods I

Final: 2010, Nov. 29th. Student: ______________________

This is an "open Textbook (Arfken & Weber), open lecture notes" take-home exam, **due 5:00 PM of Monday, 6th Dec. '10**. For full credit, show all your work. If you cannot complete one part of a calculation, a clear description of the procedure/method will still earn you partial credit. Neither collaboration nor consultation is allowed, but you may quote—with full reference—any *published* source for intermediate results that you may use.

1. For $f(\vec{r})$ and $g(\vec{r})$ two analytic functions of their respective arguments, calculate the integrals [=10+10pt]

$$\oint_S \mathrm{d}^2\vec{\sigma} \times \big(\vec{\nabla} f(\vec{r})\big) , \qquad \text{and} \qquad \oint_C \mathrm{d}\vec{r} \cdot \big(\vec{r}\, g(\vec{r})\big) ,$$

where S is the surface of the tetrahedron with vertices at (0,0,0), (1,0,0), (0,1,0) and (0,0,1) in a cartesian coordinate system, and C is the unit circle in the x-z plane centered at the origin.

2. Given are constant and perpendicular electric and a magnetic fields, so $\vec{E}\cdot\vec{B} = 0$.

a. Compute $\frac{1}{2}F_{\mu\nu}F^{\mu\nu}$ and $\frac{1}{4}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$. [=10pt]

b. Determine the velocity for a Lorentz transformation, Eq. (4.160)–(4.161), such that in the transformed system $\vec{B}' = 0$. [=10pt]

c. Determine the condition on $F_{\mu\nu}F^{\mu\nu}$ that prevents your solution to part ***b*** from contradicting special relativity (that no physical system may move faster than the speed of light in vacuum). [=5pt]

3. Consider the matrix $\mathbb{A} = \begin{bmatrix} 4 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & a \end{bmatrix}$.

a. Determine a so that at least one eigenvalue of $\mathbb{A}$ vanishes. (Should be an integer.) [=5pt]

Hint: Which easily calculable numerical characteristic of the matrix $\mathbb{A}$ *is a product of the eigenvalues?*

b. Compute all eigenvalues of $\mathbb{A}$ for the a you found in part ***a***. [=10pt]

c. Compute all normalized eigenvectors of $\mathbb{A}$ for that same a. [=15pt]

d. Compute $\sqrt[3]{\mathbb{A}}$, *i.e.*, find *a* matrix $\mathbb{M}$, such that $\mathbb{M}^3 = \mathbb{A}$. [=15pt]

4. Determine the radius of convergence of $\displaystyle\sum_{k=0}^{\infty} \frac{\Gamma(nk)\, x^k}{\Gamma(k)\,\Gamma(k+\frac{1}{3})\,\Gamma(k+\frac{2}{3})}$, depending on the positive integer n, or at least for the case $n = 3$. [=20pt]

5. Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{\mathrm{d}x\; e^{-i\alpha x}}{1+x^3}$ and $\displaystyle\int_0^{\infty} \frac{\mathrm{d}x\; x^4}{1-x^5}$ using contour integration, with $\alpha > 0$. [=15+15pt]

Hint: First find the poles of the integrand; then close the integral in the upper or lower complex plane—wherever the integrand ***decays*** *exponentially; if there are poles along the contour, determine the detour; evaluate the closed contour integral by summing the enclosed residues; finally, sort out the value of the above integrals.*