

(Fundamentalna) Fizika Elementarnih Čestica

Dan 05a: Kalibracioni princip: neabelovsko uopštenje

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Fundamentalna Fizika Elementih Čestica

Program za danas (do pauze):

- **Princip kalibracione (lokalne) simmetrije**

- Parcijalni izvod i kalibraciono-kovarijantni izvod
- Opšta transformaciona “pravila”

- **$SU(3)_c$ transformacije**

- Boja kao 3-dimenzionalan naboј
- Matrične faze i lokalna simetrija
- Matrična reprezentacija za $SU(3)$

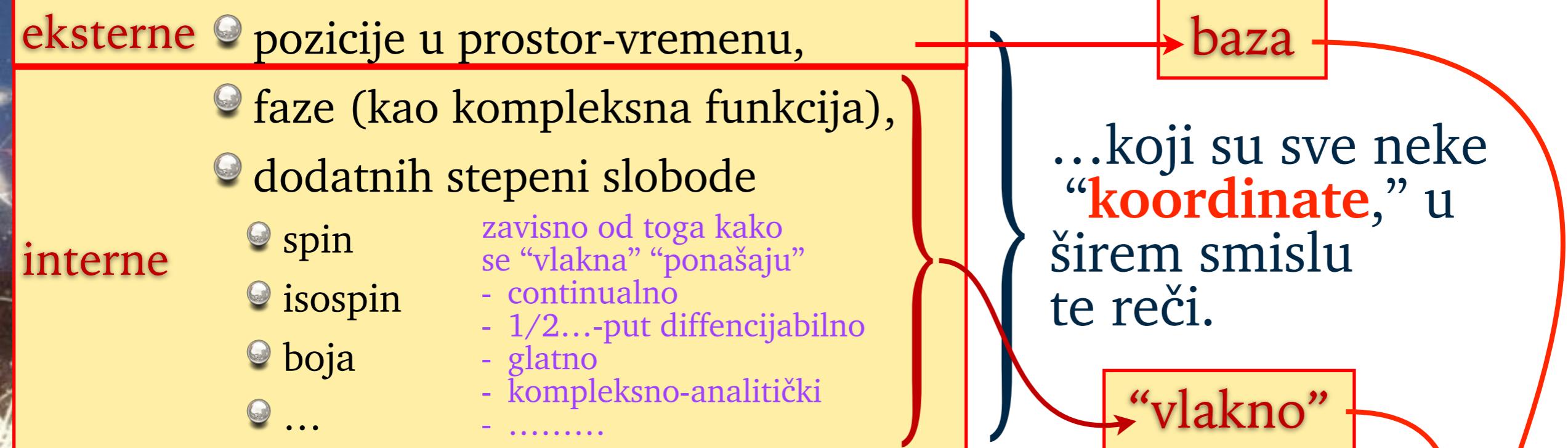
- **$SU(3)_c$ -invarijantni Lagranžian**

- Tenzor zakriviljenosti i Bianchi-ev identiteti
- Jednačine kretanja
- Očuvanje boje i jednačina kontinuiteta

Princip Kalibracione (Lokalne) Simetrije

Parcijalni i kalibraciono-kovarijantni izvod

- Pre svega:
- Matematički objekat, $\Psi(\vec{r}, t)$, koji predstavlja česticu
- zaviši (je funkcija) od promenljivih:



- Kalibracioni princip (lokalne simetrije):
- interne koordinate mogu da slobodno zavise od eksternih.

“vlaknasti svežnjevi/snopovi/...”

Princip Kalibracione (Lokalne) Simetrije

Opšta transformaciona "pravila"

- Izvodi računaju stepen/meru promene
- Stoga, ako $\Psi(\vec{r}, t)$ takođe zavisi od (\vec{r}, t) -zavisne faze,
 - onda stepen/mera promene $\Psi(\vec{r}, t)$ potiče od
 - varijacije $\Psi(\vec{r}, t)$ eksplicitno, i od
 - varijacije $\Psi(\vec{r}, t)$ implicitno, kroz varijaciju faze.
- Ako se "kalibracija" odnosi na "fiksiranje" te faze,
 - ...onda "kalibraciono-kovarijantan" izvod
 - ...mora da sadrži dva dela: $\mathcal{D}_\mu := \partial_\mu + A_\mu(\vec{r}, t)$, **kalibraciono polje**
 - gde je $A_\mu(\vec{r}, t)$ kalibracioni potencijal, tj. koneksija.
- Matematičari: $dx^\mu \mathcal{D}_\mu := dx^\mu \partial_\mu + dx^\mu A_\mu(\vec{r}, t)$ **koneksijska 1-forma**
 - daje definiciju koja je nezavisna od izbora (eksternih = prostor-vremenskih) koordinata.

