Laurent Mirror Models

(of Ricci-Flat Spaces)

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Laurent Mirror-Models

Playbill

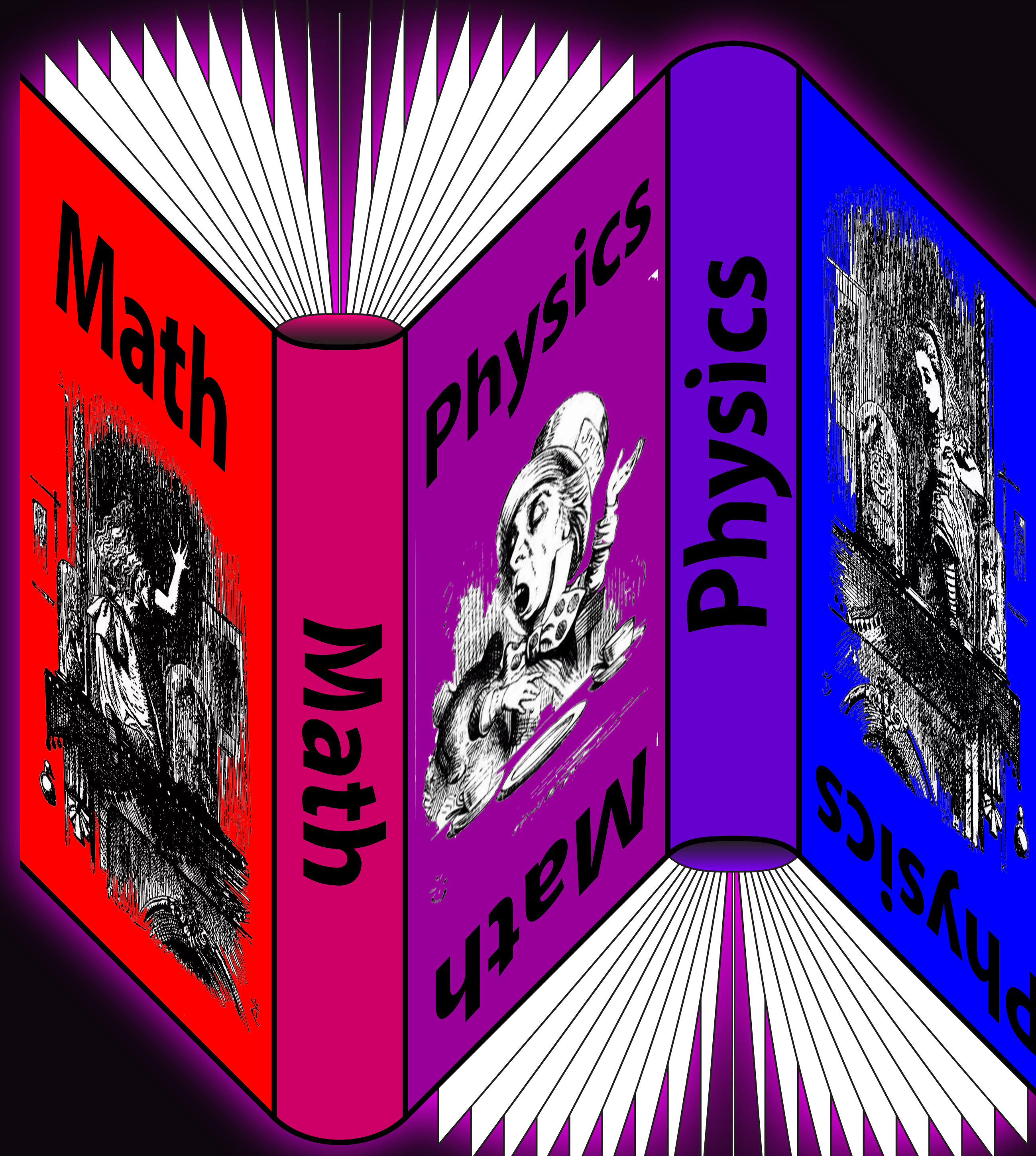
Prehistoric Prelude

Meromorphic Minuet

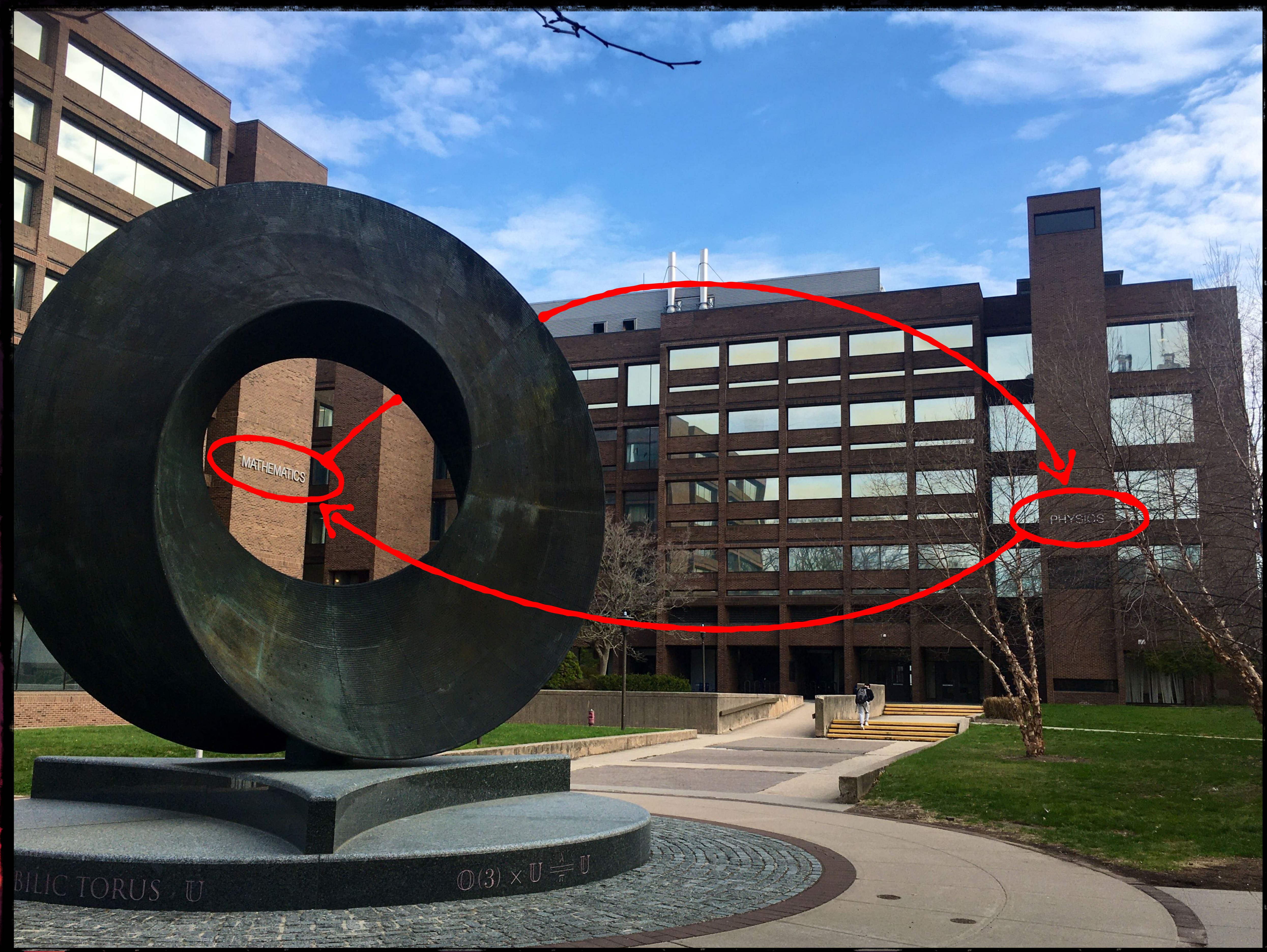
*Laurent-Toric Fugue**

Discriminant Divertimento

Mirror Motets



* "It doesn't matter what it's called, ...if it has substance."
S.-T. Yau

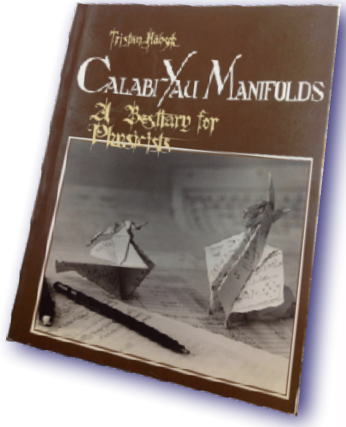

MATHEMATICS
PHYSICS
TORUS

Compactification Experimentalist

Pre-Historic Prelude

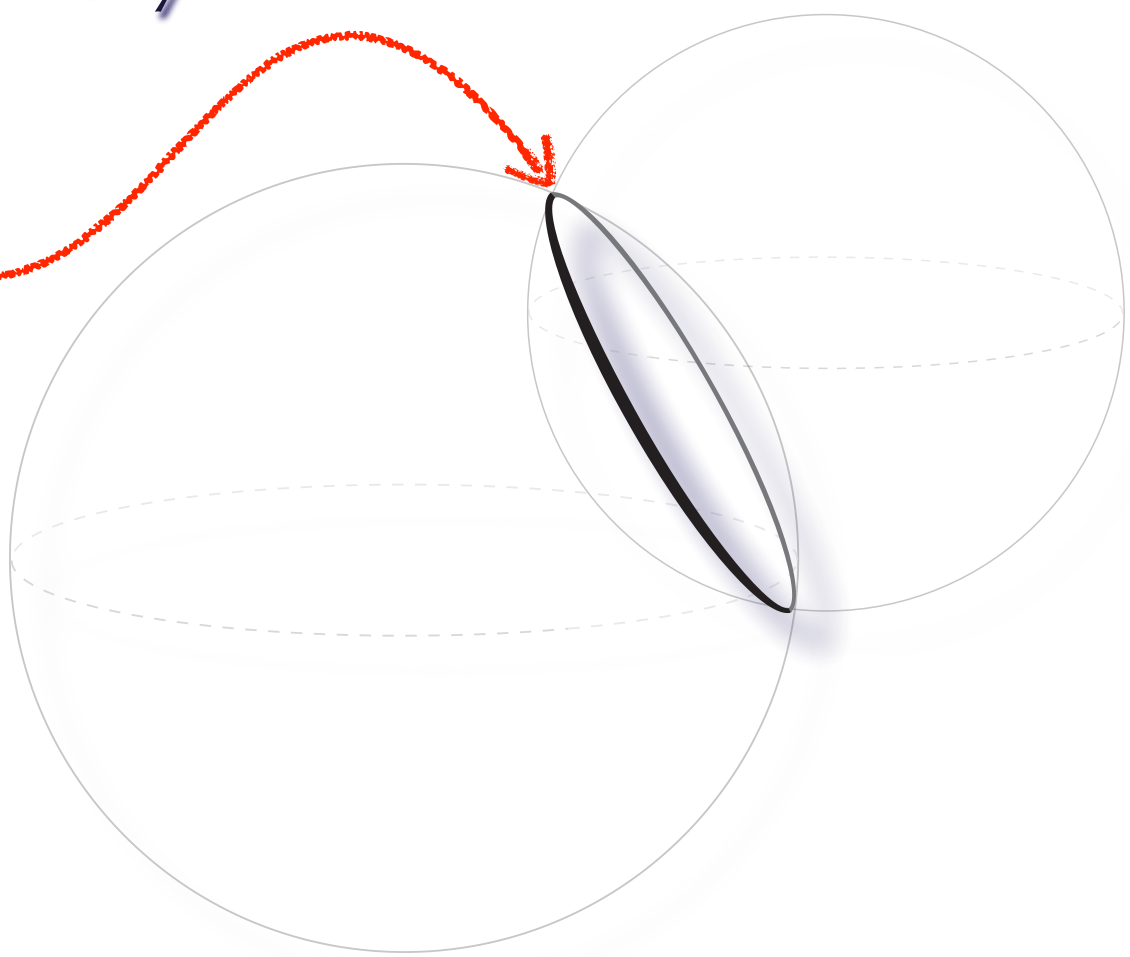
(Where are We Coming From?)

Pre-Historic Prelude



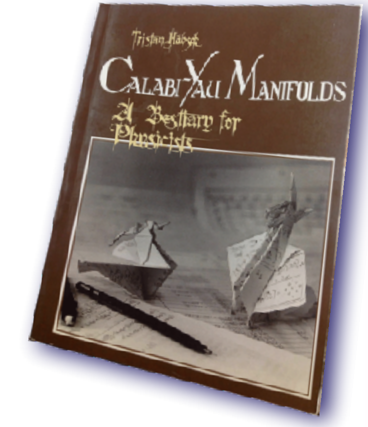
Classical Constructions — a Summary

- Complete Intersections
  - Ex. $\left.\begin{aligned}(x-x_1)^2+(y-y_1)^2+(z-z_1)^2 &= R_1^2\\ (x-x_2)^2+(y-y_2)^2+(z-z_2)^2 &= R_2^2\end{aligned}\right\}$
    - Algebraic (constraint) equations
    - …in a well-understood "ambient" (A)



- $A = \prod_i \mathbb{P}^{n_i}$, $\mathbb{P}^{n_i}_{\vec{w}}$, toric spaces, …
  - Tian-Yau: $\{\text{Fano}\}_c \smallsetminus \{\text{CY}\}_c = \{\text{CY}\}_{nc}$
  - Also: $\{\mathscr{K}^*_{X_c}\} = \{\text{CY}\}_{nc}$
- For *hypersurfaces*: $X = \{p(x) = 0\} \subset A$
  - "Functions": $[f(x)]_X = [f(x) \simeq f(x) + \lambda \cdot p(x)]_A$
  - Differentials: $[\mathrm{d}x]_X = [\mathrm{d}x \simeq \mathrm{d}x + \tilde{\lambda} \cdot \mathrm{d}p(x)]_A$
  - Homogeneity: $\mathbb{CP}^n = U(n+1) / [U(n) \times U(1)]$
    - i'th cohomology on $\mathbb{CP}^n \to U(n+1)$-tensors …*with* $U(n+1)$ *tensors*

Just like gauge transformations



Pre-Historic Prelude

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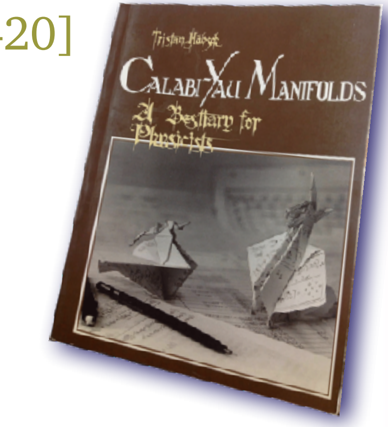
Also:
(∩) alg. *hypersurfaces*
→ nLSM w/*constraints*
→ GLSM w/*superpotentials*
→ topological A/B/½-twists
→ derived categories, *etc*.
→ *a lot happened since…*
… yet more to be done…
→ *"superspacify" (🖐GGRS)*

Just like gauge transformations

D, BV, BFV constraints in the nLSM ⇝ GLSM

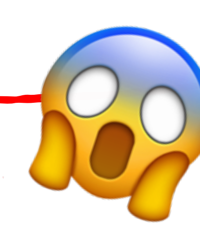
[arXiv:1606.07420]



Pre-Historic Prelude

Classical Constructions

- E.g: $X_m \in \left[\boxed{\begin{array}{c\|c} \mathbb{P}^4 & 1 \\ \mathbb{P}^1 & m \end{array}} \begin{array}{c} 4 \\ 2-m \end{array}\right]^{(2,86)}_{-168}$ $\quad$ $b_2 = 2 = h^{1,1}$ dim. space of Kähler classes; $\frac{1}{2}(b_3 - 1) = 86 = h^{2,1}$ dim. space of complex structures; $-168 = \chi = 2(h^{1,1} - h^{2,1})$ the Euler #
  - Zero-set of $\boxed{p(x,y) = 0,}$ $\deg[p] = \binom{1}{m}$, & $q(x,y) = 0$, $\deg[q] = \binom{4}{2-m}$
  - Generic $\{p=0\} \cap \{q=0\}$ smooth; $\deg_{\mathbb{P}^n}[p] + \deg_{\mathbb{P}^n}[q] = n+1 \Rightarrow \boxed{R_{\mu\nu} = 0}$
- Sequentially: $X_m \xrightarrow{q=0} \big(F_m \xrightarrow{p=0} \mathbb{P}^4 \times \mathbb{P}^1\big)$ $\quad \boxed{q(x,y) \sim \frac{q_0(x)}{y_0} + \frac{q_1(x)}{y_1}}$
  - Chern: $c = \frac{(1+J_1)^5(1+J_2)^2}{(1+J_1+mJ_2)(1+4J_1+(2-m)J_2)} = 1 + [6J_1{}^2 + (8-3m)J_1J_2] - [20J_1{}^3 - (32+15mJ_1{}^2J_2)].$
  - Wall: $\underbrace{\kappa_{111} = 2+3m, \ \kappa_{112} = 4}_{\infty\text{-vol. Yukawa}}$, so $(aJ_1 + bJ_2)^3 = [2a + 3(\underline{4b+ma})]a^2$.
  - $p_1[aJ_1 + bJ_2] = -88a - 12(\underline{4b+ma})$… the same "$4b+ma$"
  - So, $F_m \approx_{\mathbb{R}} F_{m \,(\mathrm{mod}\, 4)}$ & $X_m \approx_{\mathbb{R}} X_{m\,(\mathrm{mod}\,4)}$: 4 *diffeomorphism types*
- …but, $m = 0, 1, 2, \cdots 3 \to \deg[q] = \binom{4}{-1}$?!



Meromorphic Minuet

Why Haven't We Thought of This Before?

- $\deg[q] = \binom{4}{-1}$ holomorphic sections?! [AAGGL:1507.03235 + BH:1606.07420] [+ GvG:1708.00517]
  - Not *everywhere* on $\mathbb{P}^4 \times \mathbb{P}^1$ — (simple poles)
  - but yes on $F_3^{(4)} \subset \mathbb{P}^4 \times \mathbb{P}^1$ — in fact, 105 of 'em!

$$X_m \in \left[\begin{array}{c||c|c} \mathbb{P}^4 & 1 & 4 \\ \mathbb{P}^1 & m & 2-m \end{array}\right]^{(2,86)}_{-168}$$

for $m = 3$

- How? On $F_3^{(4)}$, $q(x,y) \simeq q(x,y) + \lambda \cdot p(x,y) \leftarrow$ equivalence class!
  - Hirzebruch, 1951 $\Rightarrow$ $p = x_0 y_0{}^3 + x_1 y_1{}^3$ & $q = c(x)\left(\frac{x_0 y_0}{y_1{}^2} - \frac{x_1 y_1}{y_0{}^2}\right)$ $\deg[c] = \binom{3}{0}$
  - So, $q_0 = q(x,y) + \frac{\lambda\, c(x)}{(y_0 y_1)^2} p(x,y) \overset{\lambda \to -1}{=\!=\!=} c(x)\left(-2\frac{x_1 y_1}{y_0{}^2}\right)$ where $y_0 \neq 0$
  - & $q_1 = q(x,y) + \frac{\lambda\, c(x)}{(y_0 y_1)^2} p(x,y) \overset{\lambda \to 1}{=\!=\!=} c(x)\left(2\frac{x_0 y_0}{y_1{}^2}\right)$ where $y_1 \neq 0$
  - & $q_1(x,y) - q_0(x,y) = 2\frac{c(x)}{(y_0 y_1)^2}\, p(x,y) = 0$, on $F_3 := \{p(x,y) = 0\}$
  - *Just as the Wu-Yang monopole avoids the "Dirac string"…*
  - *… → D, BV, BFV etc. treatment of constraints (in the nLSM → GLSM)*

Meromorphic Minuet

…in well-tempered counterpoint

[AAGGL:1507.03235 + BH:1606.07420]
+ more

- For $\Big\{\underbrace{x_0\, y_0{}^m + x_1\, y_1{}^m}_{:=\,\mathring{p}(x,y)} = -\sum_\alpha \cancel{\epsilon_\alpha}\, \delta p_\alpha(x,y)\Big\} = F^{(n)}_{m;\cancel{\epsilon}} \in \left[\begin{matrix}\mathbb{P}^n\\ \mathbb{P}^1\end{matrix}\middle\Vert\begin{matrix}1\\ m\end{matrix}\right]$
  - The central ($\epsilon = 0$) member of the family is a Hirzebruch scroll F_m:
  - <u>Directrix</u>: $S := \{\mathfrak{s}(x,y) = 0\}$, $[S] = [H_1] - m[H_2]$ & $[S]^n = -(n-1)m$;
    - where $\mathfrak{s}(x,y) := \Big(\frac{x_0}{y_1{}^m} - \frac{x_1}{y_0{}^m}\Big) + \frac{\lambda}{(y_0 y_1)^m}[x_0 y_0{}^m + x_1 y_1{}^m]$ degree $\binom{1}{-m}$
  - & $\underline{h^0(K^*)} = 3\binom{2n-1}{n} + \delta_{\epsilon,0}\vartheta_3^m\binom{2n-2}{2}(m-3)$, $\underline{h^0(T)} = n^2 + 2 + \delta_{\epsilon,0}\vartheta_1^m(n-1)(m-1)$
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  - All these "*exceptionals*" <u>*reduce*</u> (decompose) for $(\epsilon_\alpha \neq 0)$ deformations resulting in *discrete deformations* $F^{(n)}_m \to F^{(n)}_{(m_1,m_2,\cdots)}$ & ⋯ & $\approx_{\mathbb{R}} F^{(n)}_{[m\,(\mathrm{mod}\,n)]}$
- These $F^{(n)}_{(m_1,m_2,\cdots)}$'s are distinct toric varieties… w/$\{\mathfrak{s}_r,\ r \leqslant m_i\}$

Meromorphic Minuet

…in well-tempered counterpoint

[AAGGL:1507.03235 + BH:1606.07420]
+ more

- On $F^{(n)}_m$: $x_0 y_0{}^m + x_1 y_1{}^m = 0 \;\Rightarrow\; x_0 = -x_1 (y_1/y_0)^m$ & $x_1 \to X_1 = \mathfrak{S}$
  - & $(X_i,\ i=2,\cdots,n+2) = (x_2,\cdots,x_n; y_0, y_1)$
  - $\mathbb{P}^4 \times \mathbb{P}^1$ bi-degree $\to$ toric $(\mathbb{C}^\times)^2$-action:
  - Add Lagrange multiplier, $X_0\, f(X)$
  - Need $\deg[f(X)] = \binom{4}{2-m}$, with $\deg[X_1 X_{5,6}^m] = \binom{1}{0} = \deg[X_{2,3,4}]$

