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Mathematical Methods I
2nd Midterm Exam

17th Nov. '97.

Instructor: T. Hübsch

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(Student name and ID)

This is an "open Textbook (Arfken), open lecture notes" exam. For full credit, show all your work. If you cannot complete one part of a calculation, a clear description of the procedure/method will still earn you partial credit. **Budget your time**: first do what you are sure you know how; use short-cuts whenever possible (but be prepared to explain them afterwards, if necessary).

1. Test for convergence (absolute?, conditional?, uniform? — as appropriate, and for which x?): [25+25pt]

$$\sum_{n=1}^{\infty}(-1)^n \frac{n^3}{1+n^3}\ , \qquad\qquad \sum_{n=0}^{\infty}\frac{n^x}{n!}\ .$$

2. Determine the rate of convergence of the following sum, and then improve it: [10+20pt]

$$S \stackrel{\text{def}}{=} \sum_{k=0}^{\infty}\frac{1}{(k^2+1)}\ .$$

(Hint: add a *suitable* multiple of that α_p (p.297) which converges as fast as S does to obtain a sum that converges faster than S.)

3. List the locus (place) and type of all singular points of the following function: [15pt]

$$f(z) = \frac{1+e^{i\pi z}}{\sin(\pi z)}\ , \qquad |z| < \infty\ .$$

4. Evaluate the following integrals using residues: [20+20pt]

$$\int_{-\infty}^{\infty}\frac{e^{+ix}\,\mathrm{d}x}{(x^3-8)}\ , \qquad\qquad \int_0^{\infty}\frac{x^2\,\mathrm{d}x}{(x^4+1)}\ .$$

(Hint: First find the location of poles of the integrand, preferably in polar form; then close the integral by adding arc(s) at infinity in the upper or lower complex plane—wherever the integrand *decays*—and/or 'spokes' along which the integral is proportional to the original; if there are poles along the contour, determine the detour; evaluate the closed contour integral by summing the enclosed residues; finally, solve for the value of the above integrals.)