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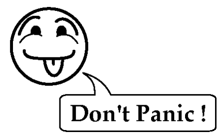

Mathematical Methods II 2nd Feb. '98.

1st Midterm Exam

Instructor: T. Hübsch ______________________ (Student name and ID)

This is an "open Textbook (Arfken), open lecture notes" exam. For full credit, show all your work. The part completed in class staple to the question sheet; complete the rest and hand it in **by Friday, 2/6/98, 3:00 pm**. Budget your time: first do what you are sure you know how; use short-cuts whenever possible (but be prepared to explain them afterwards, if necessary).

1. Consider a modification of Boyle's (differential) gas law:

$$\frac{\mathrm{d}V}{\mathrm{d}P} = -\Big(\frac{V}{P}\Big)^{\alpha}~, \qquad \alpha > 0~.$$

a. Solve the above equation for $\alpha \neq 1$ and obtain the algebraic combination of P and V which equals a constant. [=15*pt*]

b. In the solution to part a, take carefully the limit $\alpha \to 1$, and obtain the familiar Boyle gas law [see p.441]. (Hint: Recall L'Hospital's rule.) [=15*pt*]

2. Consider solving the differential equation $x^3y'' - 2\alpha xy' - \beta(1+x)y = 0$, in the series form, $y = \sum_{k=0}^{\infty} c_k x^{k+s}$.

a. Calculate the possible value(s) of s; [=5*pt*]

b. Calculate the recursion relation for the c_k's. [=10*pt*]

c. Determine if the resulting series converges. Explain why (not). [=10*pt*]

d. Determine a choice of α, β for which this solution nevertheless makes sense. [=10*pt*]

3. Given that $y(x) = x^2$ solves $x^2y'' - 2y = 0$, find and verify the second solution. [=10*pt*]

4. a. Modify $\sqrt{1+x}\,\frac{\mathrm{d}^2 f_\alpha}{\mathrm{d}x^2} + \frac{1}{\sqrt{1+x}}\frac{\mathrm{d}f_\alpha}{\mathrm{d}\theta} + \alpha f_\alpha = 0$ so as to become self-adjoint. [=5*pt*]

b. Read off the eigenvalue, the weight-function $w(x)$, and check if the limits $x_1 = -1$ and $x_2 = 1$ are compatible with self-adjointness. [=10*pt*]

c. Write down the orthogonality relation for the $f_\alpha(x)$. [=10*pt*]

(Caution: Be careful with the signs, and recall the Wronskian.)

5. With $w(x) = e^{-x}$ for $0 \leq x \leq +\infty$ and $p_0(x) = 1$, construct the next two orthogonal polynomials: $p_1(x)$—linear, and $p_2(x)$—quadratic in x. That is, with the scalar product $\langle p_i | p_j \rangle \stackrel{\text{def}}{=} \int_0^\infty \mathrm{d}x\, w(x)\, p_i^* p_j$, determine $p_1(x)$ and $p_2(x)$ up to overall constants so that they are all mutually orthogonal. (You need not normalize them!). [=30*pt*]