X_0	X_1	X_2	X_3	X_4	X_5	X_6	
-4	1	1	1	1	0	0	$\leftarrow\mathbb{P}^4$
$m-2$	$-m$	0	0	0	1	1	$\leftarrow\mathbb{P}^1$

- $f(X) = X_1{}^4 X_{5,6}^{2+3m} \oplus X_1{}^3 X_{2,3,4} X_{5,6}^{2+2m} \cdots \oplus X_1 X_{2,3,4} X_{5,6}^2 \oplus X_{2,3,4} X_{5,6}^{\boxed{2-m}}$
  - $m > 2$,

Meromorphic Minuet

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[AAGGL:1507.03235 + BH:1606.07420] + more

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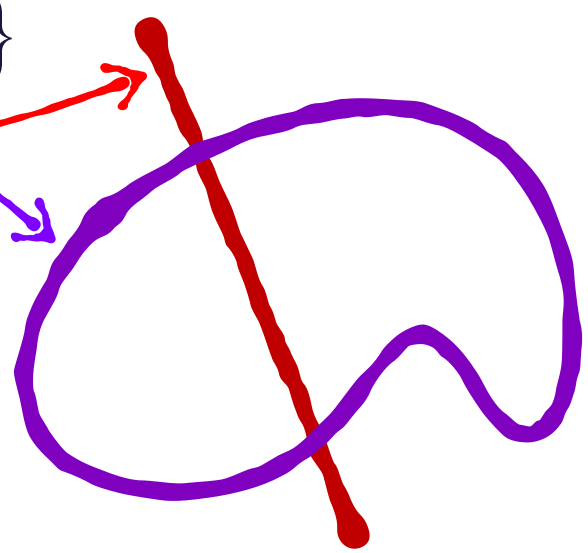
  - $m>2$, $\{f(X)=0\} = \{X_1=0\} \cup \{\oplus_k X_1{}^k X_{2,3,4}^2 X_{5,6}^{2+km} = 0\}$



Meromorphic Minuet

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+ more

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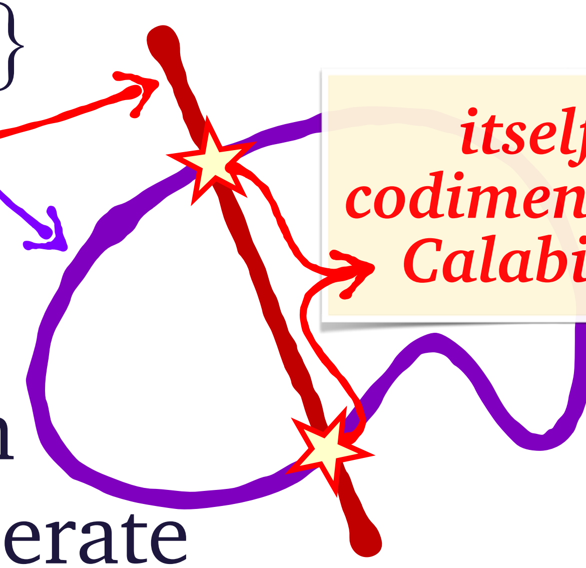
  - $\{f(X)=0\}^\sharp = \{X_1 = 0\} \cap \{\oplus_k X_1{}^k X_{2,3,4}^2 X_{5,6}^{2+km} = 0\}$

$$\left[\begin{array}{c||cc|c} \mathbb{P}^n & 1 & n-1 & 1 \\ \mathbb{P}^1 & m & 2 & -m \end{array}\right] = \left[\begin{array}{c||c|cc} \mathbb{P}^n & 1 & 1 & n-1 \\ \mathbb{P}^1 & m & -m & 2 \end{array}\right] \xrightarrow{\simeq} \left[\begin{array}{c||c} \mathbb{P}^{n-2} & n-1 \\ \mathbb{P}^1 & 2 \end{array}\right]$$

Tyurin degenerate



itself a codimension-2 Calabi-Yau

Laurent-Toric Fugue

(a *not-so-new* Toric Geometry)

A Generalized Construction of
Calabi-Yau Mirror Models
arXiv:1611.10300
+ lots more…

Meromorphic Minuet

…in well-tempered counterpoint

- On $F_m^{(n)}$: $x_0 {y_0}^m + x_1 {y_1}^m = 0 \;\Rightarrow\; x_0 = -x_1 (y_1/y_0)^m$ & $x_1 \to X_1 = \mathfrak{s}$
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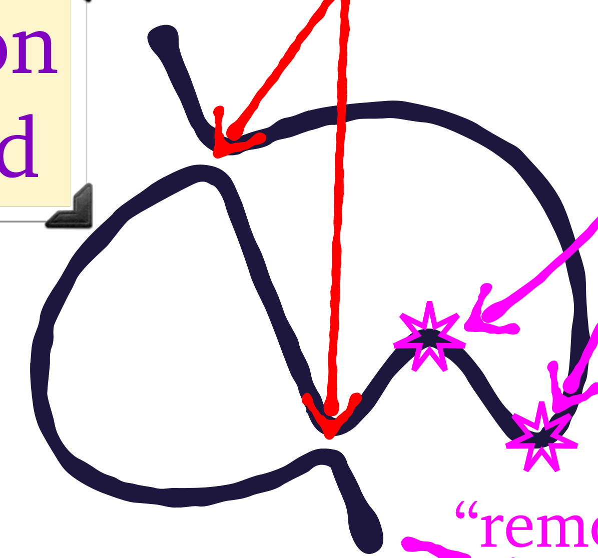
  - $\boxed{m > 2,}$ $\{f(X) = 0\}$
  - ~~$\{f(X)=0\}^\sharp$~~

SQFT/GLSM as an expansion about "classical" background

  - Embrace the Laurent terms
  - "Intrinsic limit" (L'Hôpital-"repaired") → smooth *(pre?)*complex spaces



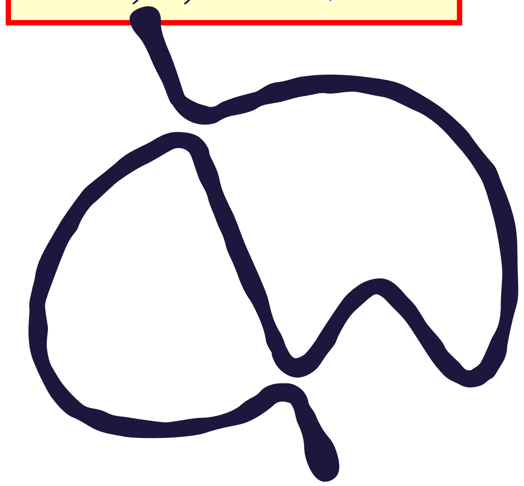
"removable" singularity

Meromorphic Minuet

arXiv:1611.10300 +much more

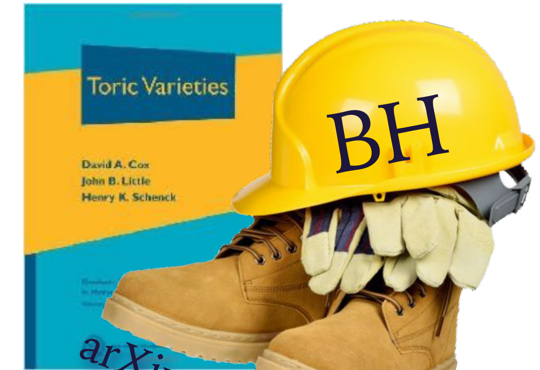
…in well-tempered counterpoint

- $f(X) = X_1^{\,4} X_{5,6}^{2+3m} \oplus X_1^{\,3} X_{2,3,4} X_{5,6}^{2+2m} \cdots \oplus X_1 X_{2,3,4} X_{5,6}^{2} \oplus \boxed{X_{2,3,4} X_{5,6}^{2-m}}$
  - $m > 2$, Laurent terms & “intrinsic limit” 😶‼️❓😕



- Virtual varieties [Francesco Severi], i.e., Weil divisors
  - E.g., $\mathbb{P}^2_{(3:1:1)}[5]$ $\;x_3{}^5 + x_4{}^5 + \frac{x_2{}^2}{x_4} = 0 \;\approx\; \frac{x_3{}^5 x_4 + x_4{}^6 + x_2{}^2}{x_4} = 0$
  - Denominator contributions subtract from those of the numerator
- Change variables [David Cox]: $(x_2, x_3, x_4) \mapsto (z_3\sqrt{z_2}, z_1{}^2, z_2)$
  - $x_3{}^5 + x_4{}^5 + \frac{x_2{}^2}{x_4} \mapsto z_1{}^{10} + z_2{}^5 + z_3{}^2$ in $\mathbb{P}^2_{(1:2:5)}[10]$
  - **Generalized to all** $F_m^{(n)}[c_1]$ ✅ — *not a fluke*
  - A *<u>desingularized</u>* *<u>finite quotient</u>* of a *<u>branched multiple cover</u>*
  - …and a variety of “general type” ($c_1 < 0$ or even $c_1 \gtrless 0$)

…there’s ∞ of those, just as of VEX polytopes!



Laurent-Toric Fugue

& Non-Convex Mirrors

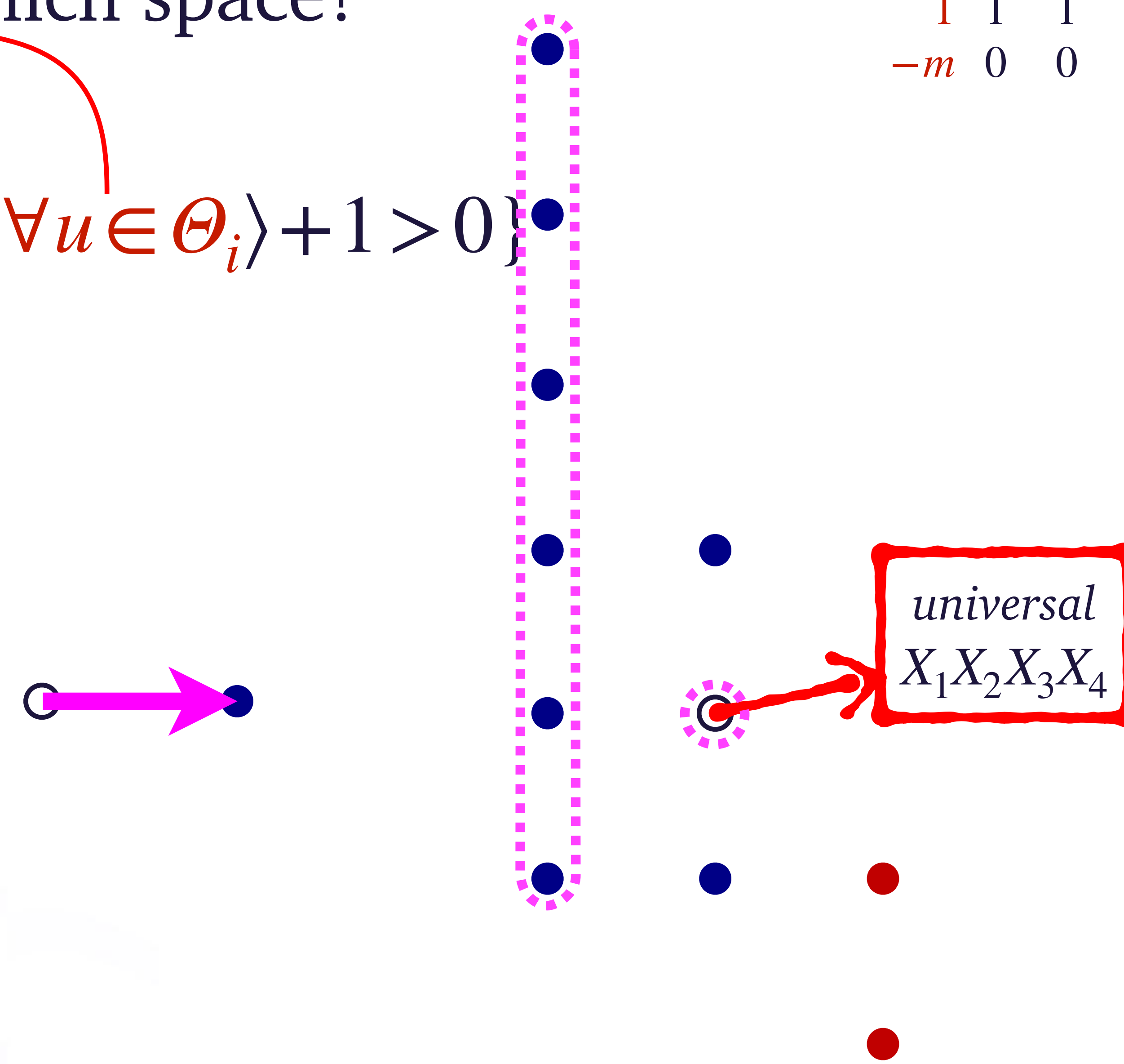
—2D *Proof-of-Concept*—

arXiv:1611.10300
+much more

- ${X_1}^2 {X_2}^0 (X_3 \oplus X_4)^{2+1m} \oplus {X_1}^1 {X_2}^1 (X_3 \oplus X_4)^{2+0m} \oplus {X_1}^0 {X_2}^2 (X_3 \oplus X_4)^{2-1m}$

X_1	X_2	X_3	X_4	X_5	X_6	
1	1	1	1	0	0	$\leftarrow \mathbb{P}^4$
$-m$	0	0	0	1	1	$\leftarrow \mathbb{P}^1$

- *Trans*polar: functions on which space?
  - $\Delta \to \bigcup_i (\text{convex } \Theta_i)$;
  - Compute $\Theta_i \to \Theta_i^{\circ} := \{v\colon \langle v | \forall u \in \Theta_i \rangle + 1 > 0\}$



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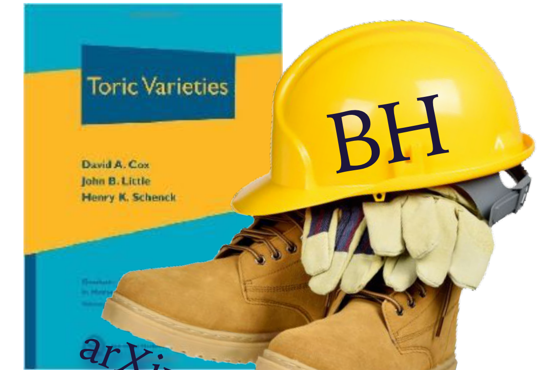

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& Non-Convex Mirrors

—2D *Proof-of-Concept*—

- $X_1{}^2 X_2{}^0 (X_3 \oplus X_4)^{2+1m} \oplus X_1{}^1 X_2{}^1 (X_3 \oplus X_4)^{2+0m} \oplus X_1{}^0 X_2{}^2 (X_3 \oplus X_4)^{2-1m}$

X_1	X_2	X_3	X_4	X_5	X_6	
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$-m$	0	0	0	1	1	$\leftarrow\mathbb{P}^1$

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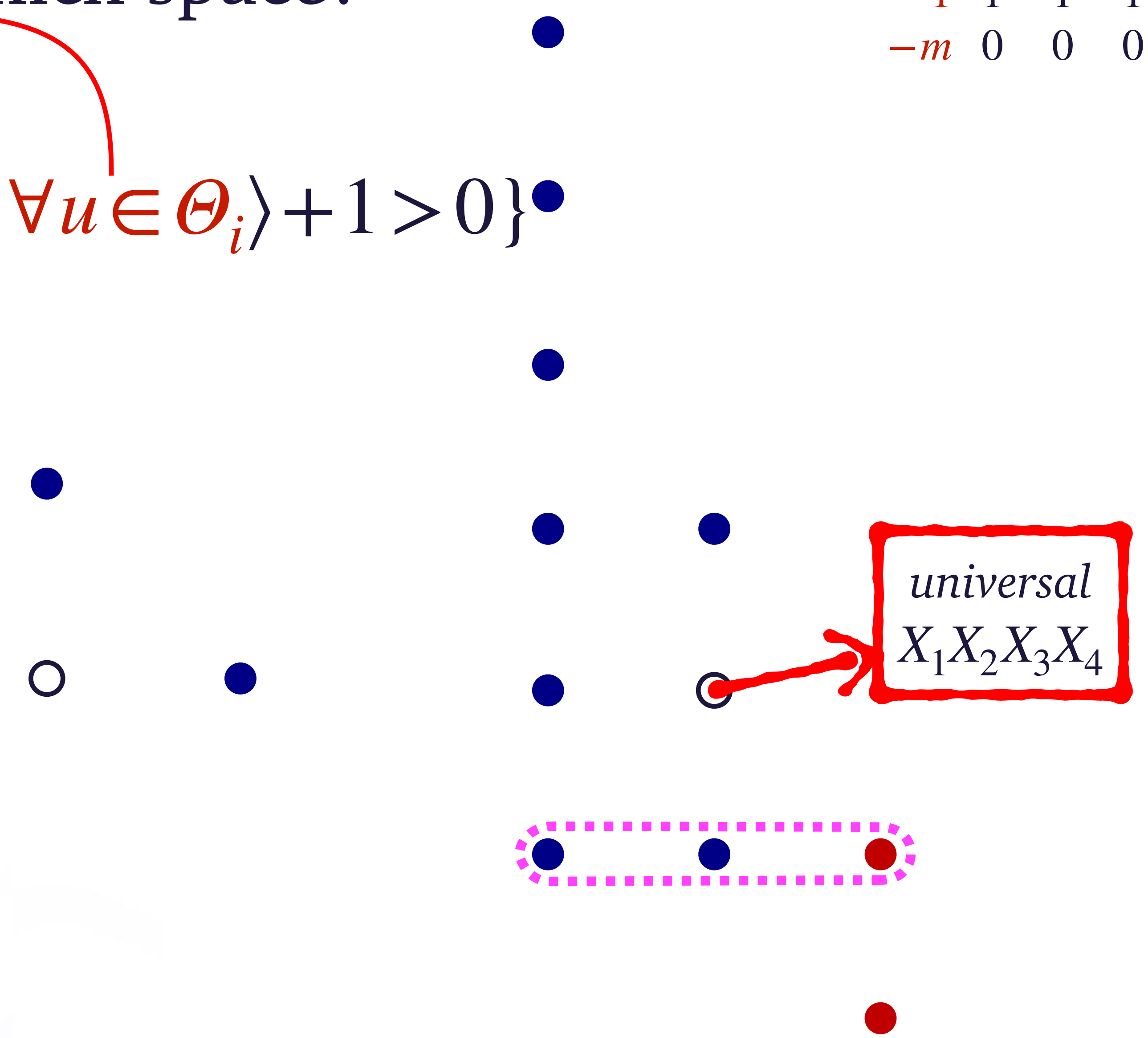
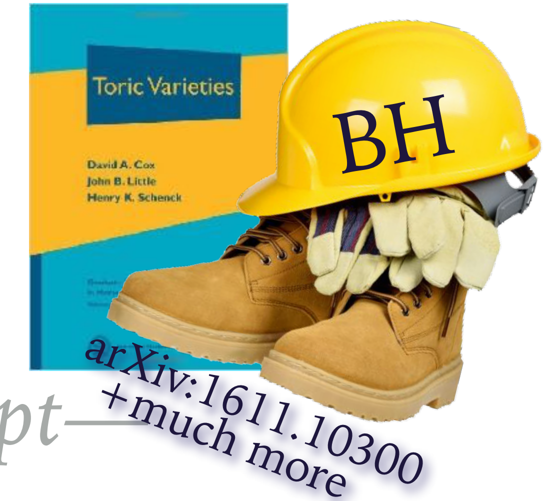