Princip Kalibracione (Lokalne) Simetrije

Opšta transformaciona "pravila"

- Unitarna kalibraciona transformacija (lokalne simetrije) je oblika
 $U_\varphi := \exp\{ i\varphi \cdot Q \}, \quad (U_\varphi^\dagger = U_\varphi^{-1})$
 - φ je niz (realnih) kalibracionih parametara,
 - Q je niz (ermitskih) generatora kalibracionih transformacija.
- Onda,
 - $\Psi(\vec{r}, t) \rightarrow U_\varphi \Psi(\vec{r}, t)$ & $\Psi(\vec{r}, t)^\dagger \rightarrow (U_\varphi \Psi(\vec{r}, t))^\dagger = \Psi(\vec{r}, t)^\dagger U_\varphi^\dagger = \Psi(\vec{r}, t)^\dagger U_\varphi^{-1}$;
 - $\mathcal{O}(\vec{r}, t) \rightarrow U_\varphi \mathcal{O}(\vec{r}, t) U_\varphi^{-1}$ za svaki operator koji deluje na $\Psi(\vec{r}, t)$;
 - ...pa stoga i: $\mathcal{D}(\vec{r}, t) \rightarrow U_\varphi \mathcal{D}(\vec{r}, t) U_\varphi^{-1}$.
- Stoga, sa $\mathcal{D}_\mu = \mathbf{1} \partial_\mu + \mathcal{A}_\mu$
 - $[\partial_\mu + \mathcal{A}_\mu(\vec{r}, t)] \rightarrow U_\varphi [\partial_\mu + \mathcal{A}_\mu(\vec{r}, t)] U_\varphi^{-1}$ implicira da elektrodinamika:
 $\vec{A} \rightarrow \vec{A} + (\vec{\nabla} \lambda)$
 - $\mathcal{A}_\mu(\vec{r}, t) \rightarrow U_\varphi \mathcal{A}_\mu(\vec{r}, t) U_\varphi^{-1} + U_\varphi (\partial_\mu U_\varphi^{-1})$
 $= U_\varphi \mathcal{A}_\mu(\vec{r}, t) U_\varphi^{-1} - i U_\varphi (\partial_\mu \varphi) \cdot Q U_\varphi^{-1}$ nehomogeni član

$SU(3)_c$ Transformacije

Boja kao 3-dimenzioni naboј

• Setimo se:

$$\left. \begin{array}{l} \Delta^{++} = [uuu], \\ \Delta^- = [ddd], \\ \Omega^- = [sss]. \end{array} \right\}$$

Spin- $\frac{3}{2}$ barioni
S-stanja; bez orbitalnog ugaonog momenta
Prostorno simetrična talasna funkcija

• Pa, ili kvarkovi nisu ni bozoni ni fermioni

$$\begin{aligned} [b_i, b_j^\dagger] &= \delta_{ij}, & [b_i, b_j] &= 0 = [b_i^\dagger, b_j^\dagger], \quad \text{bozoni}, \\ \{f_i, f_j^\dagger\} &= \delta_{ij}, & \{f_i, f_j\} &= 0 = \{f_i^\dagger, f_j^\dagger\}, \quad \text{fermoni}, \end{aligned}$$

• već parafermioni (O.W. Greenberg, 1964)

