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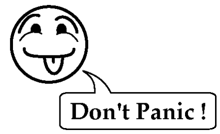

Mathematical Methods I
1st Midterm Exam

10th Oct. '97.

Instructor: T. Hübsch

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(Student name and ID)

This is an "open Textbook (Arfken), open lecture notes" exam. For full credit, show all your work. If you cannot complete one part of a calculation, a clear description of the procedure/method will still earn you partial credit. Budget your time: first do what you are sure you know how; use short-cuts whenever possible (but be prepared to explain them afterwards, if necessary).

1. Calculate $\int_S \mathrm{d}\vec{\sigma}\,[x^2+y^2+z^2]^{\frac{2}{3}}$, where S is the unit sphere, centered at the origin:

a. directly; [=10pt]

b. upon applying one of the integration/derivative identities. [=10pt]

(Hint: changing into spherical coordinates first should simplify calculations significantly.)

2. Consider a (generalized) coordinate system (ξ, η, ϑ) which is related to the cartesian system (x, y, z) through the relations

$$\xi = (x+y)\ , \qquad \eta = (x-y)\ , \qquad \vartheta = \frac{z^2}{x^2-y^2}\ .$$

Determine whether the new system is orthogonal or not. [=20pt]

(Hint: be careful about new *vs.* old coordinates in the definition of the metric tensor!!!)

3. For $i, j = 1, 2, 3$, A^i, B^j are components of two contravariant vectors and g_{ij} are the components of the metric (twice covariant) tensor; they all are some unspecified functions.

a. Determine transformation properties of $\sum_{i,j=1}^3 (A^i g_{ij} B^j)$. [=5pt]

b. Determine transformation properties of $\sum_{i,j,k=1}^3 A^k \frac{\partial}{\partial x^k}(A^i g_{ij} B^j)$. [=5pt]

c. List the three general scalars which can be made (only) from these quantities. [=15pt]

4. For the matrix $M = \begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & a \end{pmatrix}$, where a is a real number,

a. determine, without any calculation, if the eignevalues are real, and explain why; [=5pt]

b. determine a so that one of the eigenvalues would be zero; [=5pt]

c. calculate the eigenvalues; [=5pt]

d. calculate the eigenvectors; [=20pt]

e. calculate $\sqrt{M}$. [=10pt]