Laurent-Toric Fugue

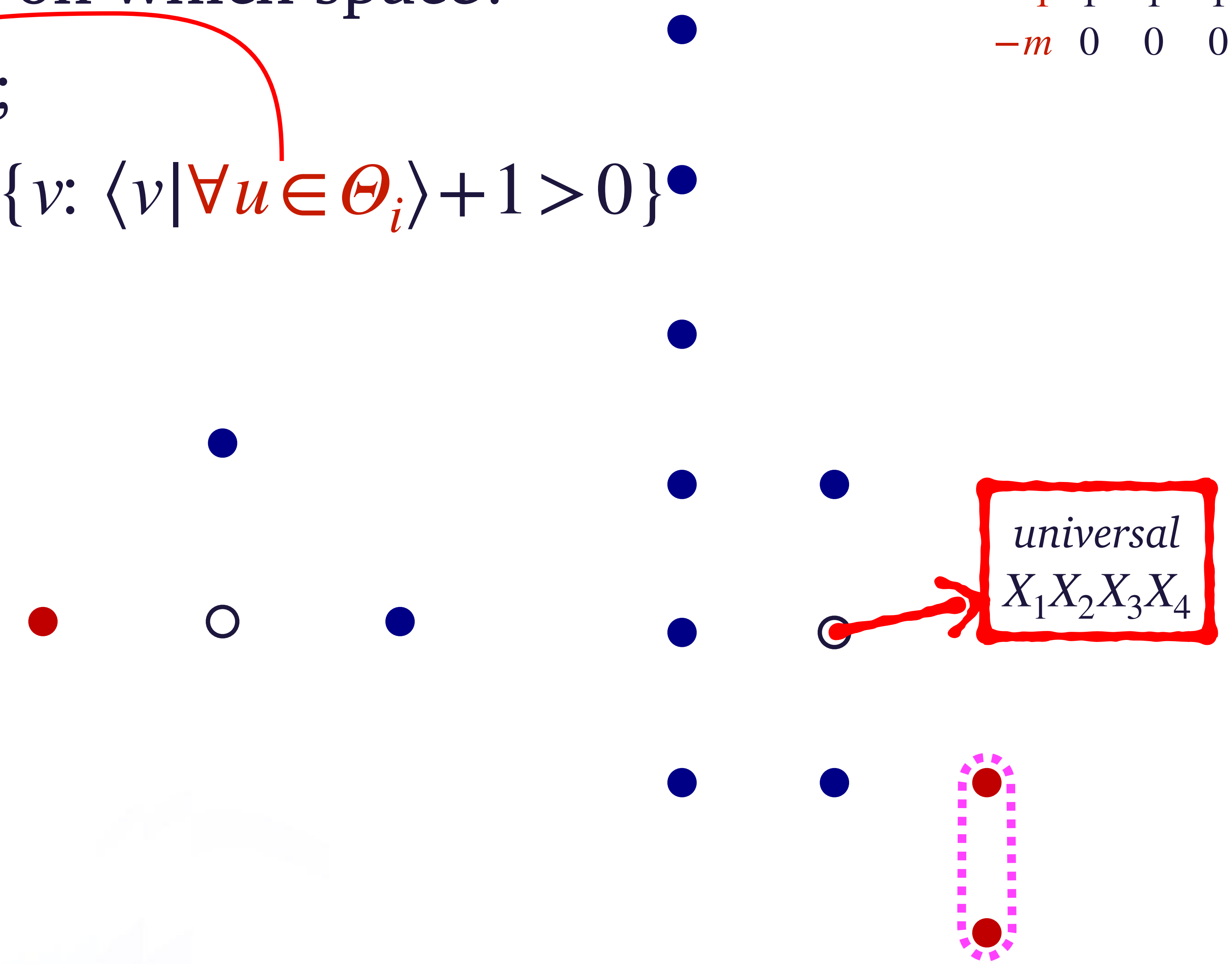
& Non-Convex Mirrors

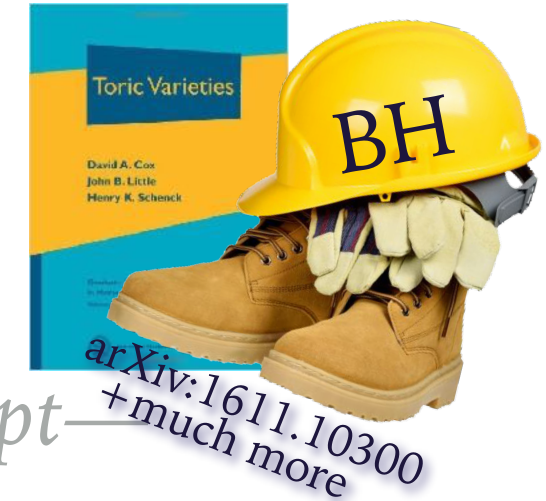
—2D *Proof-of-Concept*—



- $X_1^{\,2}X_2^{\,0}(X_3\oplus X_4)^{2+1m}\oplus X_1^{\,1}X_2^{\,1}(X_3\oplus X_4)^{2+0m}\oplus X_1^{\,0}X_2^{\,2}(X_3\oplus X_4)^{2-1m}$
- *Trans*polar: functions on which space?
  - $\Delta \to \bigcup_i(\text{convex } \Theta_i)$;
  - Compute $\Theta_i \to \Theta_i^{\circ} := \{v\!: \langle v|\forall u\in\Theta_i\rangle + 1 > 0\}$

X_1	X_2	X_3	X_4	X_5	X_6	
1	1	1	1	0	0	$\leftarrow\mathbb{P}^4$
$-m$	0	0	0	1	1	$\leftarrow\mathbb{P}^1$





arXiv:1611.10300
+much more

Laurent-Toric Fugue

& Non-Convex Mirrors

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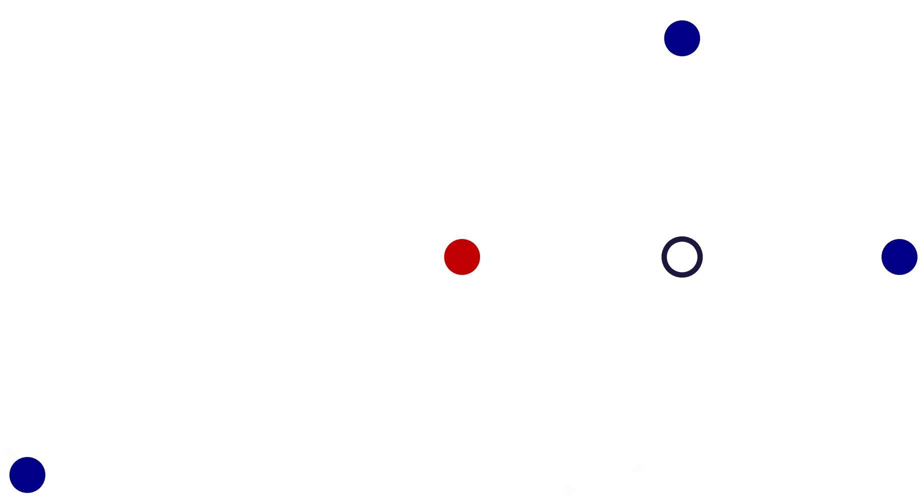
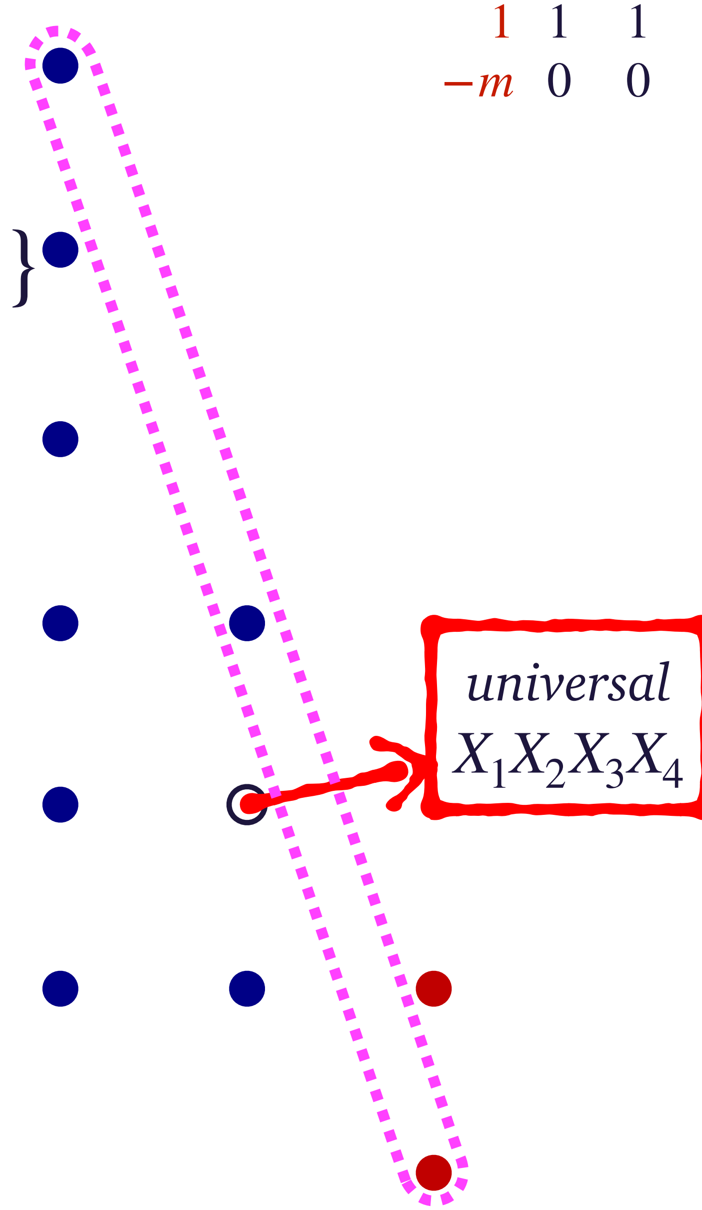

Laurent-Toric Fugue

& Non-Convex Mirrors

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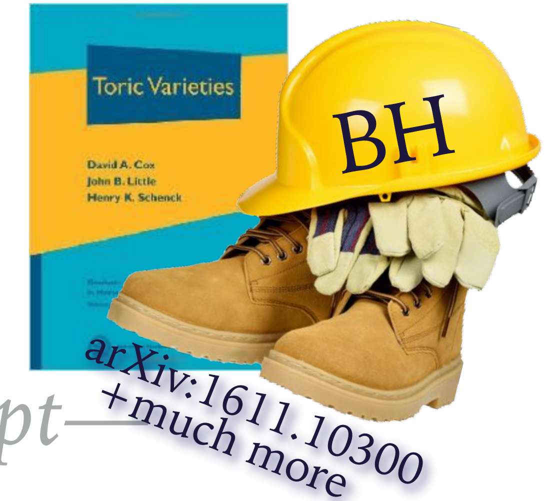

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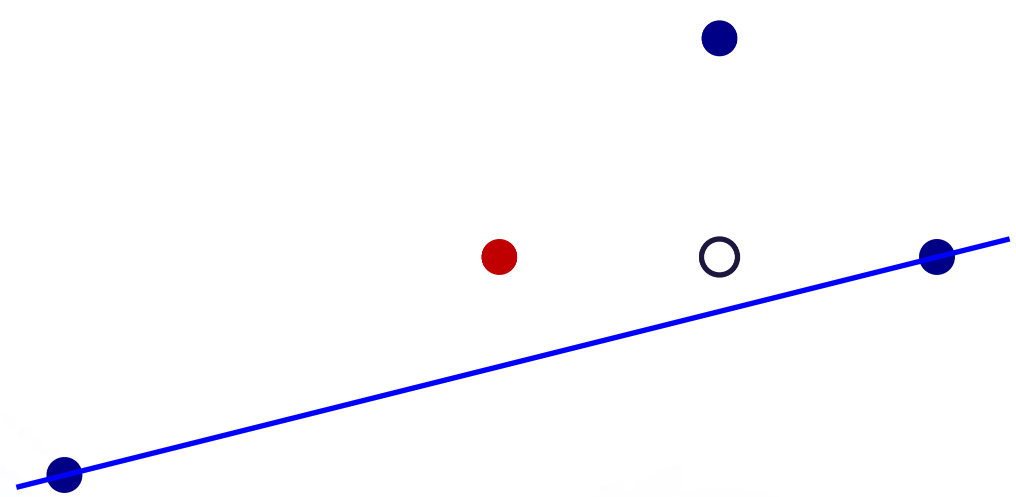

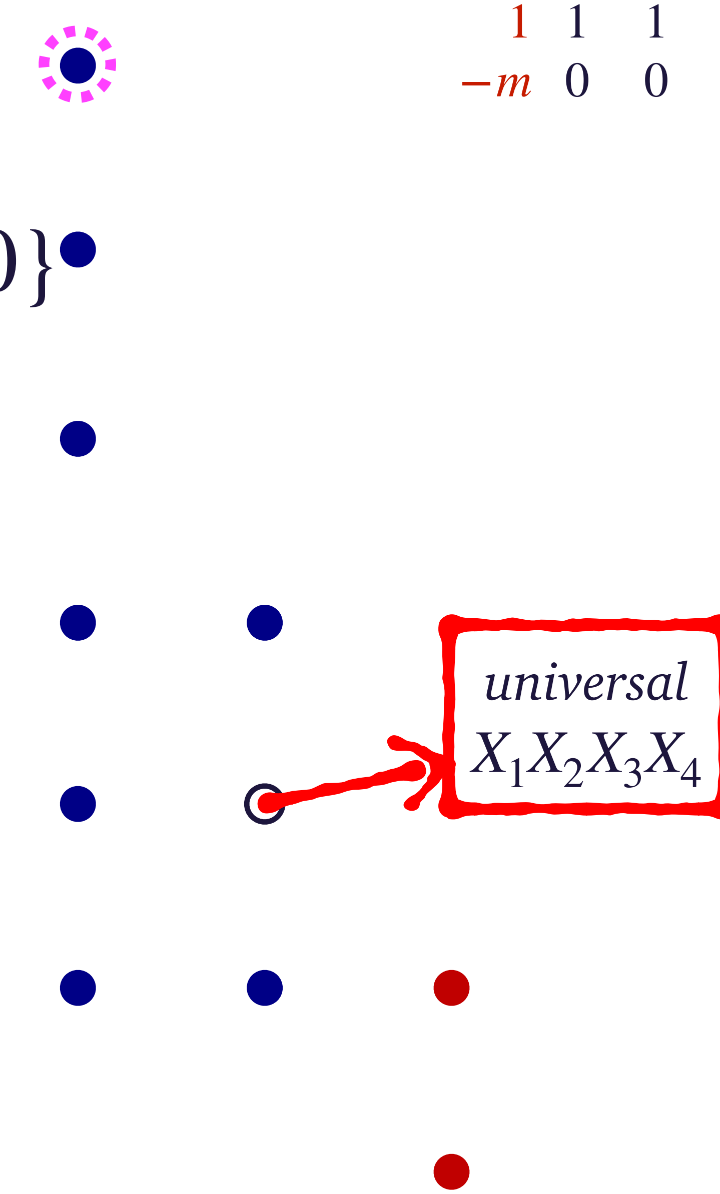

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& Non-Convex Mirrors

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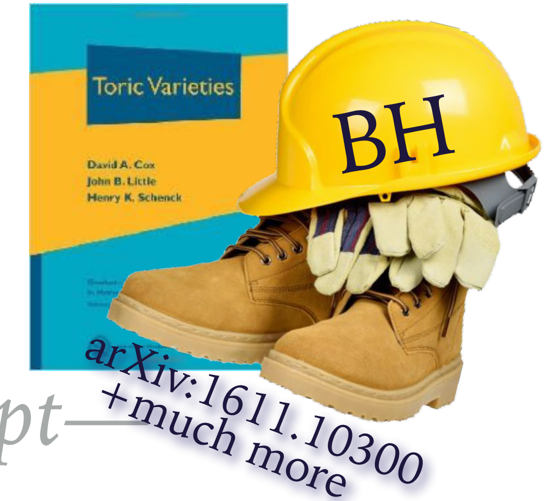

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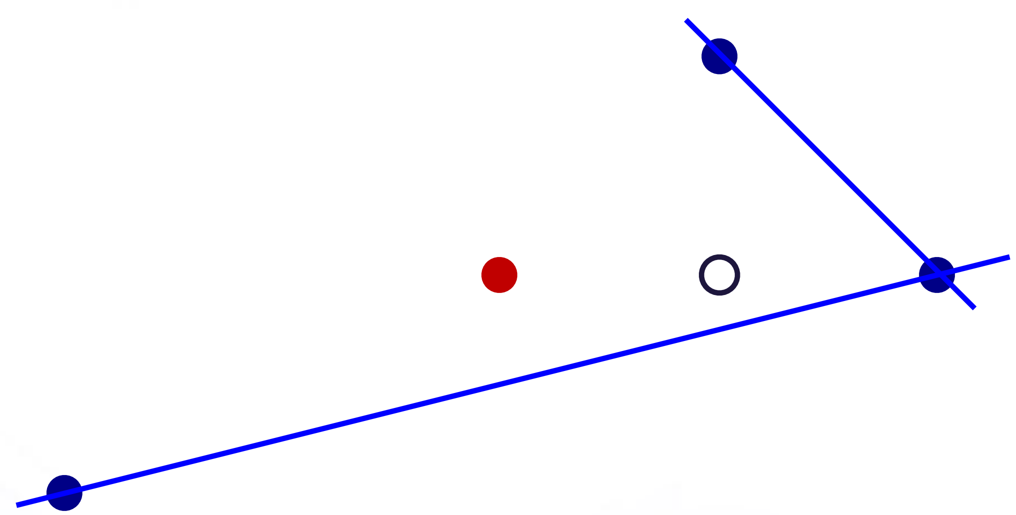

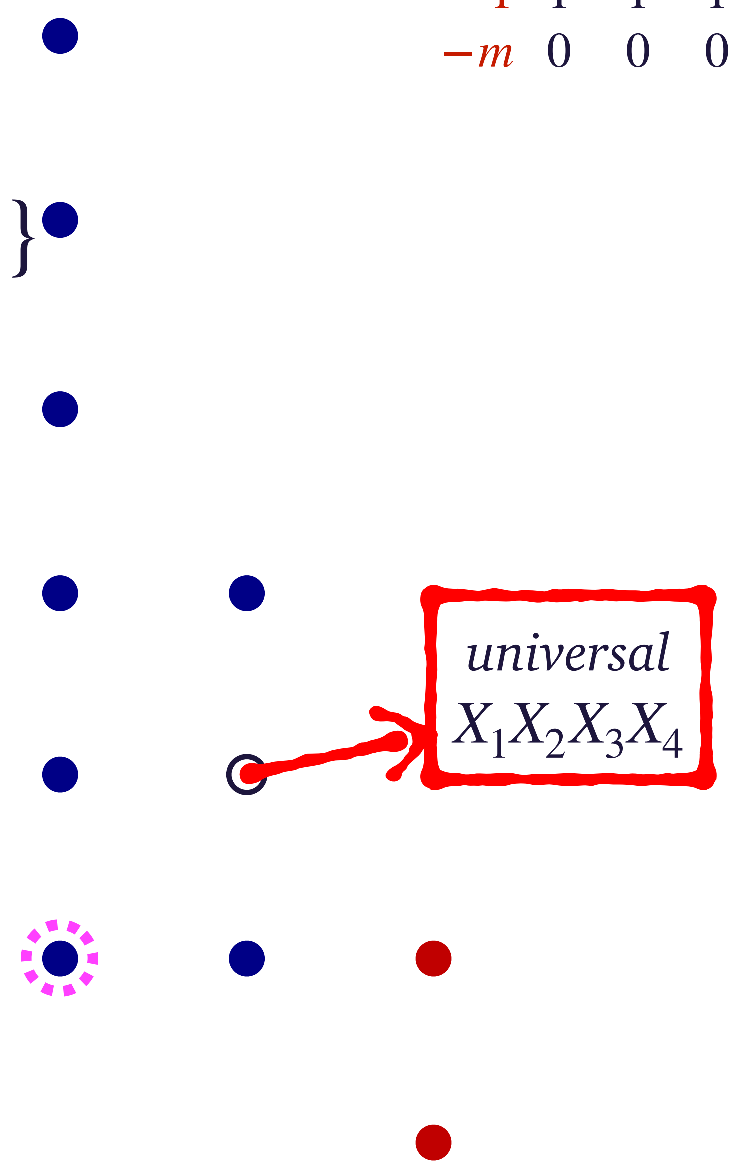

Laurent-Toric Fugue

& Non-Convex Mirrors —2D *Proof-of-Concept*—

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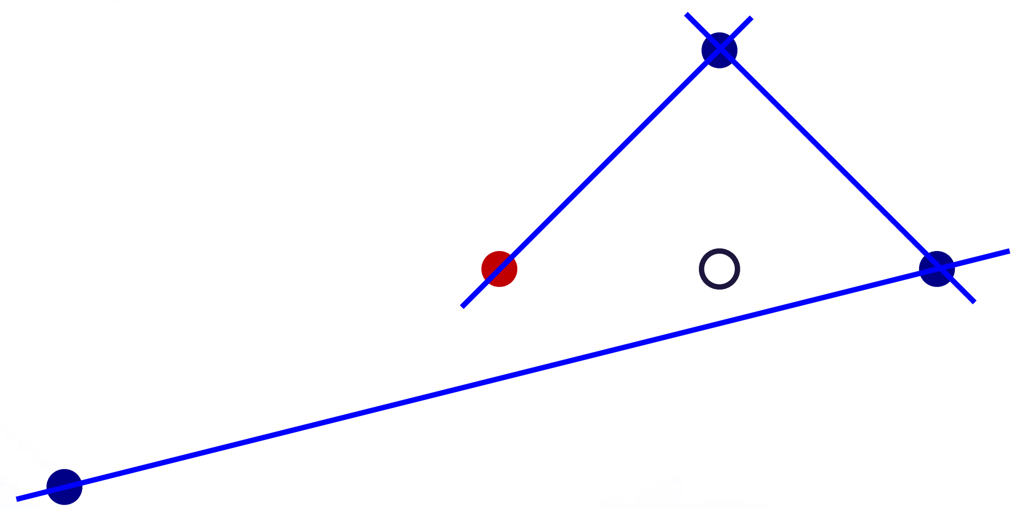

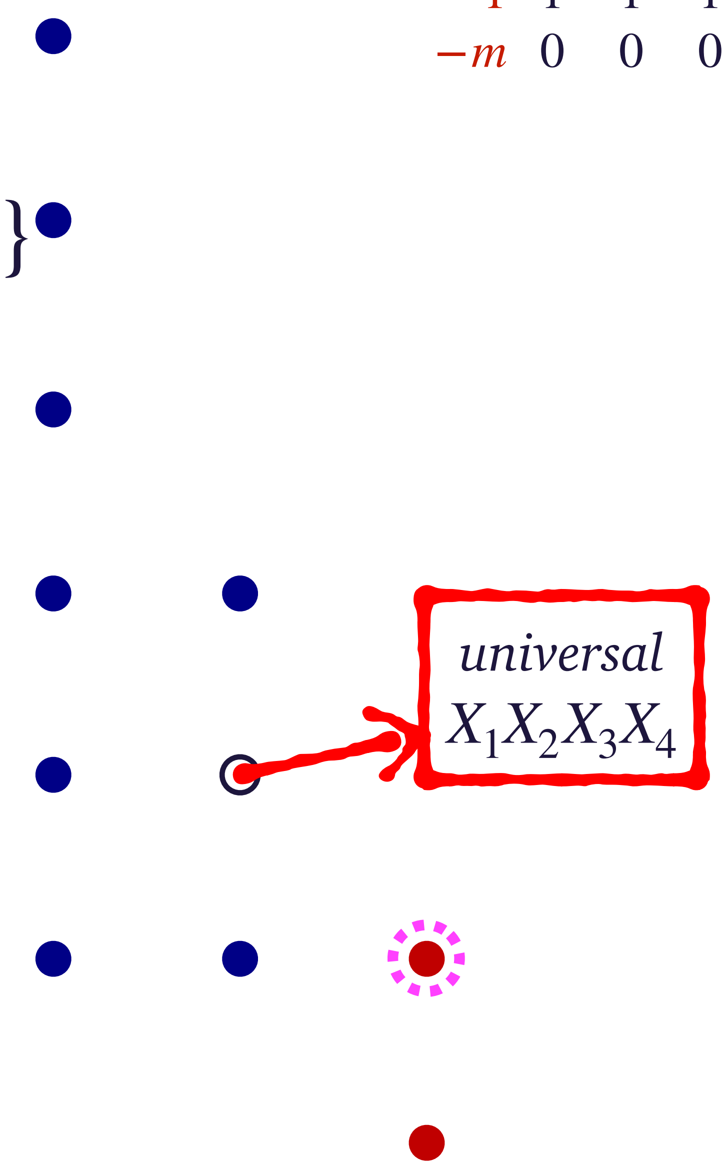

Laurent-Toric Fugue

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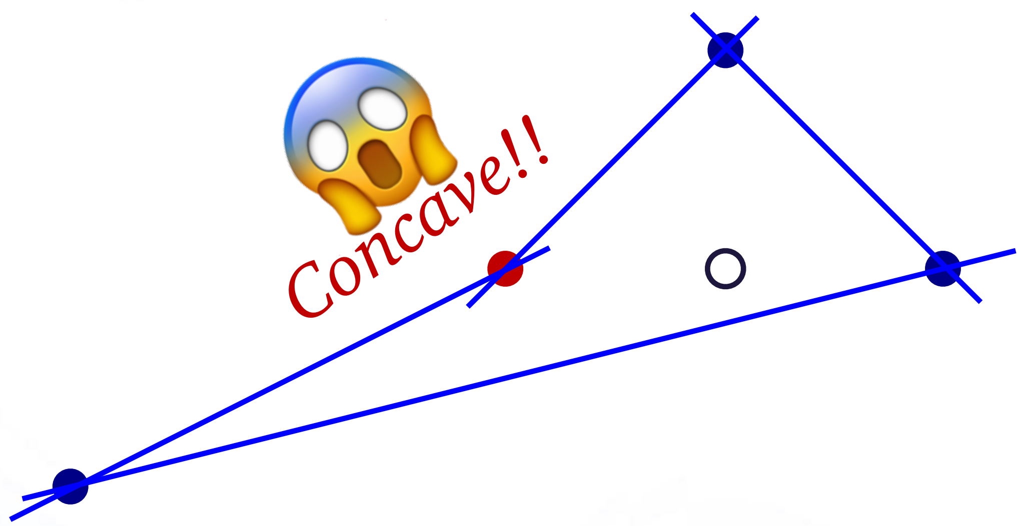

