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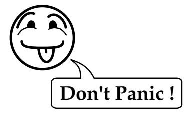
Mathematical Methods I
2nd Midterm Exam: 2010, Nov. 17



Student: __________________________

This is an "open Textbook (Arfken), open lecture notes" 90-min. in-class exam. For full credit, show all your work. The part of your solutions completed in class staple to the question sheet; then complete the *rest* of the Exam **by Wednesday, Nov. 22, 2010, 5:00 pm**, for 2/3 of the indicated credit; by signing, you attest that you have abided by this rule. **Budget your time:** do first what you are sure you know how; use shortcuts whenever possible (but be prepared to explain them afterwards, if necessary).

1. Equip the collection of real numbers in the interval $(-1,1)$ with the binary operation

$$v * v' = \frac{v+v'}{1+v\,v'}, \qquad v,v' \in (-1,1)\ . \tag{1}$$

a. Prove that $G = \{v \in (-1,1); *\}$ forms a group. [=10pt]

b. Explore the possibility of extending the group structure to thelimiting cases $(v.v') = (1,1)$, $(1,-1)$ and $(-1,-1)$, which by the $v \leftrightarrow v'$ symmetry will also include the $(-1,1)$ case. [=10pt]

2. Consider the equilateral triangle in the (x,y)-plane with vertices at $(-\frac{1}{2},0),(\frac{1}{2},0),(0,\frac{\sqrt{3}}{2})$.

a. List *all* the symmetries of this triangle. [=10pt]

b. Construct the multiplication table of rotational symmetries and prove that they form a group. [=10pt]

c. Construct the complete multiplication table of symmetries and prove that they form a group. [=10pt]

3. Determine the convergence (absolute?, conditional?, uniform? — all that are appropriate) of:

a. Test $S \stackrel{\text{def}}{=} \sum_{n=0}^{\infty} \frac{(-1)^n\,n^3}{(1-n^3)}$. [=10pt]

b. Test $S(x) \stackrel{\text{def}}{=} \sum_{n=0}^{\infty} \frac{x^n\,(x+n)^{2x}}{(2n)!}$ for $x \geq 0$, and specify the range/interval/radius of convergence. [=10pt]

c. Is the sum $\sum_{k=0}^{\infty}(-2)^k$ summable (in Hardy's "Pickwickian" sense)? If so, what is its value? [=10pt]

4. Consider the power series $S(x) \stackrel{\text{def}}{=} \sum_{k=0}^{\infty} \frac{x^k}{(k^2+1)(k+3)}$,

a. Determine the range and rate of convergence, [=10pt]

b. Improve the rate of convergence by at least one order. [=10pt]

5. Bonus Problem: Consider the matrix $\mathbb{M} = \left[\begin{smallmatrix} 2 & 3 \\ 4 & 3 \end{smallmatrix}\right]$.

a. Determine characteristic polynomial and eigenvalues of $\mathbb{M}$. [=10pt]

b. Determine the corresponding eigenvectors of $\mathbb{M}$. [=10pt]

c. Calculate $\sqrt{\mathbb{M}}$, *i.e.*, a matrix the square of which equals $\mathbb{M}$. [=10pt]