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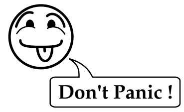

Mathematical Methods II

Exam Midterm 1; 2010, March 29. Student: ____________________

This is an "open Textbook (Arfken), open lecture notes" exam. For full credit, show all your work. The part of your solutions completed in class staple to the question sheet; then complete the *rest* of the Exam and hand it in **by Monday, 04/05/10, 5:00 pm**, for 2/3 of the indicated credit. **Budget your time:** first do what you are sure you know how; use short-cuts whenever possible (but be prepared to explain them afterwards, if necessary).

1. The generating function $g(x,t) = e^{x(t^2-x)/t} \stackrel{\text{def}}{=} \sum_{n=-\infty}^{\infty} A_n(x) t^n$, defines the $A_n(x)$.

a. Read off the differential and the contour-integral representations for $A_n(x)$. [=5+5*pt*]

b. Determine the series representation for $A_n(x)$; carefully define the ranges and the values of the summation indices. [=10*pt*]

c. Derive two recursion relations, by operating with $\frac{\partial}{\partial t}$ and with $\frac{\partial}{\partial x}$. [=5+5*pt*]

d. Combining the recursion relations, their shifts in n and/or their derivatives, derive the differential equation which the $A_n(x)$ satisfy. [=20*pt*]

e. Can your result from part d. be transformed into a self-adjoint differential equation, with an n-independent self-adjoint differential operator, so the eigenvalue would be a function of n? Justify your answer by explicit computation. [=5*pt*]

2. Consider a function $f(x)$, that is expanded in a Legendre series, $f(x) = \sum_{n=0}^{\infty} a_n P_n(x)$.

a. Prove that [=10*pt*]

$$\int_{-1}^{1} \mathrm{d}x \; [f(x)]^2 \;=\; \sum_{n=0}^{\infty} \frac{2a_n^2}{2n+1} \; .$$

b. Use your result from part a., to evaluate $\int_{-1}^{1} \mathrm{d}x \; (35x^4 + 15x^3 - 24x^2 - 7x + 2)^2$. [=10+5*pt*]

Hint: *First determine the nonzero coefficients in the Legendre expansion of* $f(x) = 35x^4 + 15x^3 - 24x^2 - 7x + 2$ *by comparing with a table of the first eight Legendre polynomials. Check your results.*

3. Consider the function

$$f(x) = \begin{cases} (1 - \frac{3|x|}{\pi}) \; , & \text{for } |x| \le \frac{\pi}{2}, \\ 0 \; , & \text{for } \frac{\pi}{2} \le |x| \le \pi. \end{cases}$$

a. Find the (sin/cos, *i.e.*, real) Fourier expansion of $f(x)$. [=10*pt*]

b. Compute the c_k's in the complex-exponential Fourer series of the same function. [=10*pt*]

c. Calculate the first derivative of the obtained series, and prove that it equals $f'(x)$. [=10*pt*]

Hint: *Sketch the function first; then determine if any of the coefficients vanish by symmetry; then determine the nonzero coefficients.)*