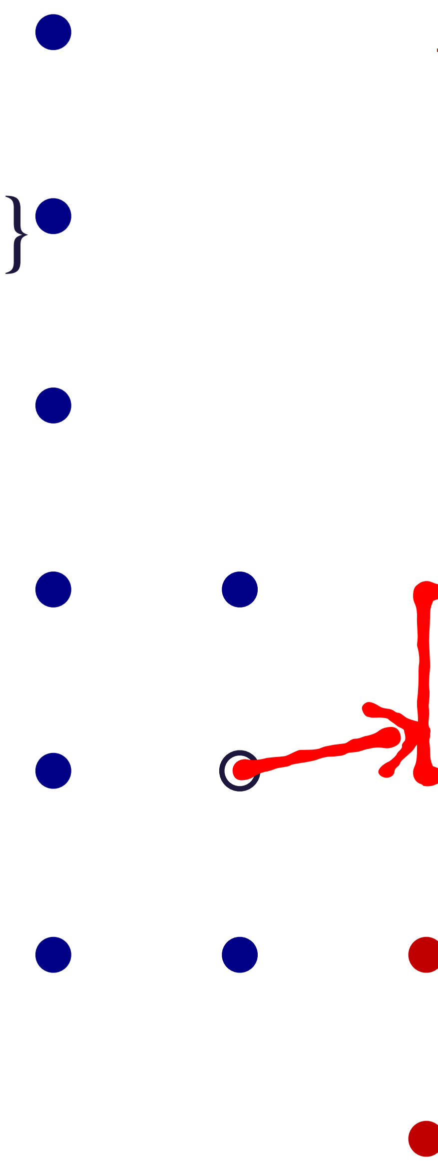

universal
$X_1X_2X_3X_4$

Laurent-Toric Fugue

& Non-Convex Mirrors

—2D *Proof-of-Concept*—

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+much more

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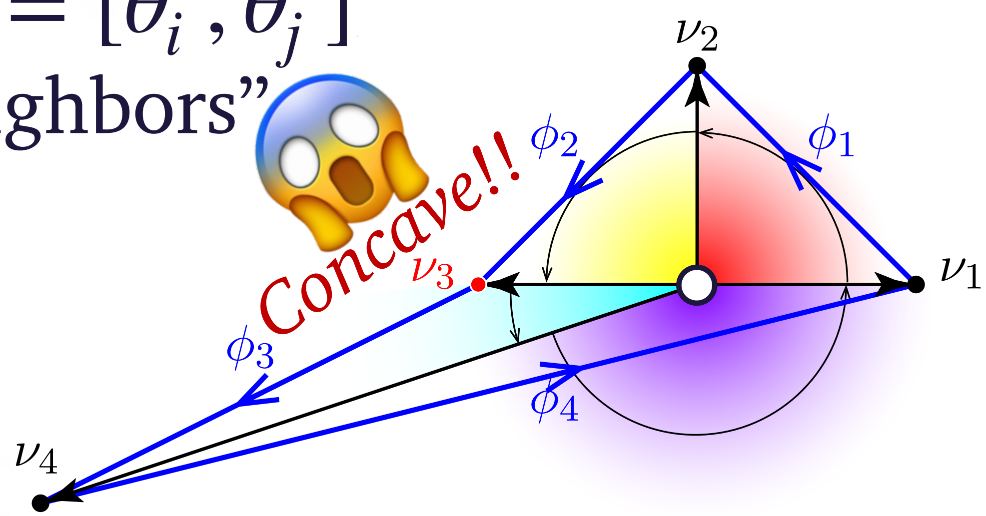

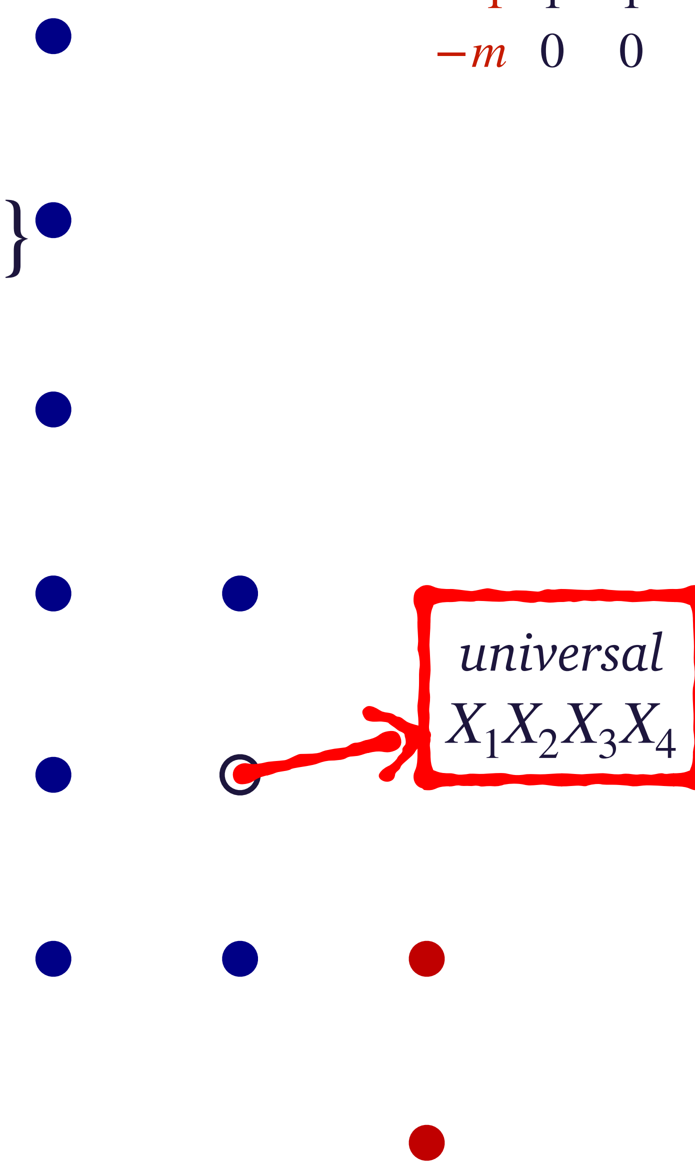

Laurent-Toric Fugue

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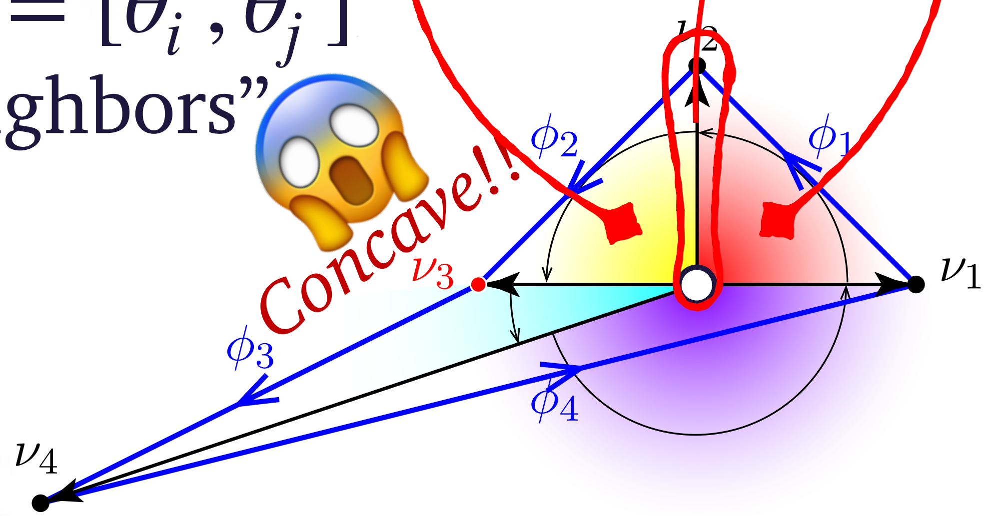
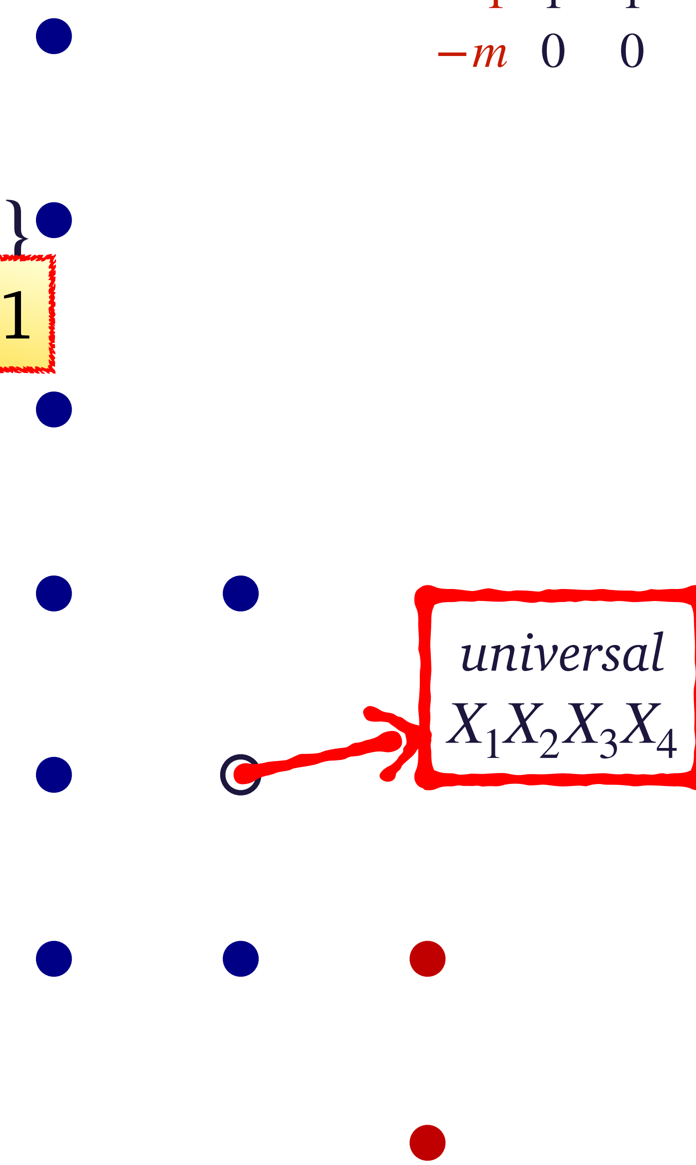
  - local chart #2
  - local chart #1
  - (Re)assemble dually
    $(\theta_i \cap \theta_j)^\circ = [\theta_i^\circ, \theta_j^\circ]$
    with "neighbors"





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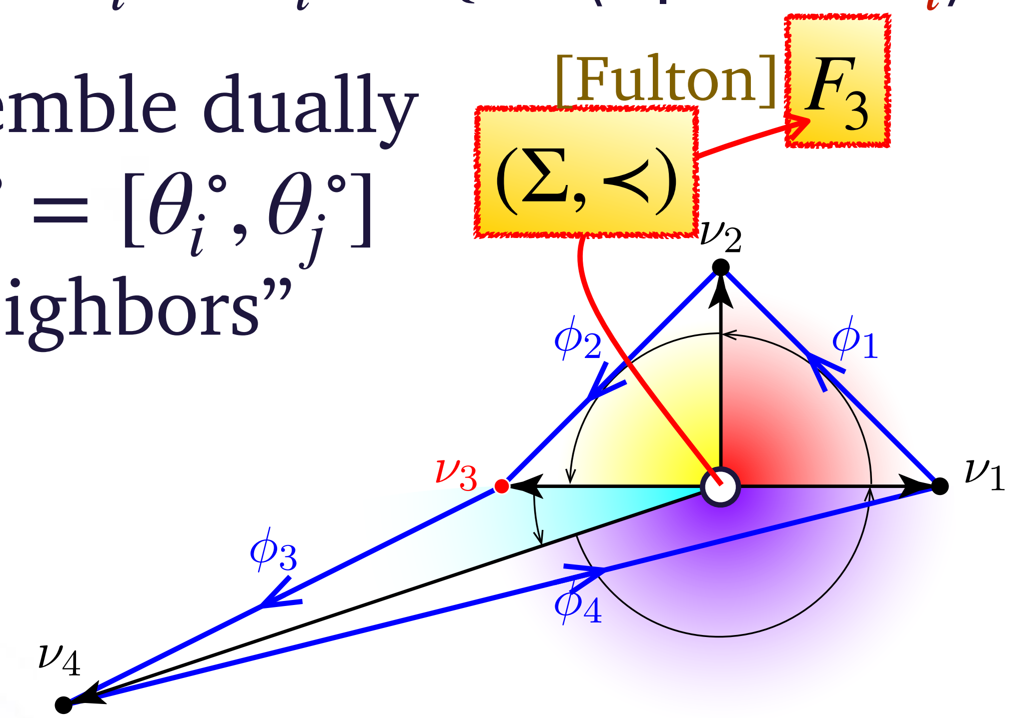

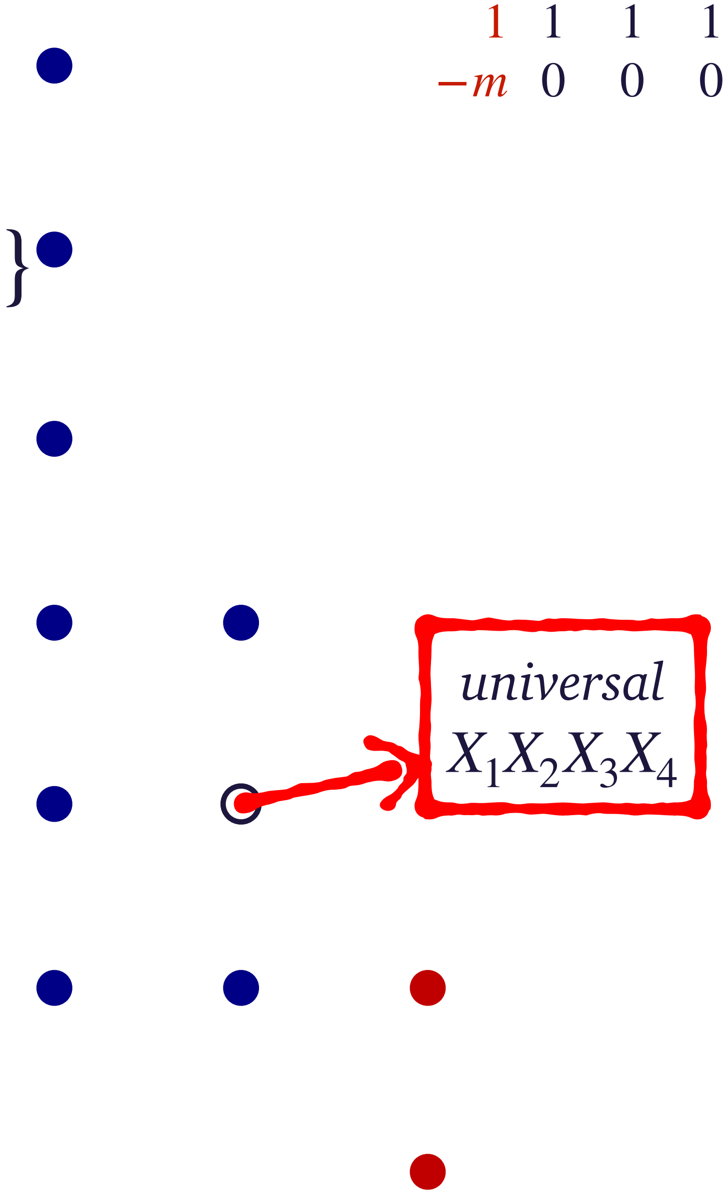

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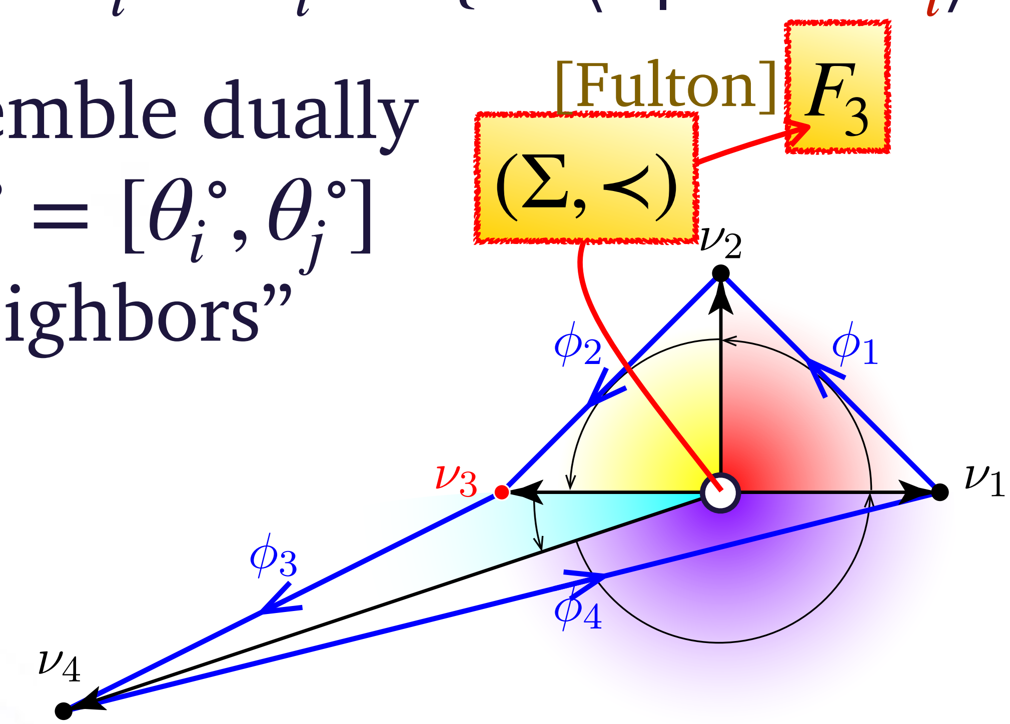

