Using the Laplace Transform

A fairly general formula for the $f(t) \leftrightarrow \widetilde{f}(s) := \mathscr{L}\{f(t)\}$ relation:

$$\widetilde{f}(s) = \sum_{\substack{i \\ z_i,\,(s-a_i)>0}} \frac{c_i}{(s-a_i)^{z_i}} + \sum_j \left[\frac{b_j\,(s-e_j)}{(s-e_j)^2 \pm \omega_j^2} + \frac{d_j\,\Omega_j}{(s-f_j)^2 \pm \Omega_j^2}\right], \tag{1}$$

$$\updownarrow$$

$$f(t) = \sum_{\substack{i \\ z_i,\,(s-a_i)>0}} \frac{c_i}{\Gamma(z_i)} e^{a_i t}\, t^{z_i - 1} + \sum_j \left[b_j\, e^{e_j t} \begin{Bmatrix} \cos(\omega_j t) \\ \cosh(\omega_j t) \end{Bmatrix} + d_j\, e^{f_j t} \begin{Bmatrix} \sin(\Omega_j t) \\ \sinh(\Omega_j t) \end{Bmatrix}\right]. \tag{2}$$

Make sure you check it completely!

Consider then the differential system

$$\dot{C}_1(t) = C_2(t) - C_1(t), \tag{3}$$
$$\dot{C}_2(t) = -C_1(t) + C_2(t) - C_3(t), \tag{4}$$
$$\dot{C}_3(t) = C_2(t) - C_3(t). \tag{5}$$

The Laplace transform of this system is

$$s\widetilde{C}_1(s) - C_1(0) = \widetilde{C}_2(t) - \widetilde{C}_1(t), \tag{6}$$
$$s\widetilde{C}_2(s) - C_2(0) = -\widetilde{C}_1(t) + \widetilde{C}_2(t) - \widetilde{C}_3(t), \tag{7}$$
$$s\widetilde{C}_3(s) - C_1(0) = \widetilde{C}_2(t) - \widetilde{C}_3(t), \tag{8}$$

or

$$\begin{bmatrix} (s{+}1) & -1 & 0 \\ 1 & (s{-}1) & 1 \\ 0 & -1 & (s{+}1) \end{bmatrix} \begin{bmatrix} \widetilde{C}_1 \\ \widetilde{C}_2 \\ \widetilde{C}_3 \end{bmatrix} = \begin{bmatrix} C_1(0) \\ C_2(0) \\ C_3(0) \end{bmatrix}, \tag{9}$$

so that the determinant of the system is

$$\det \begin{bmatrix} (s{+}1) & -1 & 0 \\ 1 & (s{-}1) & 1 \\ 0 & -1 & (s{+}1) \end{bmatrix} = s^3 + s^2 + s + 1 = (s+1)(s^2+1). \tag{10}$$

With this, the solution for $\widetilde{C}_1$ is

$$\widetilde{C}_1 = \frac{\det \begin{bmatrix} 1 & -1 & 0 \\ 0 & (s{-}1) & 1 \\ 0 & -1 & (s{+}1) \end{bmatrix}}{\det \begin{bmatrix} (s{+}1) & -1 & 0 \\ 1 & (s{-}1) & 1 \\ 0 & -1 & (s{+}1) \end{bmatrix}} = \frac{s^2}{(s+1)(s^2+1)} = \frac{1}{2}\left[\frac{1}{s+1} + \frac{s}{s^2+1} - \frac{1}{s^2+1}\right], \tag{11}$$

so that $C_1(t) = \frac{1}{2}\left[e^{-t} + \cos(t) - \sin(t)\right]$.

