product	**property**	**index not'n (*Cartesian!*)**	**physical meaning**
$\vec{A}\cdot\vec{B}$	$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$, scalar	$= A^i B_i$	projection of $\vec{A}$ on $\vec{B}$ and *vice versa*
$\vec{A}\times\vec{B}$	$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$, vector: $\perp \vec{A}, \vec{B}$	$= A_i B_j \,\varepsilon^{ijk}\, \hat{e}_k$	$\propto$ area of the rhombus spanned by $\vec{A}, \vec{B}$
$\vec{A}\cdot(\vec{B}\times\vec{C})$	$\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$	$= A_i B_j C_k \,\varepsilon^{ijk}$	volume of the rhomboid spanned by $\vec{A}, \vec{B}, \vec{C}$
$\vec{\nabla}$	vectorial derivative (nabla) *operator*	$= \hat{e}^i \frac{\partial}{\partial x^i}$	rate of change in 3-space
$(\vec{\nabla} f)$		$= \hat{e}^i \frac{\partial f}{\partial x^i}$	gradient of the function f
$\vec{\nabla}\cdot\vec{A}$	$\vec{\nabla}\cdot(\vec{A} f) = (\vec{\nabla}\cdot\vec{A})\, f + \vec{A}\cdot(\vec{\nabla} f)$	$= \frac{\partial A^i}{\partial x^i}$	divergence of the vector $\vec{A}$; … of $(\vec{A} f)$
$\vec{A}\cdot\vec{\nabla}$	$\vec{A}\cdot\vec{\nabla} = \vec{\nabla}\cdot\vec{A}$ only if $(\vec{\nabla}\cdot\vec{A}) = 0$	$= A^i \frac{\partial}{\partial x^i}$	gradient along the vector $\vec{A}$
$\vec{\nabla}\times\vec{A}$	$\vec{\nabla}\times(\vec{A} f) = (\vec{\nabla}\times\vec{A})\, f + \vec{A}\times(\vec{\nabla} f)$	$= \frac{\partial A_j}{\partial x^i} \varepsilon^{ijk} \hat{e}_k$	curl of the vector $\vec{A}$; … of $(\vec{A} f)$
$\vec{A}\times\vec{\nabla}$	$\vec{A}\times\vec{\nabla} = -\vec{\nabla}\times\vec{A}$ only if $(\vec{\nabla}\times\vec{A}) = 0$	$= \hat{e}_i \,\varepsilon^{ijk}\, A_j \frac{\partial}{\partial x^k}$	gradient perpendicular the vector $\vec{A}$
$\vec{\nabla}^2 f$	$= (\vec{\nabla}\cdot\vec{\nabla} f)$	$= \delta^{ij} \frac{\partial^2 f}{\partial x^i \partial x^j} = \sum_{i=1}^{3} \frac{\partial^2 f}{\partial x^i \partial x^i}$	Laplacian of f
$(\vec{\nabla}\times\vec{\nabla} f)$	$\equiv 0$	$= \frac{\partial^2 f}{\partial x^i \partial x^j} \,\varepsilon^{ijk}\, \hat{e}_k$	none
$\vec{\nabla}(\vec{\nabla}\cdot\vec{A})$	$= \vec{\nabla}\times(\vec{\nabla}\times\vec{A}) + \vec{\nabla}^2\vec{A}$	$= \frac{\partial^2 A^j}{\partial x^i \partial x^j}\, \hat{e}^i$	nothing in particular
$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A})$	$\equiv 0$	$= \frac{\partial^2 A_k}{\partial x^i \partial x^j}\, \varepsilon^{ijk}$	none
$\vec{\nabla}\times(\vec{\nabla}\times\vec{A})$	$= \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \vec{\nabla}^2\vec{A}$	$= \frac{\partial^2 A_j}{\partial x^l \partial x^i}\, \varepsilon^{ijk} \delta_{km} \varepsilon^{lmn}\, \hat{e}_n$	"curlcurl" of $\vec{A}$

$$\varepsilon_{ijk},\ \varepsilon^{ijk} := \begin{cases} +1 & \text{if } i,j,k \text{ is and even permutation of } 1,2,3, \\ -1 & \text{if } i,j,k \text{ is and odd permutation of } 1,2,3, \\ \ \ 0 & \text{otherwise (if two or more of } i,j,k \text{ are equal);} \end{cases}$$

$$\begin{aligned} \varepsilon^{ijk}\delta_{kl}\varepsilon^{lmn} &= \delta^{im}\delta^{jn} - \delta^{in}\delta^{jm} \ ; \\ \varepsilon^{ijk}\varepsilon_{klm} &= \delta^i_l \delta^j_m - \delta^i_m \delta^j_l \ ; \end{aligned}$$

$$\delta_{ij} := \begin{cases} 1 & \text{if } i = j, \\ 0 & \text{if } i \neq j. \end{cases}$$