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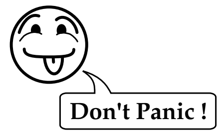

Mathematical Methods I
Final Exam

1st Dec. '04.

Instructor: T. Hübsch

(Student name and ID)

This is an "open Textbook (Arfken), open lecture notes" take-home exam, **due 5 p.m. of Monday, 8th Dec. '04**. For full credit, show all your work. If you cannot complete one part of a calculation, a clear description of the procedure/method will still earn you partial credit. Neither collaboration nor consultation is allowed, but you may quote—with full reference—any *published* source for intermediate results that you may use.

1. Calculate the integrals [15+15*pt*]

$$\int_S \mathrm{d}\vec{\sigma}\cdot\vec{r}\ , \qquad \text{and} \qquad \int_C \mathrm{d}\vec{r}\cdot\vec{\nabla} x y^2 e^{-z^2/x}\ ,$$

where S is the surface of the tetrahedron with vertices at (0,0,0), (1,0,0), (1,1,0) and (1,1,1) in a cartesian coordinate system, C is the unit circle in the x-y plane centered at the origin.

2. Given are uniform and perpendicular electric and a magnetic fields, so $\vec{E}\cdot\vec{B}=0$.

a. From $\vec{E}$ and $\vec{B}$, construct three algebraically independent scalars with respect to rotations in space (only), and find the two combinations of these which are also scalars with respect to the full Lorentz symmetry. [15*pt*]

b. Determine the velocity for a Lorentz transformation, Eq. (4.161), such that in the transformed system $\vec{B}'=0$. [15*pt*]

c. Determine a condition on $F_{\mu\nu}F^{\mu\nu}$ for the arrangement in part b. not to contradict the principle of special relativity that no physical system may move faster than the speed of light in vacuum. [10*pt*]

For a hint, express first $F_{\mu\nu}F^{\mu\nu}$ and $\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$ in terms of $\vec{E}$ and $\vec{B}$.

3. a. Determine a so that one eigenvalues of $\boldsymbol{M}=\begin{bmatrix}2&1&0\\1&0&1\\0&1&a\end{bmatrix}$ vanishes. [10*pt*]

b. Find all eigenvalues of $\boldsymbol{M}$ for the a for which $\det\boldsymbol{M}=0$. [15*pt*]

c. Find all normalized eigenvectors of $\boldsymbol{M}$ for that same a. [25*pt*]

4. Determine the radius of convergence of $\displaystyle\sum_{k=0}^{\infty}\frac{\Gamma(nk)\,x^k}{\Gamma(k)\Gamma(k+\frac{1}{3})\Gamma(k+\frac{2}{3})}$, depending on the positive integer n, or at least for $n=2$. [30*pt*]

5. Evaluate $\displaystyle\int_{-\infty}^{\infty}\frac{\mathrm{d}x\ e^{\alpha x}}{1+x^5}$ and $\displaystyle\int_0^{\infty}\frac{\mathrm{d}x\ x^2}{1-x^4}$ utilizing contour integration, with $\alpha>0$. [25+25*pt*]

(Hint: First find the poles of the integrand; then close the integral in the upper or lower complex plane—wherever the integrand *decays* exponentially; if there are poles along the contour, determine the detour; evaluate the closed contour integral by summing the enclosed residues; finally, sort out the value of the above integrals.)