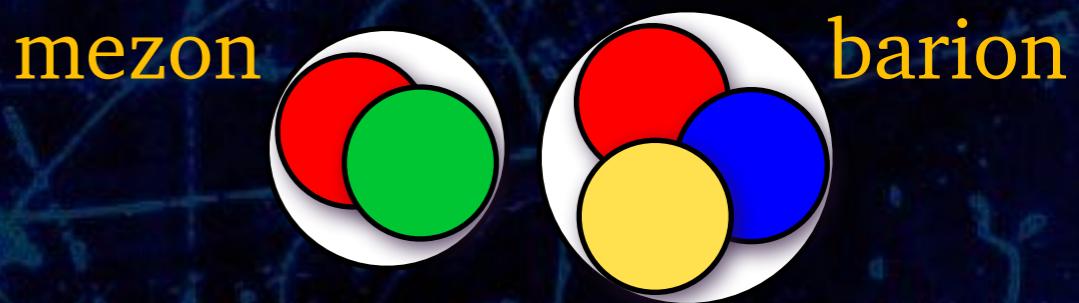
$$\begin{aligned} \{\tilde{f}_{i,\alpha}, \tilde{f}_{j,\alpha}^\dagger\} &= \delta_{ij}, & \{\tilde{f}_{i,\alpha}, \tilde{f}_{j,\beta}\} &= 0 = \{\tilde{f}_{i,\alpha}^\dagger, \tilde{f}_{j,\alpha}^\dagger\}, \\ [\tilde{f}_{i,\alpha}, \tilde{f}_{j,\beta}^\dagger] &= \delta_{ij}, & [\tilde{f}_{i,\alpha}, \tilde{f}_{j,\beta}] &= 0 = [\tilde{f}_{i,\alpha}^\dagger, \tilde{f}_{j,\beta}^\dagger], \quad \alpha \neq \beta, \end{aligned} \quad \left. \right\} \text{parafermioni,}$$

• ...ili...

$SU(3)_c$ Transformacije

Boja kao 3-dimenzioni naboј

- ... ili: Kvarkovi jesu fermioni,
- ...ali imaju dodatni stepen slobode.
 - Januar 1965: Boris V. Struminsky, Dubna (Moskva, Russia)
 - ...tada sa N. Bogolyubov-im + Albert Tavchelidze-om
 - Maj 1965, A. Tavchelidze: ICTP, Trieste (Italy)
 - Decembar 1965, Moo-Young Han + Yoichiro Nambu
 - celobrojno naelektrisani, obojeni kvarkovi + 8 (boja-antiboja) gluoni
 - Konačna verzija (sa razlomački naelektrisanim kvarkovima):
1974, William Bardeen, Harald Fritzsch & Murray Gell-Mann
 - Kvark: $\Psi_n^{\alpha A}(\vec{r}, t)$, gde:
 - $n = u, d, s, c, b, t$ označava "ukus"
 - $\alpha = \text{crveno, žuto, plavo}$ označava "boju"
 - $A = 1, 2, 3, 4$ osnačava komponentu Dirac-ovog spinora
 - P.S.: Greenberg naknadno dokazuje ekvivalentnost...



$SU(3)_c$ Transformacije

Matrične faze i lokalna simetrija

$$U_\varphi := \exp\{ i\varphi \cdot Q \}$$

- Bez pisanja Dirac-ovih komponenti,

$$\Psi_n(x) = \hat{e}_\alpha \Psi_n^\alpha(x) = \hat{e}_r \Psi_n^r(x) + \hat{e}_y \Psi_n^y(x) + \hat{e}_b \Psi_n^b(x) = \begin{bmatrix} \Psi_n^r(x) \\ \Psi_n^y(x) \\ \Psi_n^b(x) \end{bmatrix},$$

- ...gde $n = u, d, s, c, b, t$ naznačava "ukus."

- Pisanjem boja u matričnom formatu,

- faza talasne funkcije kvarka postaje 3×3 matrica,
 • kao i operator unitarne transformacije faze, U_φ

$$\Psi_n(x) \rightarrow e^{i(\varphi/\hbar) \cdot (g_c Q)} \Psi_n(x), \quad \varphi \cdot Q := \varphi^a Q_a$$

- gde su Q_a 3×3 matrice

- ermitske, da bi U_φ bile unitarne,

- bez traga, da bi U_φ bile unimodularne.

