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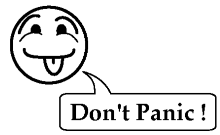

Mathematical Methods II 17th April '98.

The Final Exam

Instructor: T. Hübsch

________________________ (Student name and ID)

This is an "open Textbook (Arfken), open lecture notes and handouts" take-home exam, due by 5:00 p.m. of Monday, 27th April '98. For full credit, **show all your work**. If you cannot complete one part of a calculation, a clear description of the procedure/method will still earn you partial credit. No collaboration or consultation is allowed, and you may rely only on results from the textbook, lecture notes and class-handouts.

1. For the *inhomogeneous* differential equation $x^3y'' + (x^2{+}2)y' - \alpha xy = x$:

a. Find each singular point and whether it is essential or nonessential. [7+7*pt*]

b. Find a power-series solution of the homogeneous equation and specify the possible choices of α for which this solution makes sense. [7+7*pt*]

c. Find the associated self-adjoint homogeneous equation, determine the range $x_1 \leq x \leq x_2$ within which the associated differential operator is self-adjoint and specify the orthogonality relation for the solutions y_α. [7+7+7*pt*]

d. Specify the singularity structure expected of the "second" homogeneous solution, as defined in Arfken's (8.127). [7*pt*]

e. Find a low integral value of α for which the particular solution of the above *inhomogeneous* equation is a low-degree polynomial; find this particular solution. [7+7*pt*]

(Recall: if y_p is a particular solution and $y_{h,1}, y_{h,2}$ two independent solutions of the homogeneous part, the general solution of the inhomogeneous equation is $y_p + C_1 y_{h,1} + C_2 y_{h,2}$.)

2. Solve the diffusion equation $\frac{\partial}{\partial t}T = k\vec{\nabla}^2 T$ by separating variables, using standard solutions and fitting the boundary conditions. $T(\vec{r}, t)$ is confined within a quarter-spherical cavity (1st and 2nd octant) and vanishes on: the flat vertical walls at $\varphi = 0$ and $\varphi = 180°$, the flat horizontal floor at $\theta = 90°$, and the curved dome at $r = a$, respectively. [4×7*pt*]

3. Using Laplace transformation, solve the system of differential equations

$$\frac{\mathrm{d}}{\mathrm{d}t}C_1 = -C_1 + 2C_2\ , \qquad C_1(0) = 0\ ,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}C_2 = C_1 - C_2 + 2C_3\ , \qquad C_2(0) = 0\ ,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}C_3 = +2C_2 + C_3\ , \qquad C_3(0) = 1\ ,$$

subject to the specified initial conditions. [4×7*pt*]