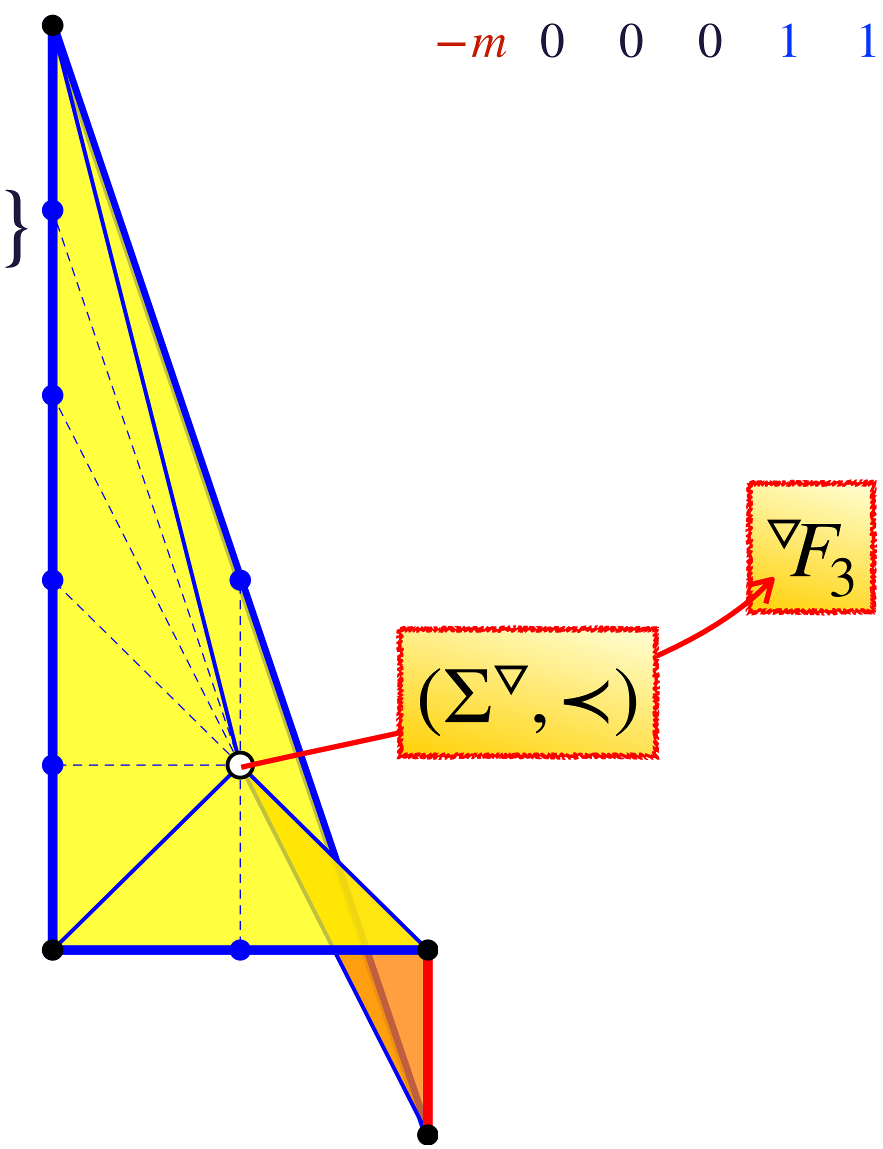

Laurent-Toric Fugue

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arXiv:1611.10300 +much more

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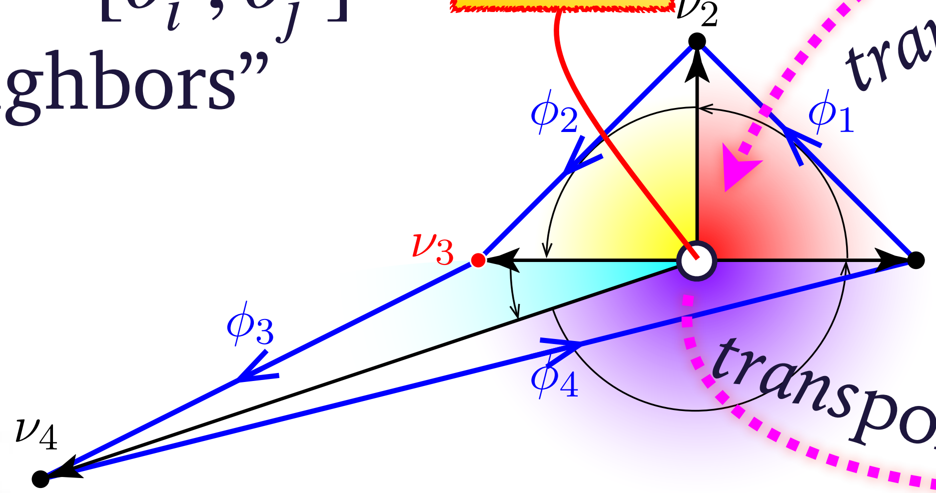
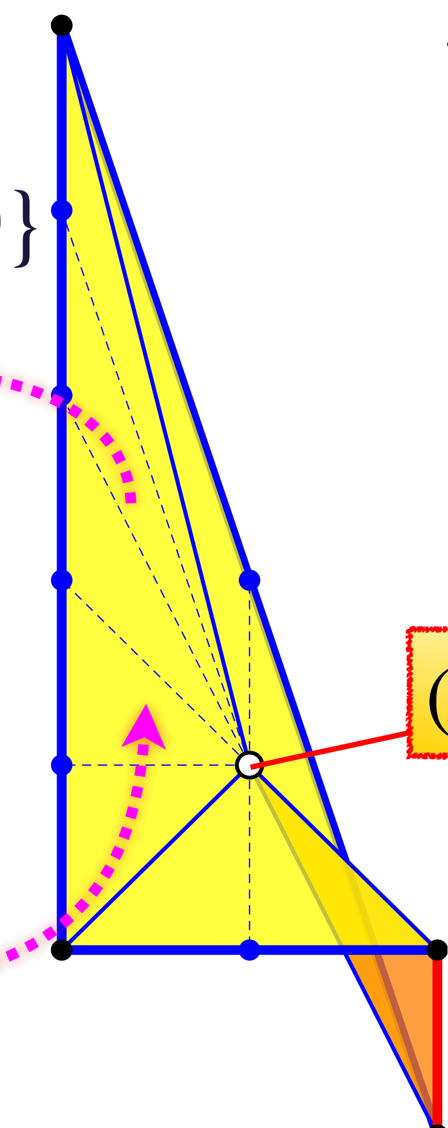
[Fulton] F_3

$(\Sigma, \prec)$



transpolar

transpolar



$^{\nabla}F_3$

$(\Sigma^{\nabla}, \prec)$

poset $(\Sigma, \prec)$	$\leftrightarrow$	dual poset $(\Sigma^{\nabla}, \prec)$

$F_3[c_1] \overset{\text{MM}}{\longleftrightarrow} {}^{\nabla}F_3[c_1]$

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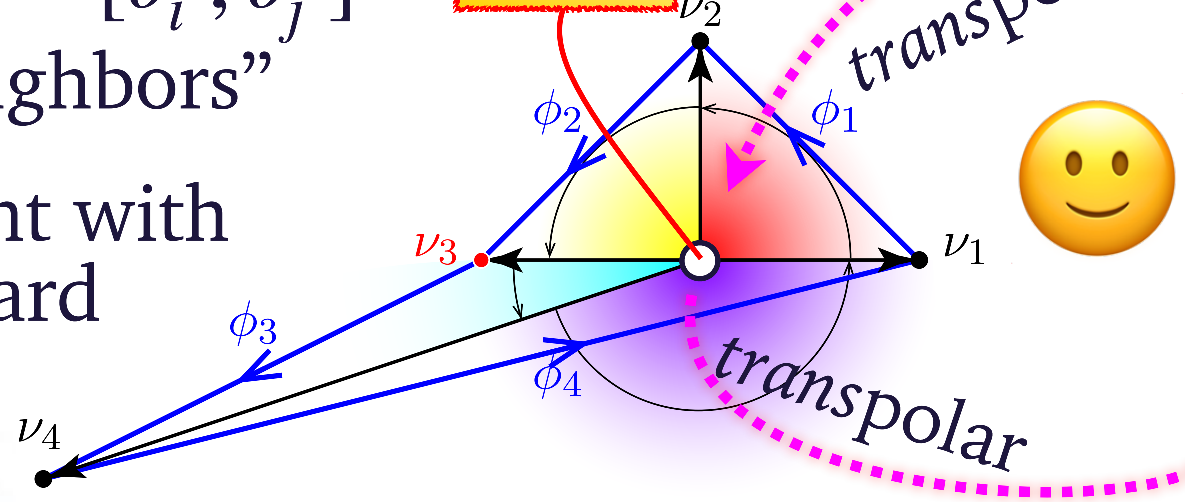
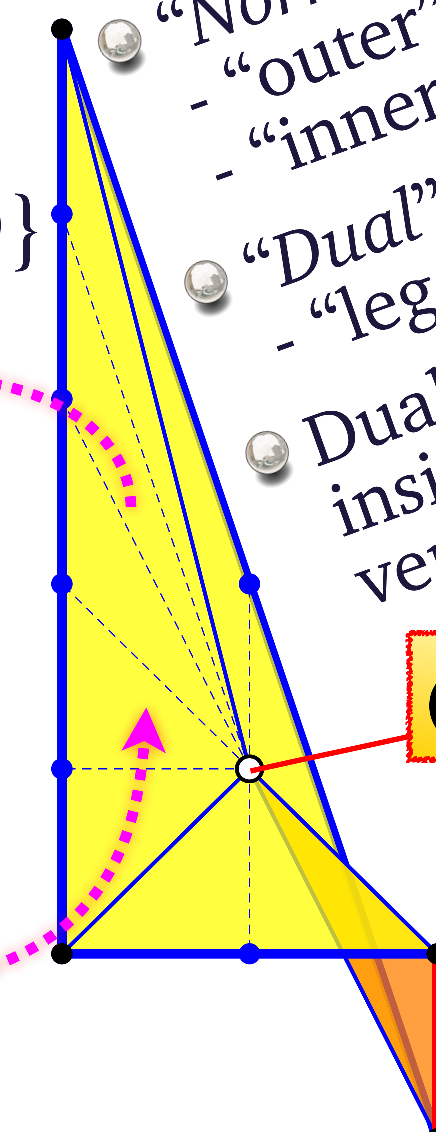
[Fulton] F_3

$(\Sigma, \prec)$





- "Normal fan"
  - "outer" [GE]
  - "inner/local" [C,L&S]
- "Dual"
  - "legal loops" [P&RV]
- Dual cones ↦ inside opening vertex-cones [?BH]

${}^{\nabla}F_3$

$(\Sigma^{\nabla}, \prec)$

'92: Khovanskii
+Pukhlikov
'93: Karshon
+Tolman
'99: Hattori
+Masuda
+lots of
(pre)symplectic geometry

poset $(\Sigma, \prec)$	$\leftrightarrow$	**dual poset** $(\Sigma^{\nabla}, \prec)$

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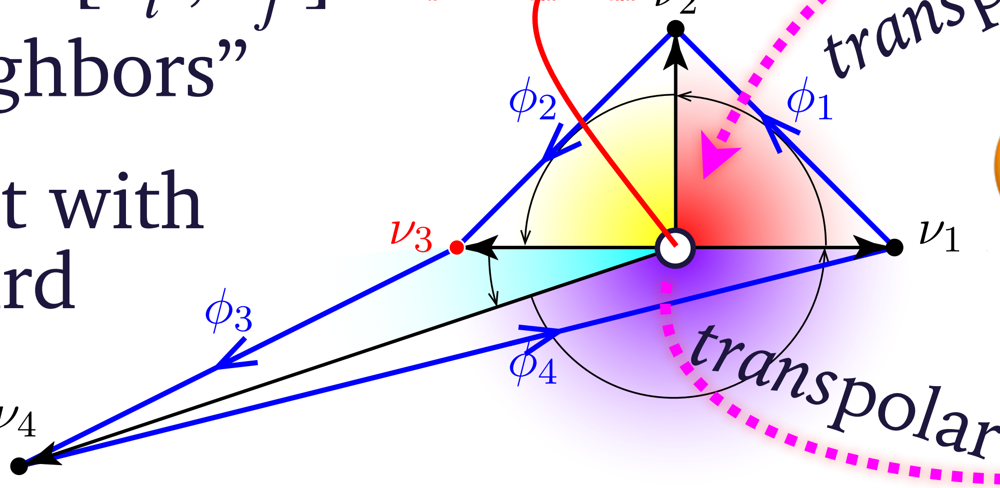
    (pre)complex algebraic geometry

[Fulton] F_3

$(\Sigma, \prec)$



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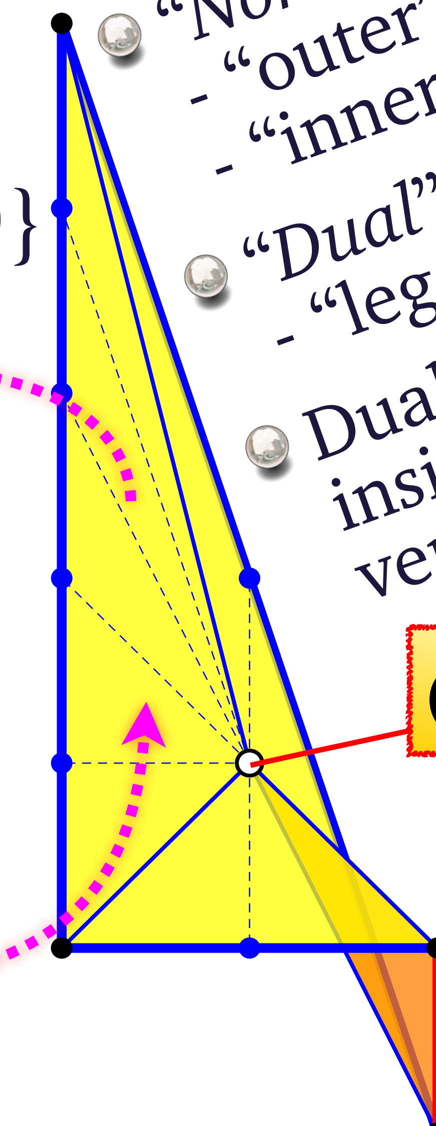
$^\nabla F_3$

$(\Sigma^\nabla, \prec)$



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poset $(\Sigma, \prec)$ ⟷ **dual poset** $(\Sigma^\nabla, \prec)$

$F_3[c_1] \overset{\text{MM}}{\longleftrightarrow} {}^\nabla F_3[c_1]$ → *(pre)symplectic geometry*

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arXiv:1611.10300
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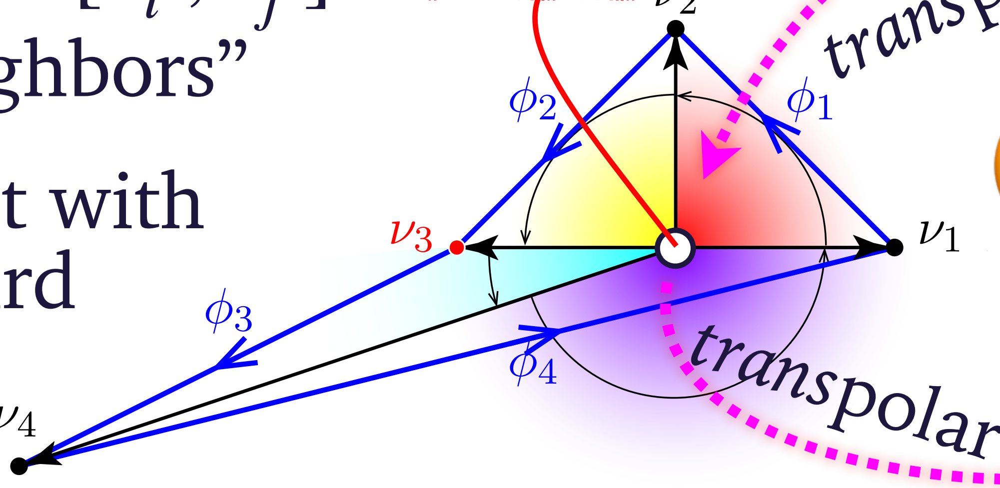
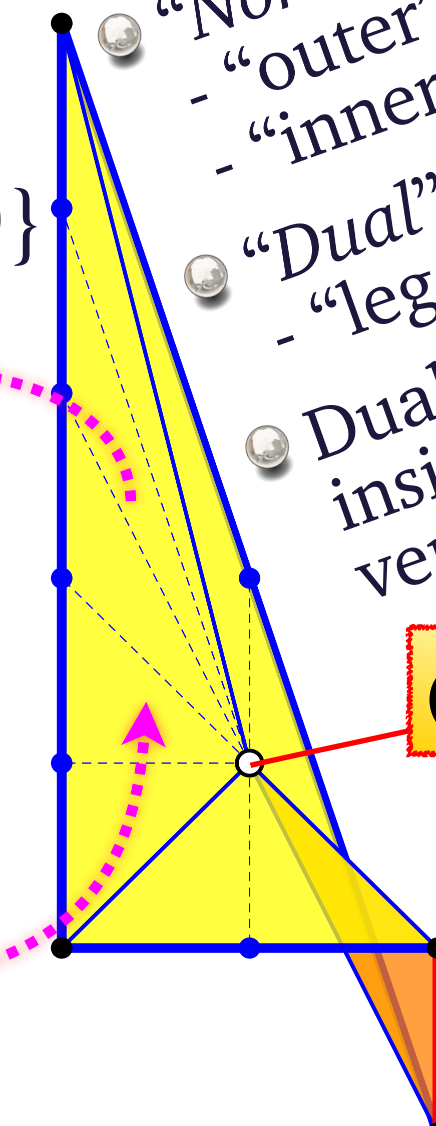
[Fulton] F_3

$(\Sigma, \prec)$





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(pre)complex *algebraic geometry*

poset $(\Sigma, \prec)$ $\longleftrightarrow$ **dual poset** $(\Sigma^\nabla, \prec)$

$F_3[c_1] \overset{\text{MM}}{\longleftrightarrow} {}^\nabla F_3[c_1]$ → (pre)symplectic *geometry*

Laurent-Toric Fugue

& Non-Convex Mirrors

—*Proof-of-Concept*—

arXiv:1611.10300 +much more

- K3 in $F_3^{(3)}$, one of *two* “cornerstone” mirror pairs:

$$a_1\, x_4^8 + a_2\, x_3^8 + a_3\, \frac{x_1^3}{x_3} + a_5\, \frac{x_2^3}{x_3} : \quad \exp\left\{ 2i\pi \begin{bmatrix} \frac13 & \frac23 & 0 & 0 \\ \frac1{24} & \frac1{24} & \frac18 & 0 \\ \hline \frac38 & \frac38 & \frac18 & \frac18 \end{bmatrix} \right\} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} : \quad \begin{cases} G = \mathbb{Z}_3 \times \mathbb{Z}_{24}, \\ Q = \mathbb{Z}_8. \end{cases}$$

$$\begin{bmatrix} 0 & 0 & 0 & 8 \\ 0 & 0 & 8 & 0 \\ 3 & 0 & -1 & 0 \\ 0 & 3 & -1 & 0 \end{bmatrix} \qquad \mathbb{P}^3_{(3:3:1:1)}[8] \qquad \mathbb{P}^3_{(3:5:8:8)}[24]/\mathbb{Z}_3$$

B³H²K

$$b_1\, y_3^3 + b_2\, y_5^3 + b_3\, \frac{y_2^8}{y_3\, y_5} + b_4\, y_1^8 : \quad \exp\left\{ 2i\pi \begin{bmatrix} \frac18 & 0 & 0 & 0 \\ 0 & 0 & \frac13 & \frac23 \\ \hline \frac3{24} & \frac5{24} & \frac13 & \frac13 \end{bmatrix} \right\} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_5 \end{bmatrix} : \quad \begin{cases} G^\nabla = \mathbb{Z}_8 \\ Q^\nabla = \mathbb{Z}_{24} \times \mathbb{Z}_3 \end{cases}$$

$$\frac{|G|}{|Q|} = \frac{3{\cdot}24}{8} = 9 = \frac{d(\Delta_{\mathcal{F}_3})}{d(\Delta^\star_{\mathcal{F}_3})} = \frac{54}{6}$$

- The Hilbert space & interactions restricted by the symmetries
  - Analysis: classical, semi-classical, quantum corrections…
  - …in spite of the manifest singularity in the (super)potential

Laurent-Toric Fugue

& Non-Convex Mirrors

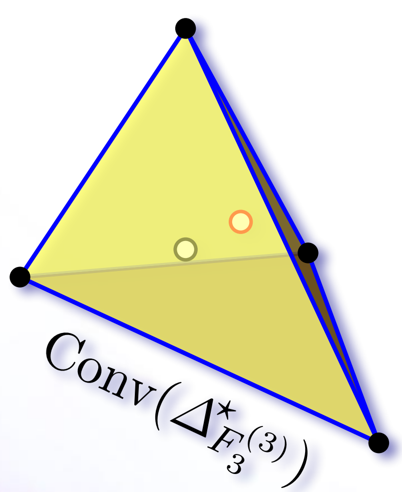
—*Proof-of-Concept*—

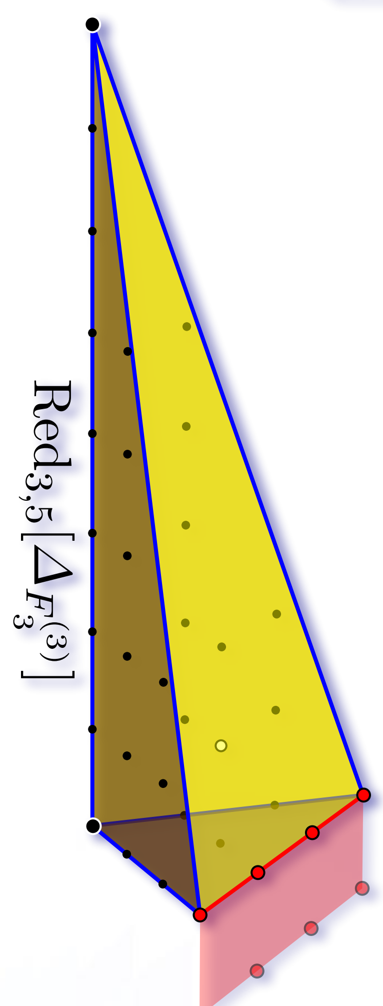
- Toric transposition:

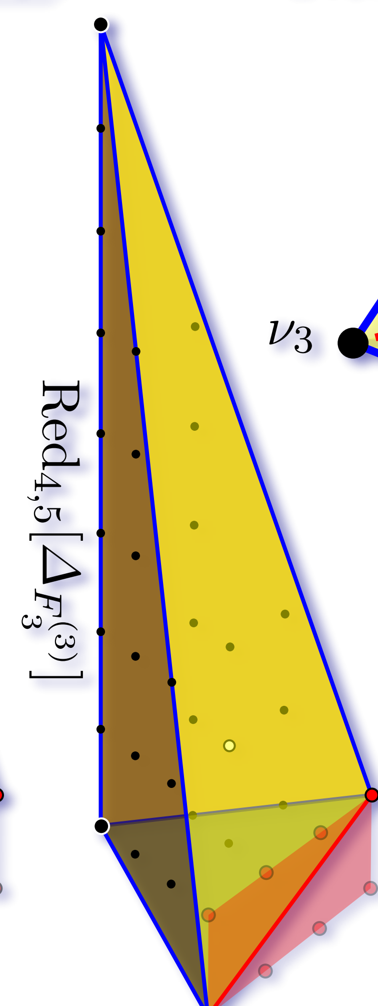
$$f(x;\Delta_{F_m^{(3)}}) = a_1\, x_1{}^3\, x_4{}^{2m+2} + a_2\, x_1{}^3\, x_5{}^{2m+2} + \underline{a_3\, \frac{x_2{}^3}{x_4{}^{m-2}}} + a_4\, \frac{x_2{}^3}{x_5{}^{m-2}} + \underline{a_5\, \frac{x_3{}^3}{x_4{}^{m-2}}} + a_6\, \frac{x_3{}^3}{x_5{}^{m-2}}$$