Dijagonalna transformacija, sa $Q_0=1$,
 deluje podjednako na sve boje,
 kao elektromagnetizam...
 ...samo što $g_c \neq g_e$

$SU(3)_c$ Transformacije

Matrične faze i lokalna simetrija

- Kalibracione (lokalno simetrijske) transformacije

$$[i\hbar \not{D} - mc] \Psi_n(x) = 0 \rightarrow [i\hbar \not{D}' - mc] \Psi'_n(x) = 0$$

$$\not{D} := \gamma^\mu \mathcal{D}_\mu, \quad \mathcal{D}_\mu \rightarrow \mathcal{D}'_\mu := U_\varphi \mathcal{D}_\mu U_\varphi^{-1}$$

Dirac-ove matrice

$$U_\varphi = e^{i(\varphi^a/\hbar)(g_c Q_a)}$$

- Multi-komponentnost: $\not{D} \Psi_n = \gamma^\mu \mathcal{D}_\mu \Psi_n$ “boja” komponenta Dirac-ovog spinora
- $$(\not{D}_\mu \Psi_n)^\alpha = \gamma^\mu \mathcal{D}_\mu{}^\alpha{}_\beta \Psi_n^\beta \quad (\not{D}_\mu \Psi_n)^{\dot{\alpha} A} = [\gamma^\mu]^A{}_B \mathcal{D}_\mu{}^\alpha{}_\beta \Psi_n^{\beta B}$$

- ...koji se obično ne pišu eksplisitno.

- U opštem slučaju: $\not{D} = \gamma^\mu \left[1 \partial_\mu + \frac{ig_c}{\hbar c} A_\mu^a Q_a \right]$

- gde forma operatora Q_a zavisi od objekta na koji deluje

- Q_a je 3×3 matrica ako je Ψ 3-komponentna matrica-kolona

$SU(3)_c$ Transformacije

Matrična reprezentacija $SU(3)$

$$U_\varphi = e^{i(\varphi^a/\hbar)(g_c Q_a)}$$

$$\not{D} = \gamma^\mu \left[\mathbf{1} \partial_\mu + \underbrace{\frac{ig_c}{\hbar c} A_\mu^a Q_a}_{\mathcal{A}_\mu} \right]$$

- Iz “opšteg” formalizma sledi da je:

$$\mathcal{A}'_\mu = U_\varphi \mathcal{A}_\mu U_\varphi^{-1} + U_\varphi (\partial_\mu U_\varphi^{-1}) = \frac{ig_c}{\hbar c} [A_\mu^a - \textcolor{red}{c}(\partial_\mu \varphi^a)] U_\varphi Q_a U_\varphi^{-1}$$

- pa $A'_\mu{}^a Q_a = [A_\mu^a - \textcolor{red}{c}(\partial_\mu \varphi^a)] \textcolor{blue}{U}_\varphi Q_a \textcolor{blue}{U}_\varphi^{-1}$.
- Q_a su “generatori”: $[Q_a, Q_b] = if_{ab}{}^c Q_c$, $f_{ab}{}^c$ = “strukturna const.”
- Q_a su 3×3 matrice koje deluju na (kvark) 3-vektor boje.
- Ali, kako Q_a deluje na osam 4-vektorskih potencijala A_μ^a ?

$$\begin{aligned} A'_\mu{}^a Q_a &= [A_\mu^a - \textcolor{red}{c}(\partial_\mu \varphi^a)] \left(1 + \frac{ig_c}{\hbar} \varphi^b Q_b + \dots \right) Q_a \left(1 - \frac{ig_c}{\hbar} \varphi^b Q_b + \dots \right) \\ &= A_\mu^a Q_a + \frac{ig_c}{\hbar} A_\mu^a \varphi^b Q_b Q_a - \frac{ig_c}{\hbar} A_\mu^a \varphi^c Q_a Q_c - \textcolor{red}{c}(\partial_\mu \varphi^a) Q_a + \dots \\ \delta A_\mu^a Q_a &= -\textcolor{red}{c}(\partial_\mu \varphi^a) Q_a - \frac{ig_c}{\hbar} A_\mu^b ([Q_b, Q_c] = (if_{bc}{}^a) Q_a) \varphi^c \\ \delta A_\mu^a &= -\textcolor{red}{c} [\delta_c^a \partial_\mu + \frac{g_c}{\hbar c} A_\mu^b ([\tilde{Q}_b]_c{}^a := (f_{bc}{}^a))] \varphi^c = -\textcolor{red}{c} (\mathcal{D}_\mu \varphi)^a \end{aligned}$$