The solution for $\widetilde{C}_2$ is

$$\widetilde{C}_2 = \frac{\det\begin{bmatrix}(s{+}1) & 1 & 0\\ 1 & 0 & 1\\ 0 & 0 & (s{+}1)\end{bmatrix}}{\det\begin{bmatrix}(s{+}1) & -1 & 0\\ 1 & (s{-}1) & 1\\ 0 & -1 & (s{+}1)\end{bmatrix}} = -\frac{s+1}{(s+1)(s^2+1)} = -\frac{1}{s^2+1}, \tag{12}$$

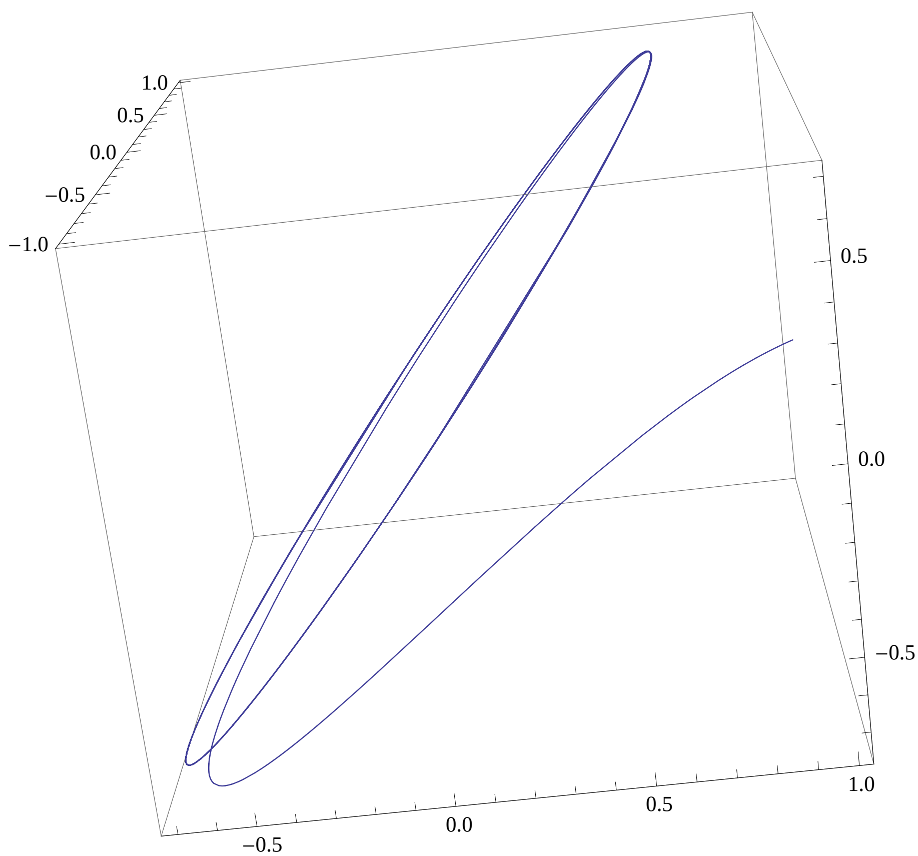
so that $C_2(t) = -\sin(t)$.

Finally, the solution for $\widetilde{C}_3$ is

$$\widetilde{C}_3 = \frac{\det\begin{bmatrix}(s{+}1) & -1 & 1\\ 1 & (s{-}1) & 0\\ 0 & -1 & 0\end{bmatrix}}{\det\begin{bmatrix}(s{+}1) & -1 & 0\\ 1 & (s{-}1) & 1\\ 0 & -1 & (s{+}1)\end{bmatrix}} = \frac{-1}{(s+1)(s^2+1)} = \frac{1}{2}\Big[\frac{-1}{s+1} + \frac{s}{s^2+1} - \frac{1}{s^2+1}\Big], \tag{13}$$

and $C_3(t) = \frac{1}{2}\big[-e^{-t} + \cos(t) - \sin(t)\big]$.

The resulting "flow" in the (C_1, C_2, C_3)-space is depicted below:



It starts far right, at $(1, 0, 0)$, and ends up quickly but asymptotically approaching the tilted loop.