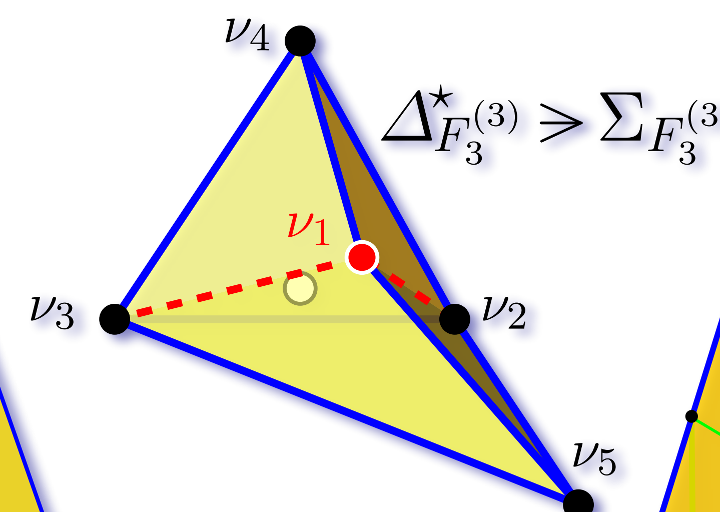
$$g(y;\Delta^\star_{F_m^{(3)}}) = \underbrace{b_1\, y_1{}^3\, y_2{}^3}_{\nu_1} + b_2\, \underline{y_3{}^3}\, y_4{}^3 + b_3\, \underline{y_5{}^3}\, y_6{}^3 + b_4\, \frac{y_1{}^{2m+2}}{(\underline{y_3\, y_5})^{m-2}} + b_5\, \frac{y_2{}^{2m+2}}{(y_4\, y_6)^{m-2}}$$

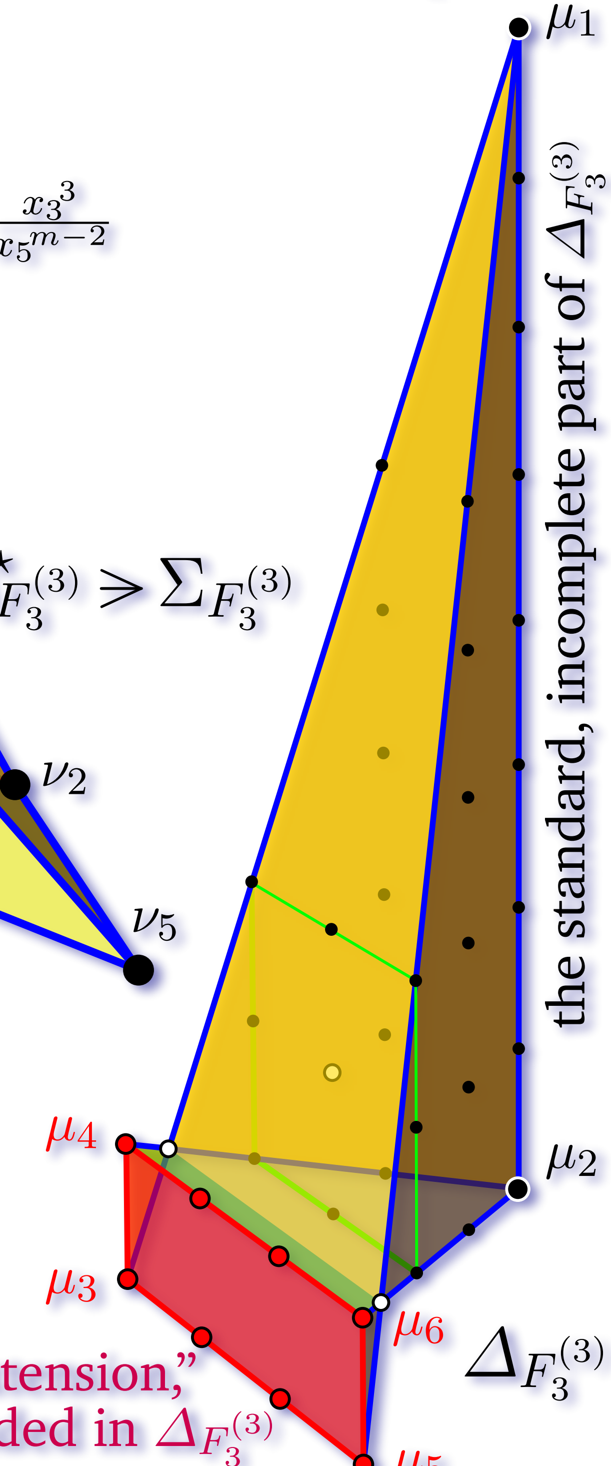
$$\mathbb{E} = \begin{bmatrix} 3 & 0 & 0 & 2m{+}2 & 0 \\ 3 & 0 & 0 & 0 & 2m{+}2 \\ 0 & 3 & 0 & 2{-}m & 0 \\ 0 & 3 & 0 & 0 & 2{-}m \\ 0 & 0 & 3 & 2{-}m & 0 \\ 0 & 0 & 3 & 0 & 2{-}m \end{bmatrix}$$









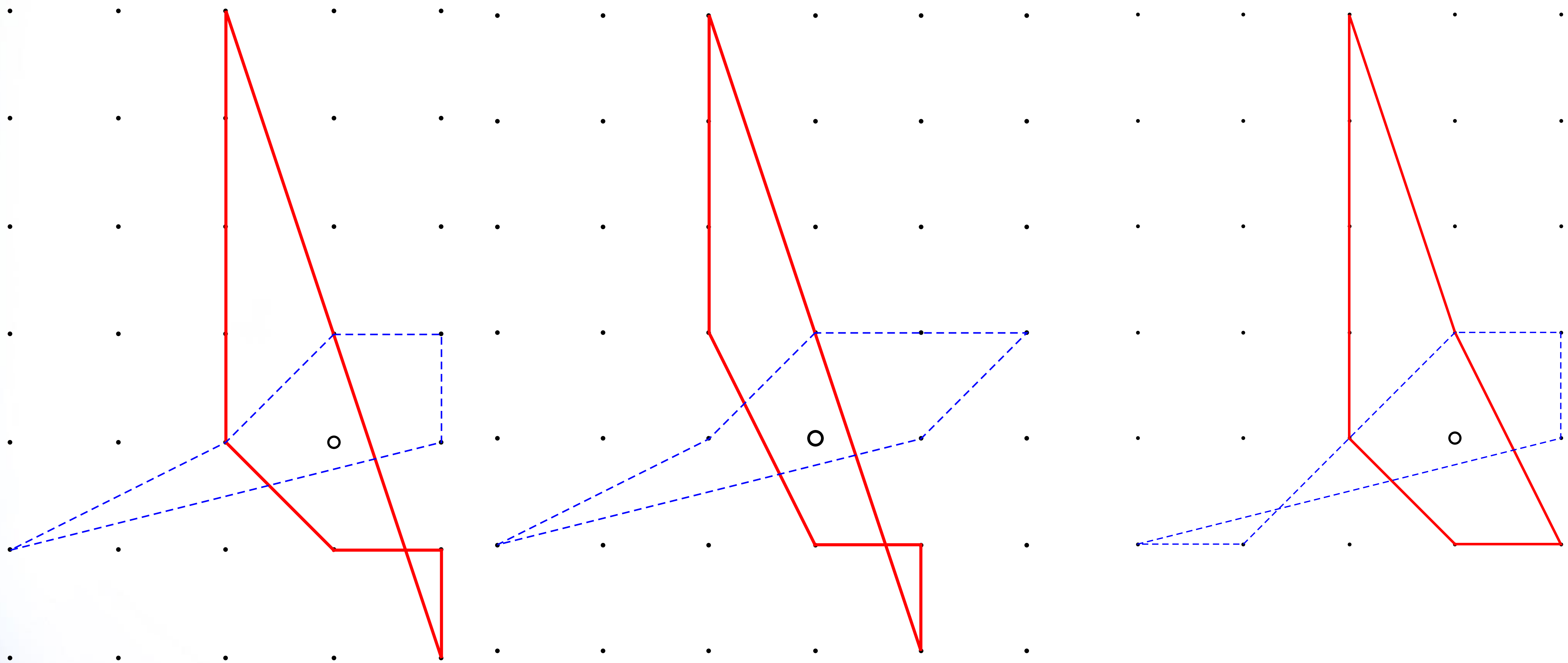


Laurent-Toric Fugue

& Non-Convex Mirrors

—*Proof-of-Concept*—

- Not just Hirzebruch n-folds, either:
  - Buckets of 2-dimensional polygons, invented to test $\nabla: \Delta^{\star} \overset{1\text{–}1}{\longleftrightarrow} \Delta$



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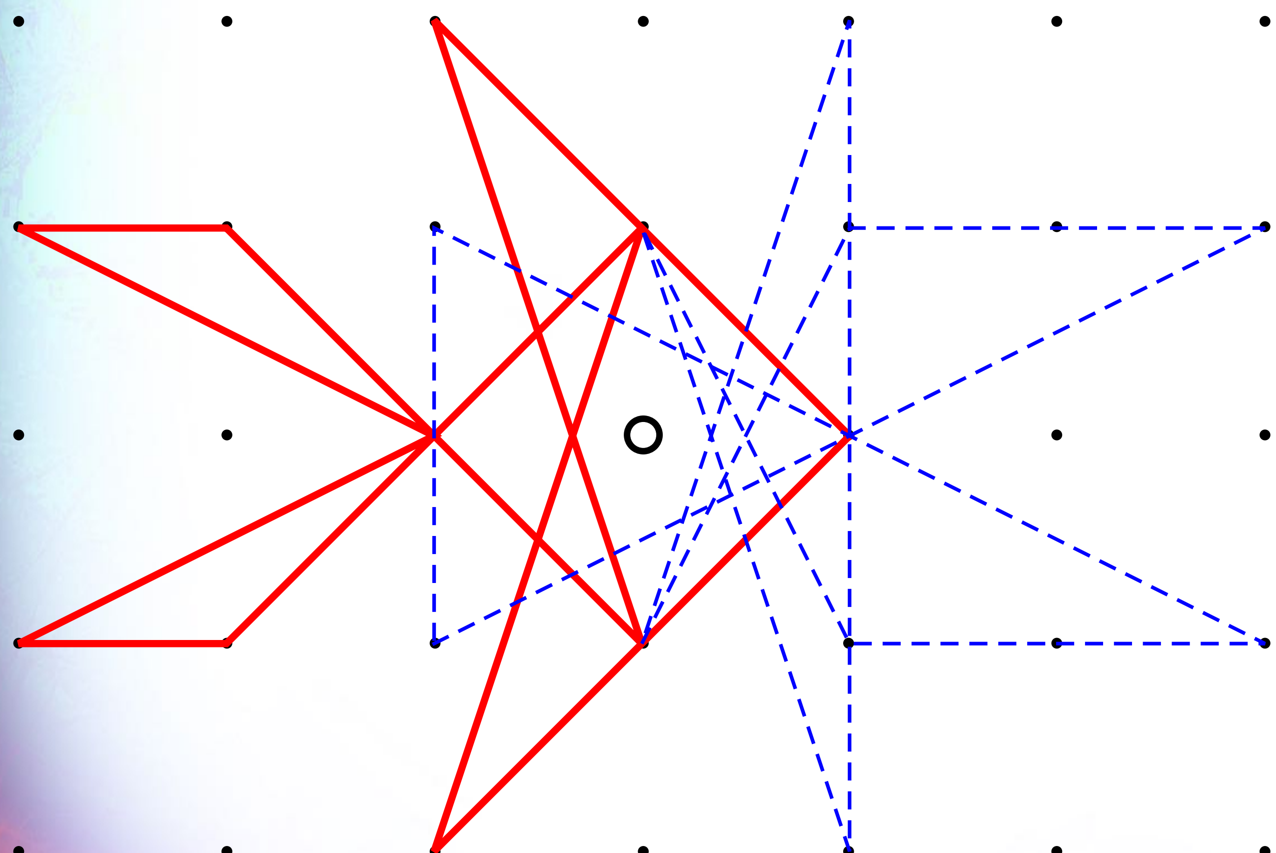

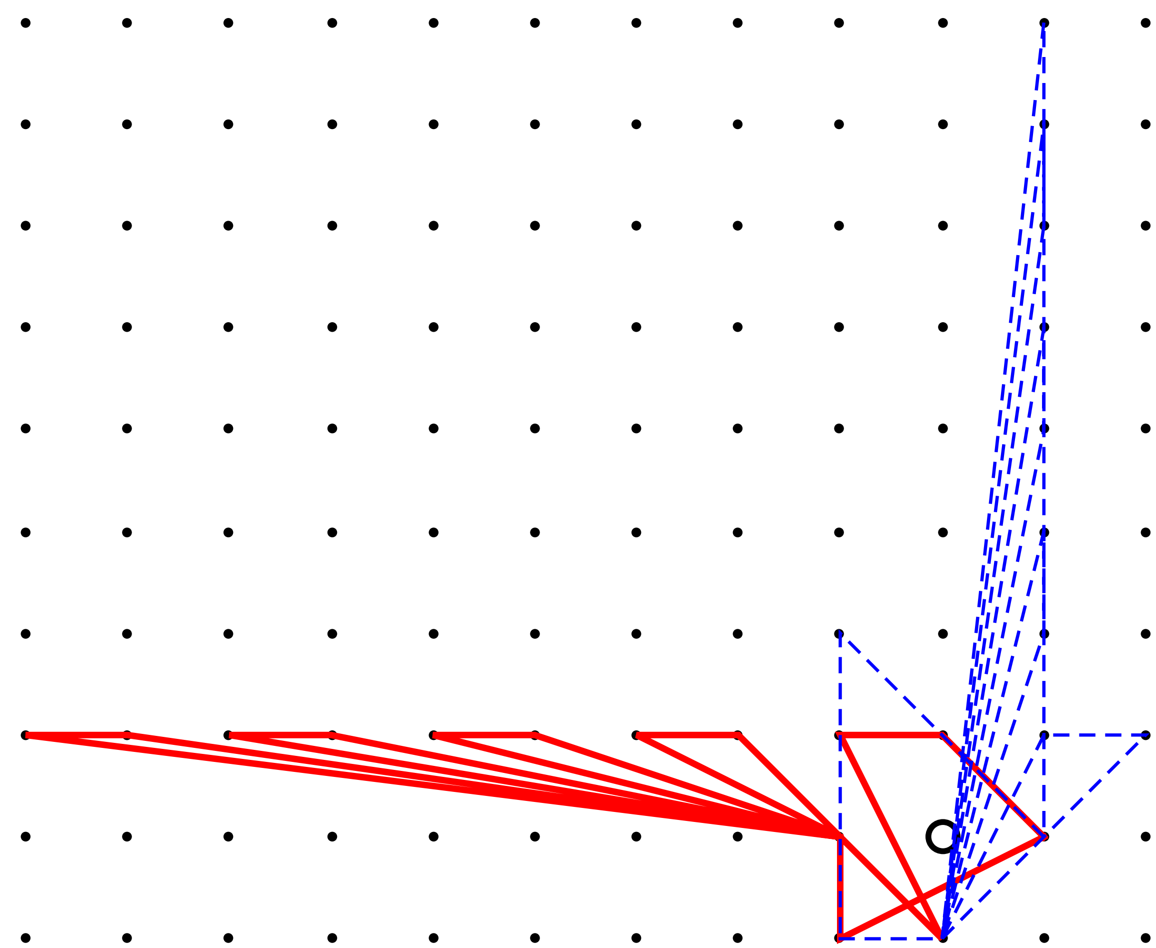

Laurent-Toric Fugue

& Non-Convex Mirrors

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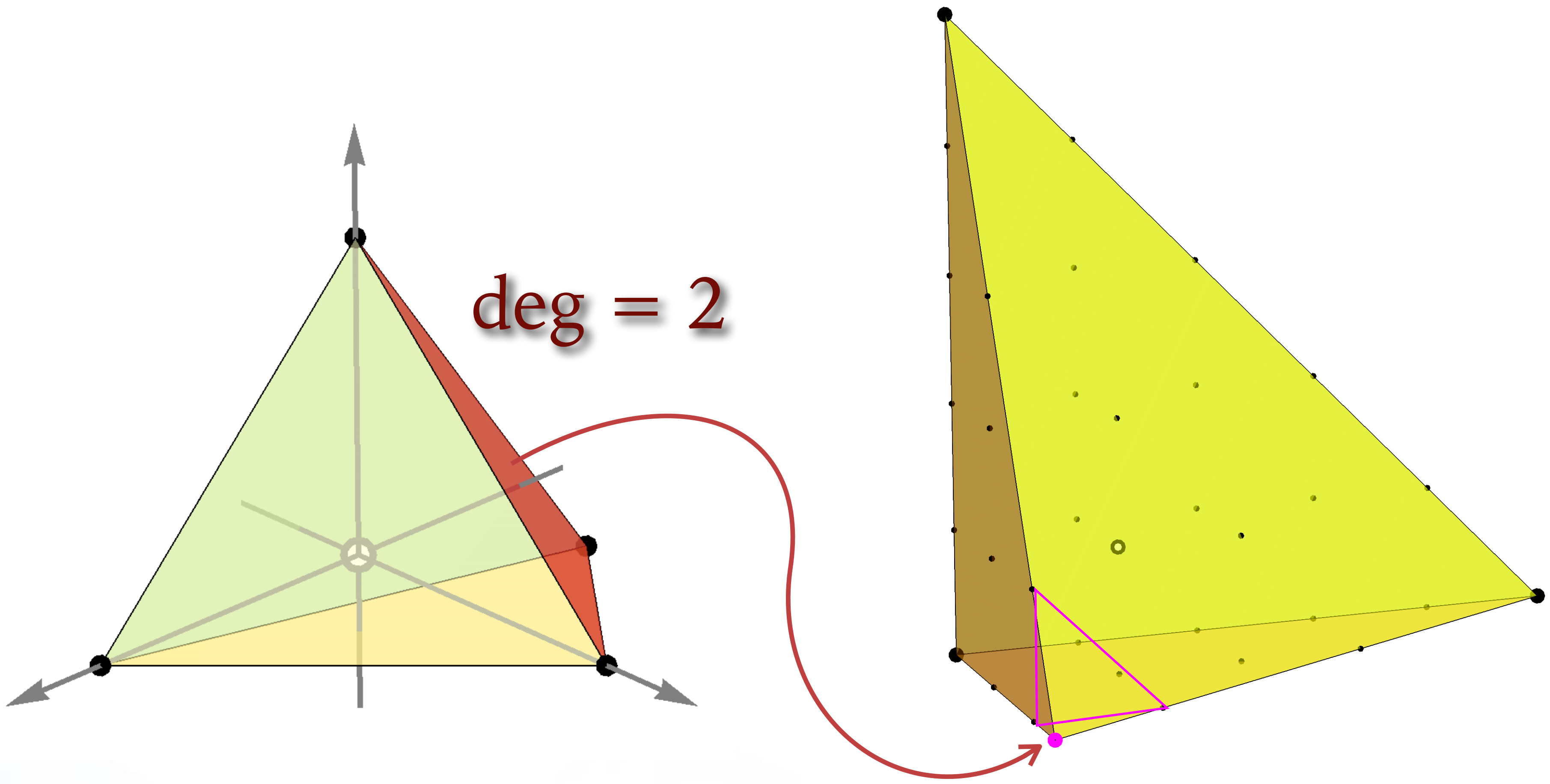
- Not just Hirzebruch n-folds, either:
  - Buckets of 2-dimensional polygons, invented to test $\nabla : \Delta^\star \xleftrightarrow{1\text{–}1} \Delta$
  - And, plenty of 3-dimensional polyhedra:



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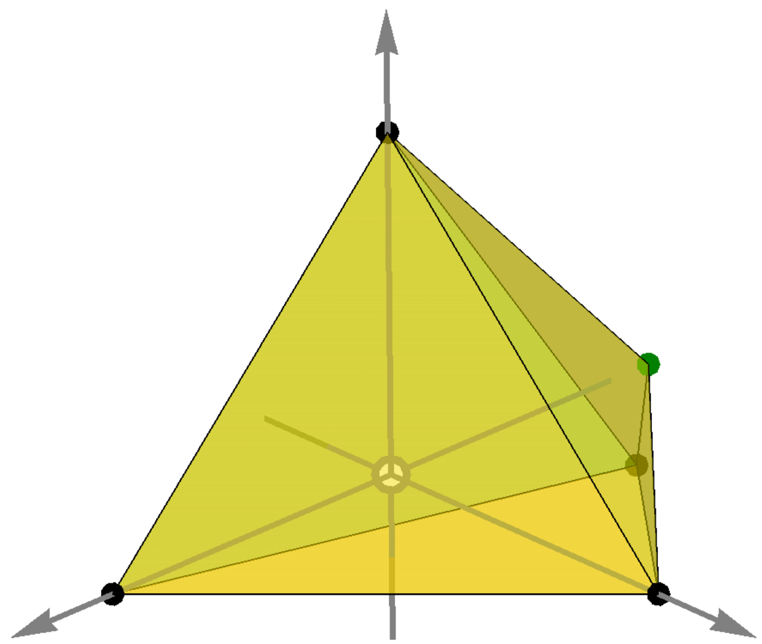

