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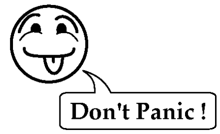

Mathematical Methods II
2nd Midterm Exam

20th March '98.

Instructor: T. Hübsch

______________________ (Student name and ID)

This is an "open Textbook (Arfken), open lecture notes" exam. For full credit, show all your work. Hand in the part completed in class by Fri., 3/20/98, 10:00 am; **complete** the rest by end of the **Wed., 4/2/98, 9:00 am** class. The take-home part **supplements and supersedes** the in-class part, but is valued at 2/3 of the points. Budget your time: first do what you are sure you know how; use short-cuts whenever possible (but be prepared to explain them afterwards).

1. The generating function $g(x,t) = e^{x(t+x/2t)} \stackrel{\text{def}}{=} \sum_{n=0}^{\infty} A_n(x)t^n$, defines the $A_n(x)$.

a. Read off the differential and the contour-integral representation for $A_n(x)$. [5+5*pt*]

b. Determine the series representation for $A_n(x)$. [5*pt*]

c. Derive two recursion relations, by operating with $\frac{\partial}{\partial t}$ and with $\frac{\partial}{\partial x}$. [5+5*pt*]

d. Derive the differential equation which the $A_n(x)$ satisfy. [10*pt*]

2. An infinite cylindrical shell of radius a is kept at the potential $V(r{=}a,\phi,z){=}V_0\sin(2\phi)$. Knowing that $\vec{\nabla}^2 V{=}0$, with no other charges or fields present,

a. write down the general solution for the potential $V(r,\phi,z)$ both inside $(r<a)$ and outside $(r>a)$ the shell; [10*pt*]

b. determine all constants and summations from the boundary and periodicity conditions. [10*pt*]

(Hint: $V(r,\phi,z){<}\infty$ at $r=0$, and $(r\,V\frac{\partial}{\partial r}V){<}\infty$ as $r\to\infty$. Trig identities may be useful.)

3. The vibrations (V) of a perfectly elastic jelly contained in a rigid sphere of radius a, wherein it cannot slip, obeys the equation: $\big[\vec{\nabla}^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\big]h = 0$.

a. Write the general solution as $V(r,\theta,\phi,t) = \sum_{p,q,s} c_{p,q,s} R_{p,q}(r) F_{q,s}(\theta,\phi) e^{i\omega t}$, and determine the general solutions $R_{p,q}(r)$ and $F_{q,s}(\theta,\phi)$. [10*pt*]

b. Impose the natural boundary conditions (no radial or angular displacement at the containing sphere) and specify the ranges of the p,q,s-summations. [10*pt*]

c. Determine the list of frequencies ω in terms of the zeros of $R_{p,q}$ and $F_{q,s}$, and determine the lowest frequency *exactly*. [10*pt*]

d. Is it possible to hear the type of vibration? That is, does knowing only the frequency specify the function $R_{p,q}F_{q,s}e^{i\omega t}$ *unambiguously*? [10*pt*]

e. Two vertical non-slip partitions are inserted, crossing at the vertical symmetry axis and at an angle of $2\pi/N$ between them; N is an integer. Determine the effect of this modification on the allowed values of p,q,s and the frequencies ω. [10*pt*]

(Hint: Compare the present problem with the example in § 11.7.

4. Find the (sin/cos) Fourier expansion of $f(x) = 1 - (\frac{x}{\pi})^2$, for $|x| < \pi$. [15*pt*]

(Hint: Sketch the function first; then determine if any of the coefficients vanish by symmetry; then determine the nonzero coefficients.)