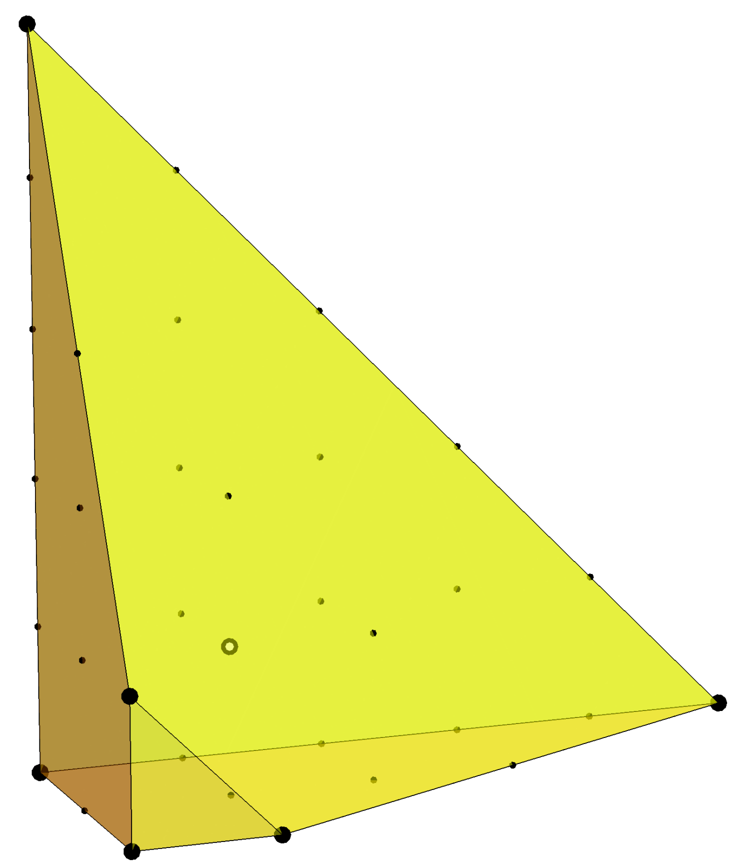

Laurent-Toric Fugue

& Non-Convex Mirrors

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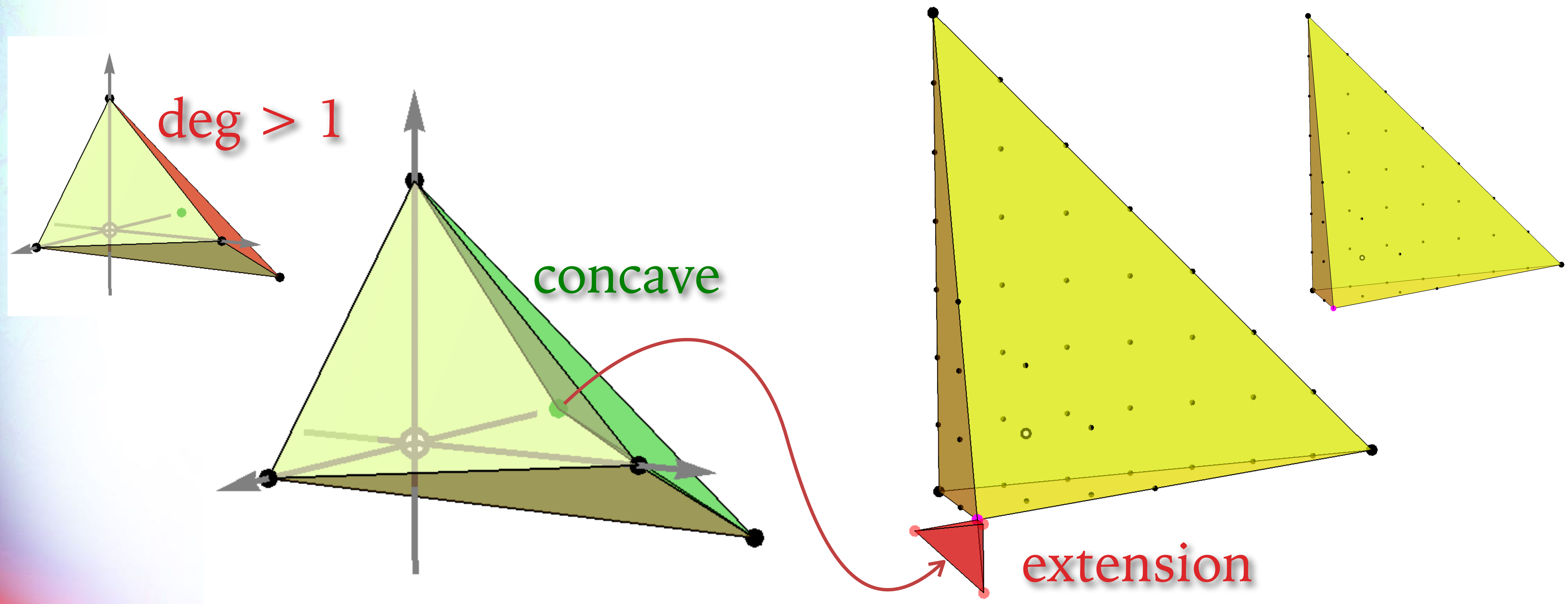

Laurent-Toric Fugue

& Non-Convex Mirrors

—*Proof-of-Concept*—

arXiv:1611.10300
+much more

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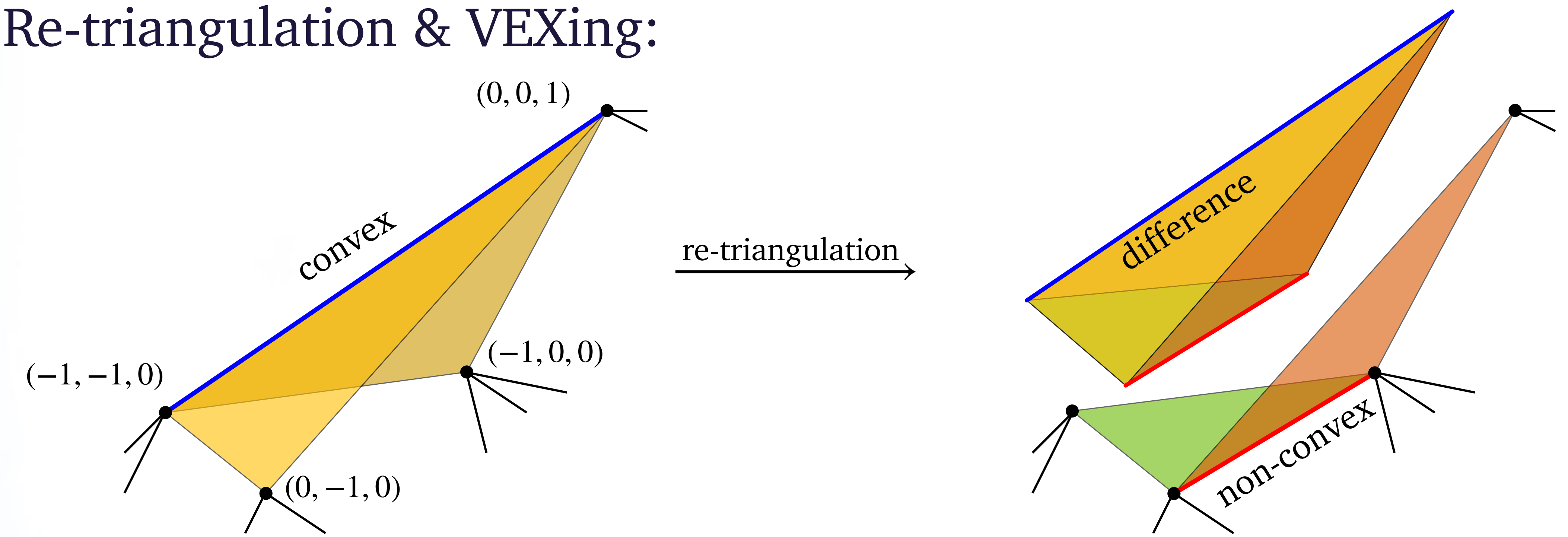
  - And, plenty of 3-dimensional polyhedra:
  - Re-triangulation & VEXing:



Laurent-Toric Fugue

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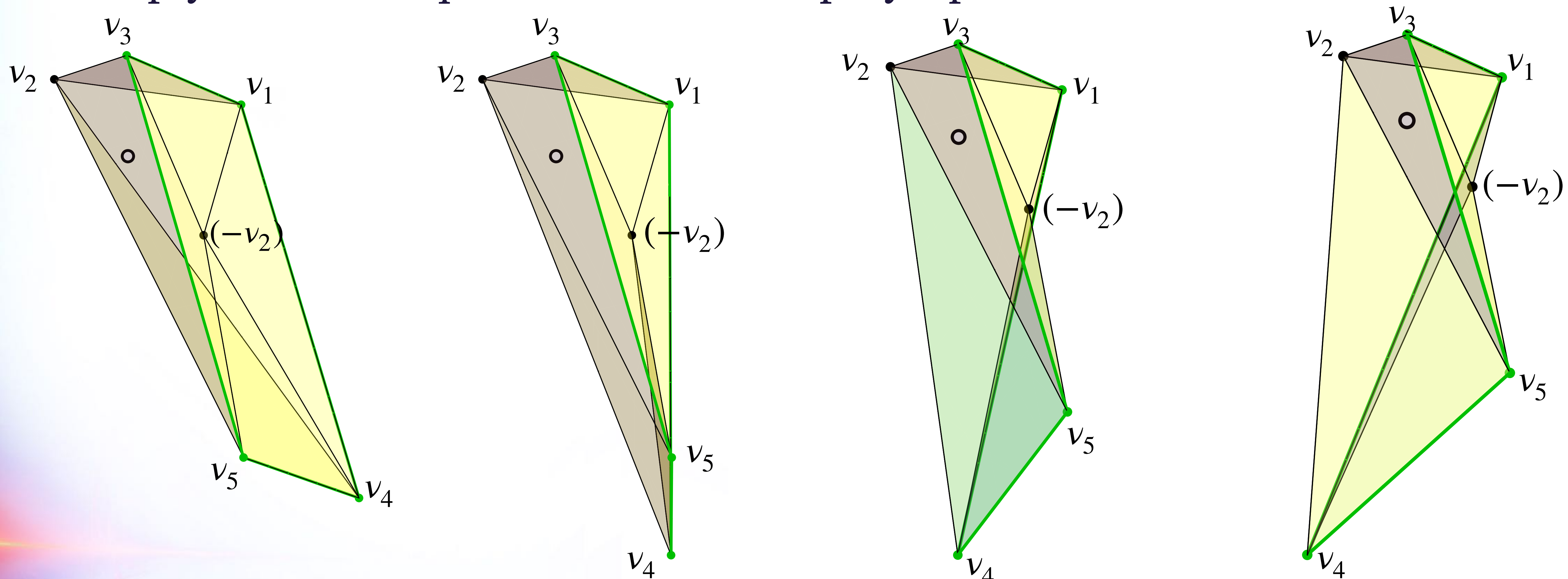
  - And, plenty of 3-dimensional polyhedra:
  - Re-triangulation & VEXing:
  - Multiply infinite sequences of twisted polytopes:



Laurent-Toric Fugue

& Non-Convex Mirrors

—*Proof-of-Concept*—

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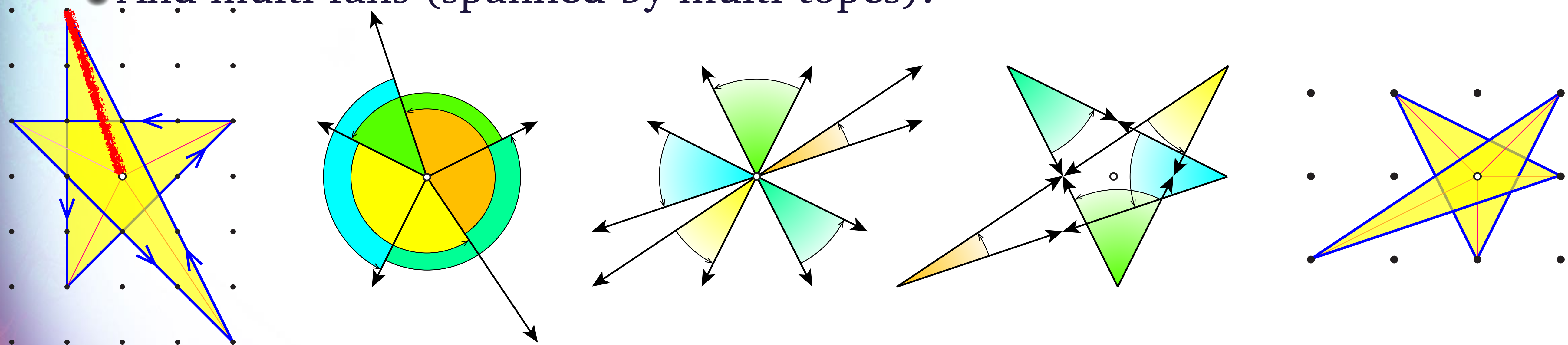
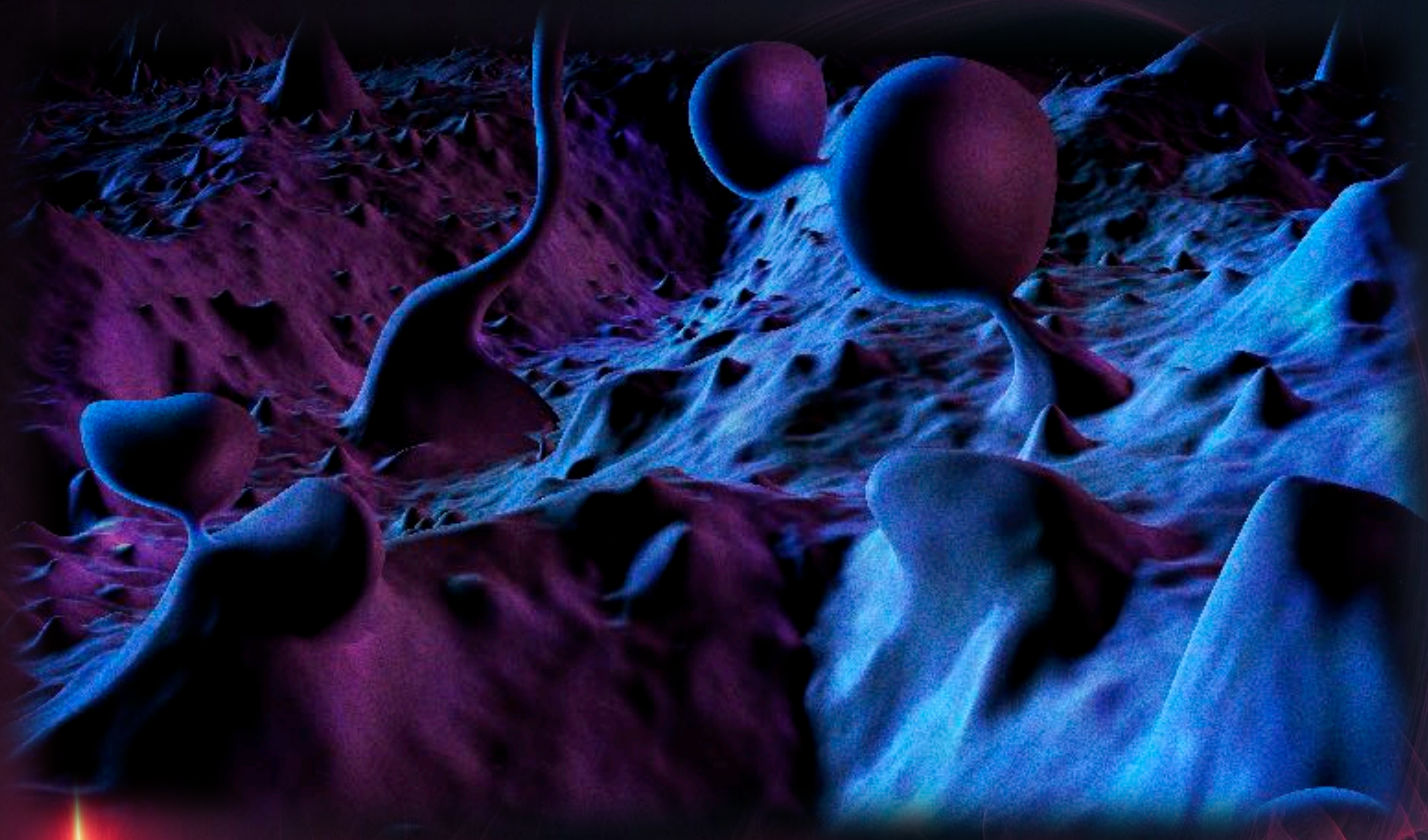
- Not just Hirzebruch n-folds, either:
  - Buckets of 2-dimensional polygons, invented to test $\nabla: \Delta^\star \xleftrightarrow{1\text{–}1} \Delta$
  - And, plenty of 3-dimensional polyhedra:
  - Re-triangulation & VEXing:
  - Multiply infinite sequences of twisted polytopes:
  - And multi-fans (spanned by multi-topes):



winding number (multiplicity, Duistermaat-Heckman fn.) $= 2$

[A. Hattori+M. Masuda" *Theory of Multi-Fans*, Osaka J. Math. **40** (2003) 1–68]



Discriminant Divertimento

(How Small Can We Go?)

Discriminant Divertimento

arXiv:1611.10300
+much more

The Phase-Space

—*Proof-of-Concept*—

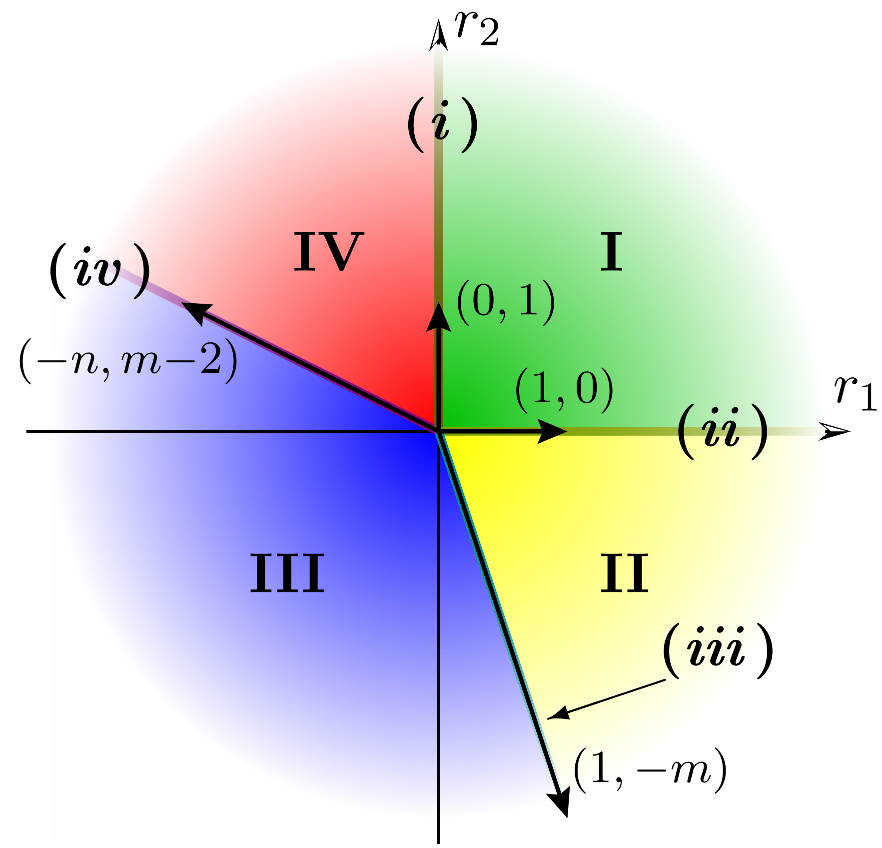
- The (super)potential: $W(X) := X_0 \cdot f(X)$,

$$f(X) := \sum_{j=1}^{2} \Big(\sum_{i=2}^{n} (a_{ij}\, X_i^n) X_{n+j}^{2-m} + a_j\, X_1^n X_{n+j}^{(n-1)m+2} \Big)$$

	X_0	X_1	X_2	$\cdots$	X_n	X_{n+1}	X_{n+2}
Q^1	$-n$	1	1	$\cdots$	1	0	0
Q^2	$m-2$	$-m$	0	$\cdots$	0	1	1

- The possible vevs

	$\lvert x_0\rvert$	$\lvert x_1\rvert$	$\lvert x_2\rvert$	$\cdots$	$\lvert x_n\rvert$	$\lvert x_{n+1}\rvert$	$\lvert x_{n+2}\rvert$
i	0	0	0	$\cdots$	0	$*$	$*$
I	0	$*$	$*$	$\cdots$	$*$	$*$	$*$
ii	0	0	$*$	$\cdots$	$*$	0	0
II	0	$\lvert x_1\rvert = \sqrt{\frac{\sum_j \lvert x_{n+j}\rvert^2 - r_2}{m}} = \sqrt{r_1 - \sum_{i=2}^{n} \lvert x_i\rvert^2} > 0$	$*$	$\cdots$	$*$	$*$	$*$
iii	0	$\sqrt{r_1}$	0	$\cdots$	0	0	0
III	$\sqrt{\frac{mr_1+r_2}{(n-1)m+2}}$	$\sqrt{\frac{(m-2)r_1+nr_2}{(n-1)m+2}}$	0	$\cdots$	0	0	0
iv	$\sqrt{-r_1/n}$	0	0	$\cdots$	0	0	0
IV	$\sqrt{-r_1/n}$	0	0	$\cdots$	0	$*$	$*$

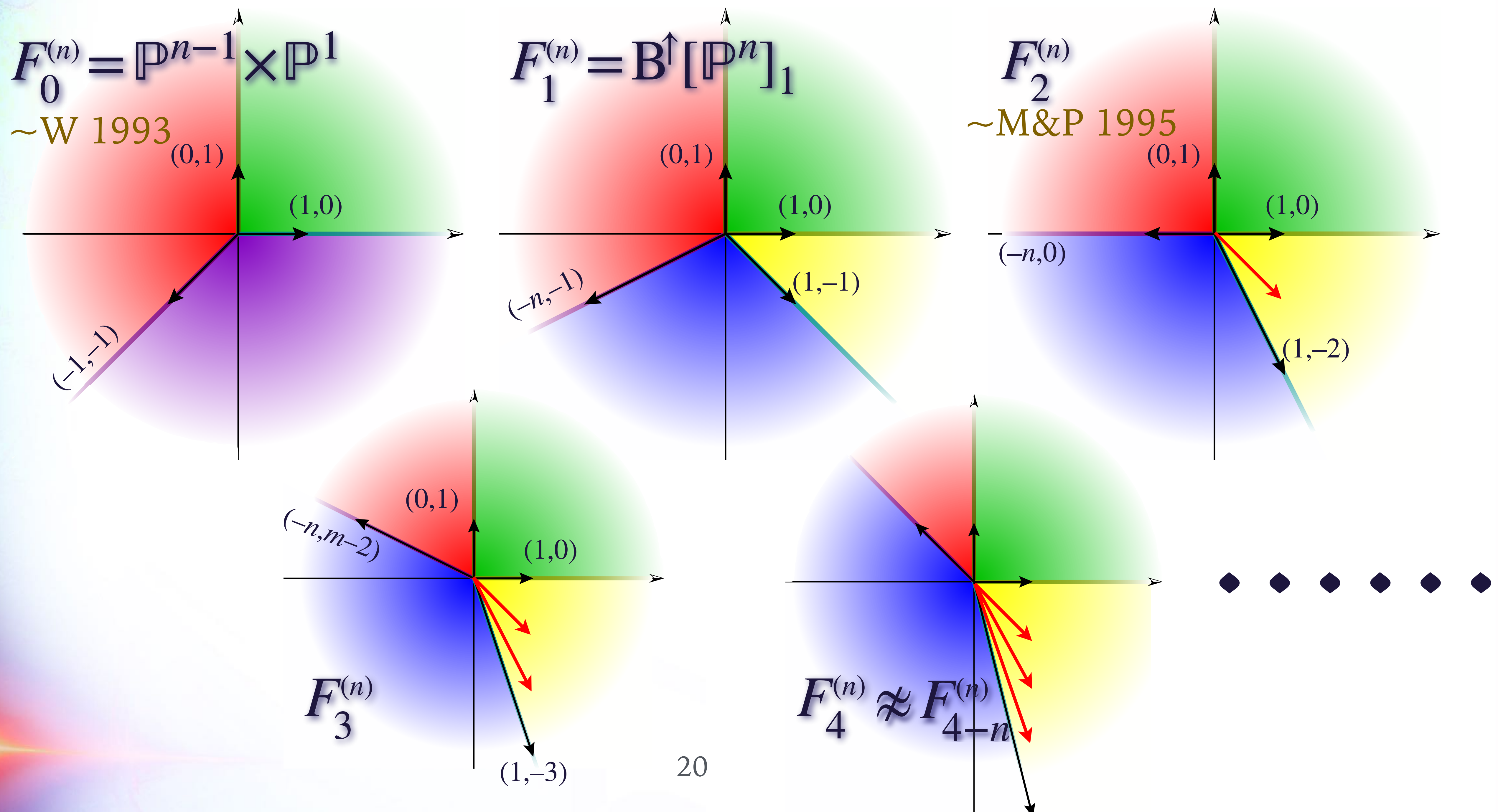


Discriminant Divertimento

The Phase-Space

—*Proof-of-Concept*—

- Varying m in $F_m^{(n)}$:



Discriminant Divertimento

arXiv: "real soon"

The Discriminant

—Proof-of-Concept—

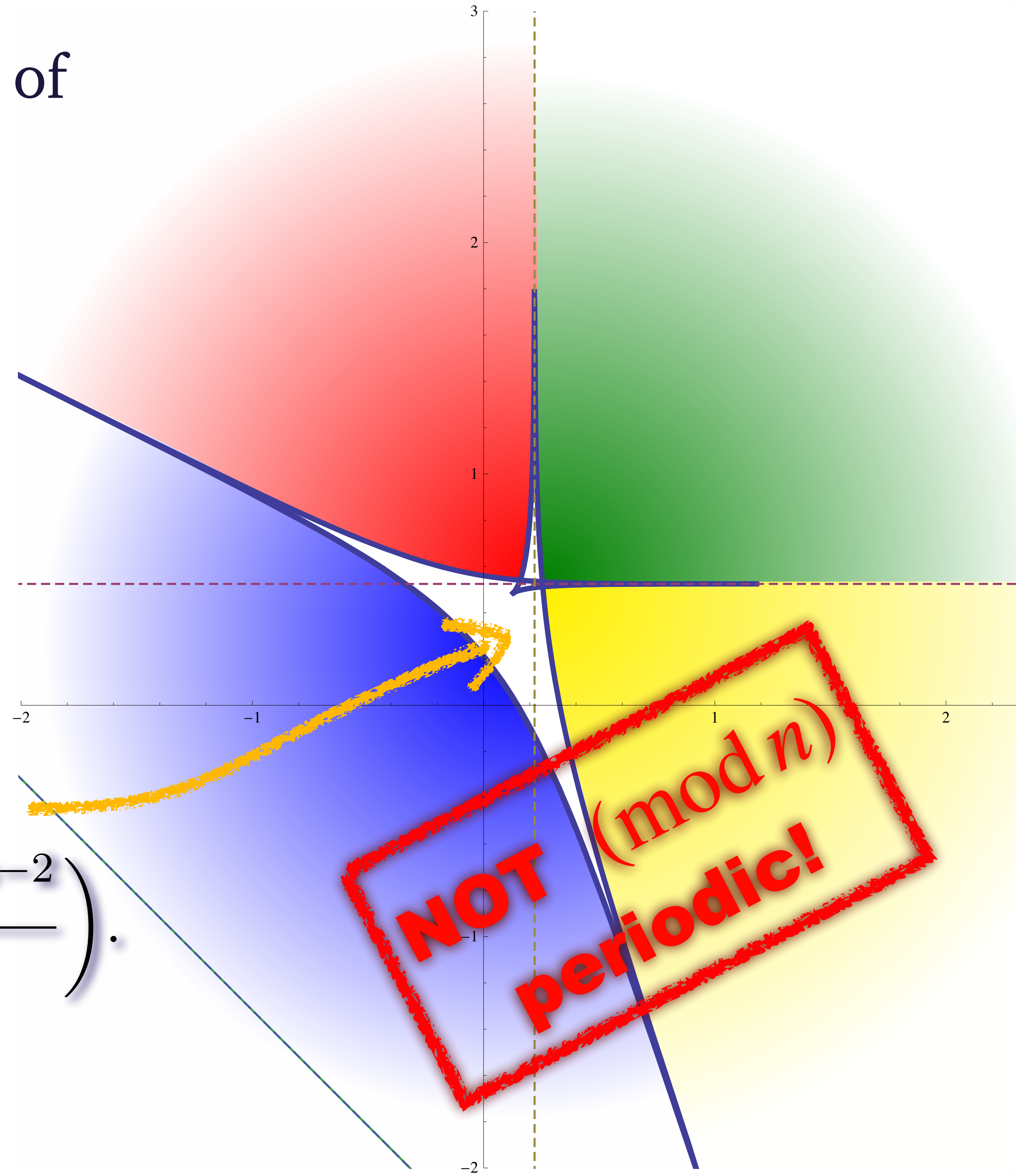
- Now add worldsheet instantons:
  - Near $(r_1, r_2) = (0,0)$, classical analysis of Kähler (metric) phase-space fails [M&P: arXiv:hep-th/9412236]
- With

	X_0	X_1	X_2	$\cdots$	X_n	X_{n+1}	X_{n+2}
Q^1	$-n$	1	1	$\cdots$	1	0	0
Q^2	$m-2$	$-m$	0	$\cdots$	0	1	1

- the instanton resummation gives:

$$r_1 + \frac{\hat{\theta}_1}{2\pi i} = -\frac{1}{2\pi} \log\left(\frac{\sigma_1^{n-1}(\sigma_1 - m\,\sigma_2)}{[(m-2)\sigma_2 - n\sigma_1]^n}\right),$$

$$r_2 + \frac{\hat{\theta}_2}{2\pi i} = -\frac{1}{2\pi} \log\left(\frac{\sigma_2^2\,[(m-2)\sigma_2 - n\sigma_1]^{m-2}}{(\sigma_1 - m\,\sigma_2)^m}\right).$$



...and a Mirror Motet

(Yes, *the* B^3H^2K-mirrors)

Mirror Motets

The Discriminant

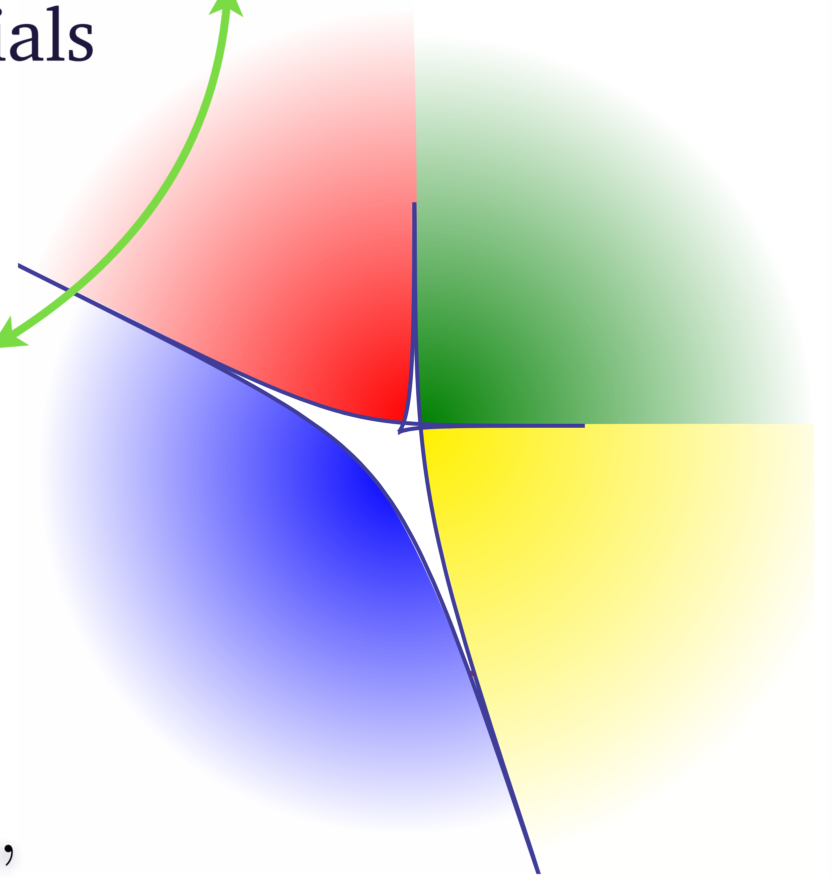
—Proof-of-Concept—

- Now compare with the complex structure of the B³H²K-mirror
  - Restricted to the "cornerstone" defining polynomials

$$f(x) = a_0 \prod_{\nu_i \in \Delta^\star} (x_{\nu_i})^{\langle \nu_i, \mu_0 \rangle + 1} + \sum_{\mu_I \in \Delta} a_{\mu_I} \prod_{\nu_i \in \Delta^\star} (x_{\nu_i})^{\langle \nu_i, \mu_I \rangle + 1}$$

transpose

$$g(y) = b_0 \prod_{\mu_I \in \Delta} (y_{\mu_I})^{\langle \mu_I, \nu_0 \rangle + 1} + \sum_{\nu_i \in \Delta^\star} b_{\nu_i} \prod_{\mu_I \in \Delta} (y_{\mu_I})^{\langle \mu_I, \nu_i \rangle + 1}$$



- In particular,

$$g(y) = \sum_{i=0}^{n+2} b_i\, \phi_i(y) \;\; = b_0\, \phi_0 + b_1\, \phi_1 + b_2\, \phi_2 + b_3\, \phi_3 + b_4\, \phi_4,$$

$$\phi_0 := y_1 \cdots y_4, \quad \phi_1 := y_1^2\, y_2^2, \quad \phi_2 := y_3^2\, y_4^2, \quad \phi_3 := \frac{y_1^{m+2}}{y_3^{m-2}}, \quad \phi_4 := \frac{y_2^{m+2}}{y_4^{m-2}},$$

$$z_1 = -\frac{\beta\,[(m-2)\beta + m]}{m+2}, \quad z_2 = \frac{(2\beta+1)^2}{(m+2)^2\,\beta^m}, \qquad \beta := \left[\frac{b_1\,\phi_1}{b_0\,\phi_0} \Big/ {}^{A}\mathscr{J}(g)\right],$$

$\phi_0^2 = \phi_1 \phi_2$ etc.

Mirror Motets

The Discriminant

—Proof-of-Concept—

- Now compare with the complex structure of the B³H²K-mirror
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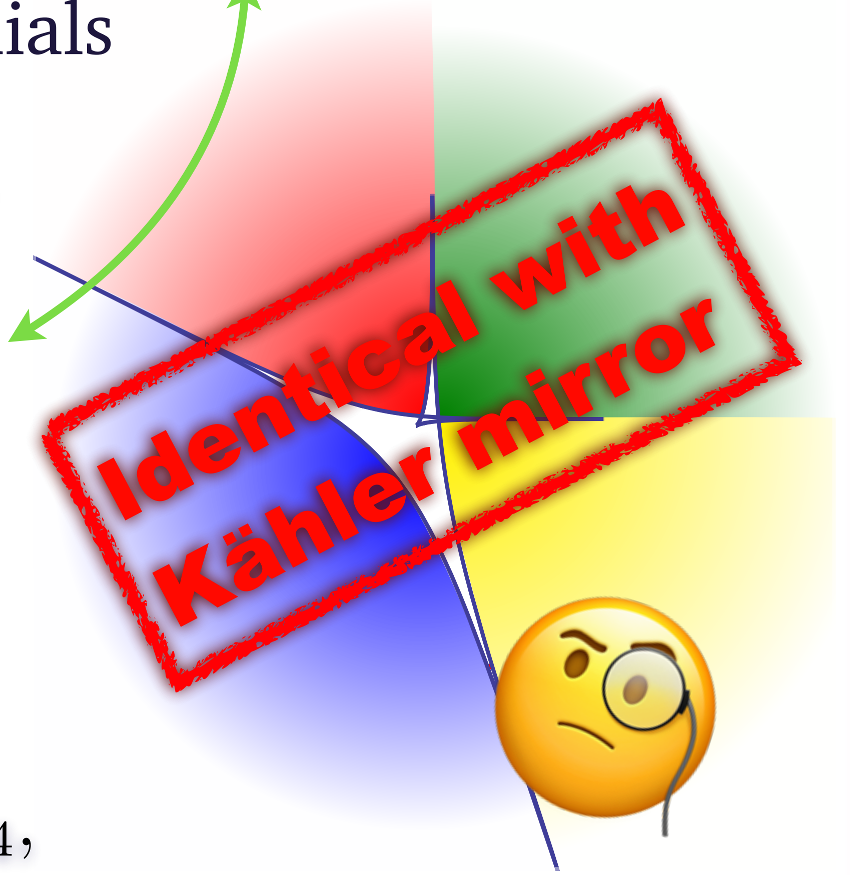
$$f(x) = a_0 \prod_{\nu_i \in \Delta^\star} (x_{\nu_i})^{\langle \nu_i, \mu_0 \rangle + 1} + \sum_{\mu_I \in \Delta} a_{\mu_I} \prod_{\nu_i \in \Delta^\star} (x_{\nu_i})^{\langle \nu_i, \mu_I \rangle + 1}$$

transpose

$$g(y) = b_0 \prod_{\mu_I \in \Delta} (y_{\mu_I})^{\langle \mu_I, \nu_0 \rangle + 1} + \sum_{\nu_i \in \Delta^\star} b_{\nu_i} \prod_{\mu_I \in \Delta} (y_{\mu_I})^{\langle \mu_I, \nu_i \rangle + 1}$$



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$\phi_0^2 = \phi_1 \phi_2$ etc.

Mirror Motets

The Discriminant

—Proof-of-Concept—

- So: $\mathscr{M}({}^{\nabla}\!F^{(n)}_m[c_1]) \overset{\text{mm}}{\approx} \mathscr{W}(F^{(n)}_m[c_1])$ — easy: 2-dimensional
- In fact, also: $\mathscr{W}({}^{\nabla}\!F^{(n)}_m[c_1]) \overset{\text{mm}}{\approx} \mathscr{M}(F^{(n)}_m[c_1])$

✅…restricted to no (MPCP) blow-ups & “cornerstone” polynomial

- Then, $\dim \mathscr{W}({}^{\nabla}\!F^{(n)}_m[c_1]) = n = \dim \mathscr{M}(F^{(n)}_m[c_1])$
- Same methods:

$$e^{2\pi i\,\tilde\tau_\alpha} = \prod_{I=0}^{2n}\Big(\sum_{\beta=1}^{2}\tilde Q_I^\beta\,\tilde\sigma_\beta\Big)^{\tilde Q_I^\alpha}$$

$$\tilde z_a = \prod_{I=0}^{2n}(a_I\,\varphi_I(x))^{\tilde Q_I^\alpha}\Big/{}^{A}\mathscr{J}$$

$n=4$

I	$(\sum_\beta \tilde Q_I^\beta\,\tilde\sigma_\beta)$	$(a_I\,\varphi_I)/{}^{A}\mathscr{J}_{(210)}(f)$
0	$-2(m{+}2)(\tilde\sigma_1+\tilde\sigma_2)$	$-2\big((a_3\,\varphi_3)+(a_4\,\varphi_4)\big)$
1	$m\,\tilde\sigma_1+2\,\tilde\sigma_2$	$\frac{m\,(a_3\,\varphi_3)+2\,(a_4\,\varphi_4)}{m+2}$
2	$2\,\tilde\sigma_1+m\,\tilde\sigma_2$	$\frac{2\,(a_3\,\varphi_3)+m\,(a_4\,\varphi_4)}{m+2}$
3	$(m{+}2)\,\tilde\sigma_1$	$(a_3\,\varphi_3)$
4	$(m{+}2)\,\tilde\sigma_2$	$(a_4\,\varphi_4)$

🥳

Laurent GLSM Coda

arXiv: "real soon"

—*Proof-of-Concept*—

Summary

- CY($n-1$)-folds in Hirzebruch n-folds
  - Euler characteristic ✅
  - Chern class, term-by-term ✅
  - Hodge numbers ✅ (*jump @* $^\sharp\mathscr{X}$)
  - Cornerstone polynomials & mirror ✅
  - Phase-space regions & mirror ✅
  - Phase-space discriminant & mirror ✅
  - The "other way around" ✅ (*limited!*)
  - Yukawa couplings ✅
  - World-sheet instantons ✅
  - Gromov-Witten invariants 🔜?✅
    - *Will there be anything else? …being ML-datamined*

- Oriented polytopes
- Trans-polar$^\triangledown$ constr.
- Newton $\varDelta_X := (\varDelta^\star_X)^\triangledown$
- VEX polytopes s.t.: $((\varDelta)^\triangledown)^\triangledown = \varDelta$
- Star-triangulable w/flip-folded faces
- Polytope *extension* ⇔ Laurent monomials

B^3H^2K mirrors

Toric & GLSM textbooks to be 😇😈 …*extended*

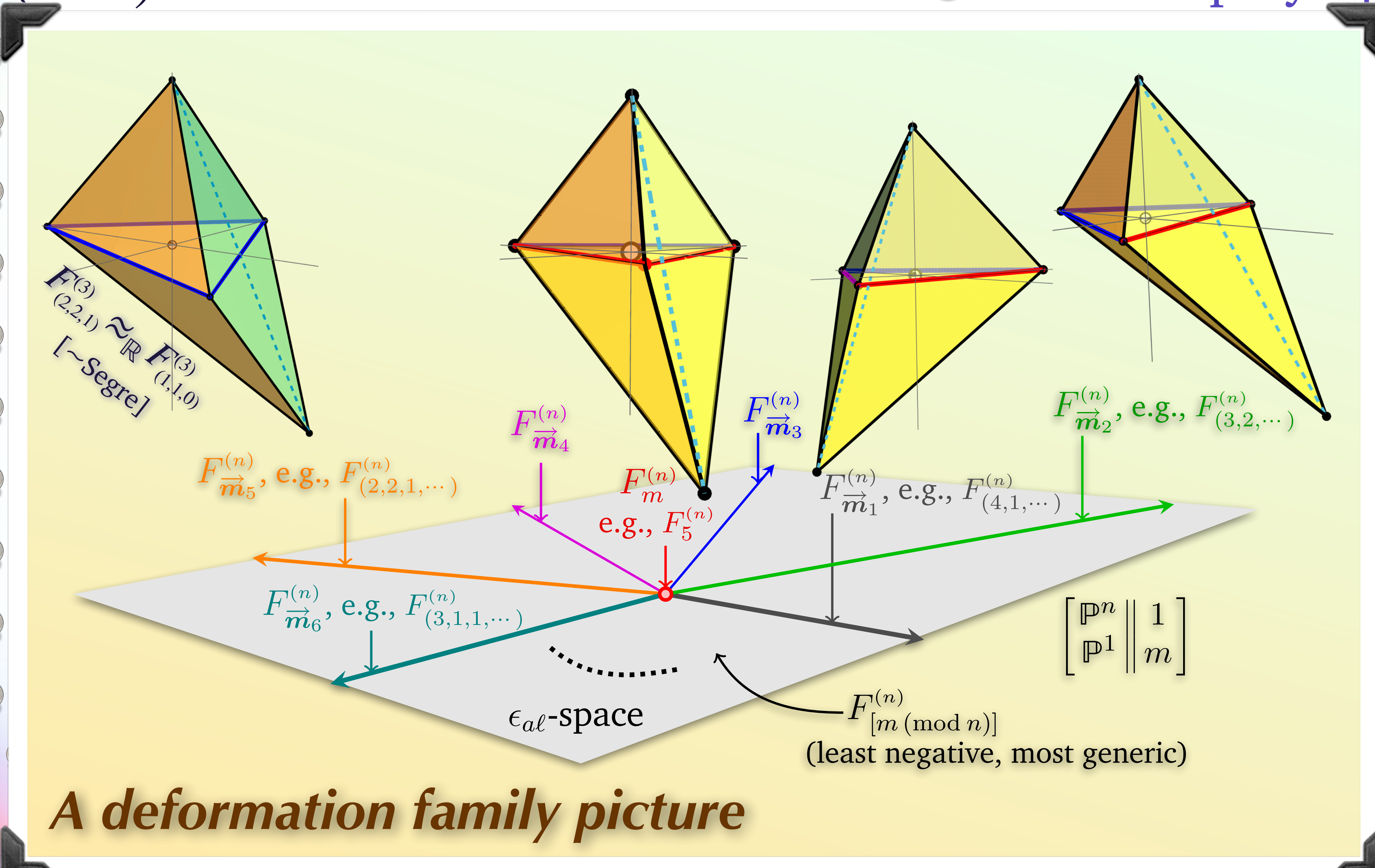
$d(\theta^{(k)}) := k!\,\mathrm{Vol}(\theta^{(k)})$ [BH: *signed* by orientation!]

Laurent GLSM Coda

Summary

—Proof-of-Concept—

- CY(n−1)-folds in Hirzebruch n-folds
- Oriented polytopes



…str.

…$\star_X)^\nabla$

B³H²K mirrors

…ces

…on

…omials

…ks to be *…xtended*


Thank You!
https://tristan.nfshost.com/
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Department of Physics, Faculty of Natural Sciences, Novi Sad University, Serbia
Department of Mathematics, University of Maryland, College Part, MD