$SU(3)_c$ -invarijantni Lagranžijan

Tenzor zakrivljenosti i Bianchi-ev identitet

- Abelovsko/neabelovske razlike: $\mathcal{D}'_\mu = U_\varphi \mathcal{D}_\mu U_\varphi^{-1}$ implicira

- $A'_\mu = A_\mu - (\partial_\mu \varphi)$ za elektrodinamiku

- ali $A'^a_\mu = A^a_\mu - \textcolor{red}{c}(\mathcal{D}_\mu \varphi)^a = A^a_\mu - \textcolor{red}{c}(\partial_\mu \varphi^a) - \frac{g_c}{\hbar} f_{bc}{}^a A^b_\mu \varphi^c$

nelinearno

- Takodje, za elektrodinamiku je

- $F'_{\mu\nu} = F_{\mu\nu}$, tako da su \vec{E} i \vec{B} su kalibraciono invarijantni!

- $F'_{\mu\nu} = [\partial_\mu A'_\nu - \partial_\nu A'_\mu] = [(\partial_\mu A_\nu - \partial_\mu (\partial_\nu \varphi)) - (\partial_\nu A_\mu - \partial_\nu (\partial_\mu \varphi))] = [\partial_\mu A_\nu - \partial_\nu A_\mu]$

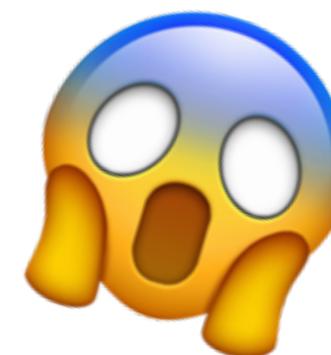
- Ali, u neabelovskom (matričnom) slučaju:

- $[\partial_\mu A'_\nu - \partial_\nu A'_\mu] \neq [\partial_\mu A_\nu - \partial_\nu A_\mu]$

- $[\partial_\mu A'_\nu - \partial_\nu A'_\mu] \neq U_\varphi [\partial_\mu A_\nu - \partial_\nu A_\mu] U_\varphi^{-1}$

- ...čak ni samo za infinitezimalne

kalibracione transformacije $U_\varphi \approx 1 + \frac{ig_c}{\hbar} \varphi^a Q_a$



$[Q_a, Q_b] = if^{ab}{}_c Q_c$
sve zbog

$SU(3)_c$ -invarijantni Lagranžijan

Tenzor zakrivljenosti i Bianchi-ev identitet

• U elektrodinamici:

- $[\mathcal{D}_\mu, \mathcal{D}_\nu] f(\mathbf{x}) = [\partial_\mu + \frac{iq}{\hbar c} A_\mu, \partial_\nu + \frac{iq}{\hbar c} A_\nu] f(\mathbf{x}) = \frac{iq}{\hbar c} [(\partial_\mu A_\nu - \partial_\nu A_\mu) = F_{\mu\nu}] f(\mathbf{x})$

- Ovo mora da bude komutator, da rezultat u uglastoj zagradi ne bi bio diferencijalni operator, već “obična” funkcija.

- A komutator upravo računa razliku u... pa, ...u komutaciji.

• U opštem: $[\mathcal{D}, \mathcal{D}] = (\text{torzija}) \cdot \mathcal{D} + \boxed{(\text{zakrivljenost})}$

• Stoga računamo:

$$\begin{aligned} \mathbb{F}_{\mu\nu} &:= \frac{\hbar c}{ig_c} [\mathcal{D}_\mu, \mathcal{D}_\nu] = \frac{\hbar c}{ig_c} [\partial_\mu + \frac{ig_c}{\hbar c} A_\mu^b Q_b, \partial_\nu + \frac{ig_c}{\hbar c} A_\nu^c Q_c] \\ &= (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a) Q_a + \frac{\hbar c}{ig_c} \left(\frac{ig_c}{\hbar c}\right)^2 A_\mu^b A_\nu^c ([Q_b, Q_c] = if_{bc}{}^a Q_a) \\ \text{pa je } \mathbb{F}_{\mu\nu} &:= F_{\mu\nu}^a Q_a \quad \text{i} \quad F_{\mu\nu}^a := (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a) - \frac{g_c}{\hbar c} f_{bc}{}^a A_\mu^b A_\nu^c \end{aligned}$$

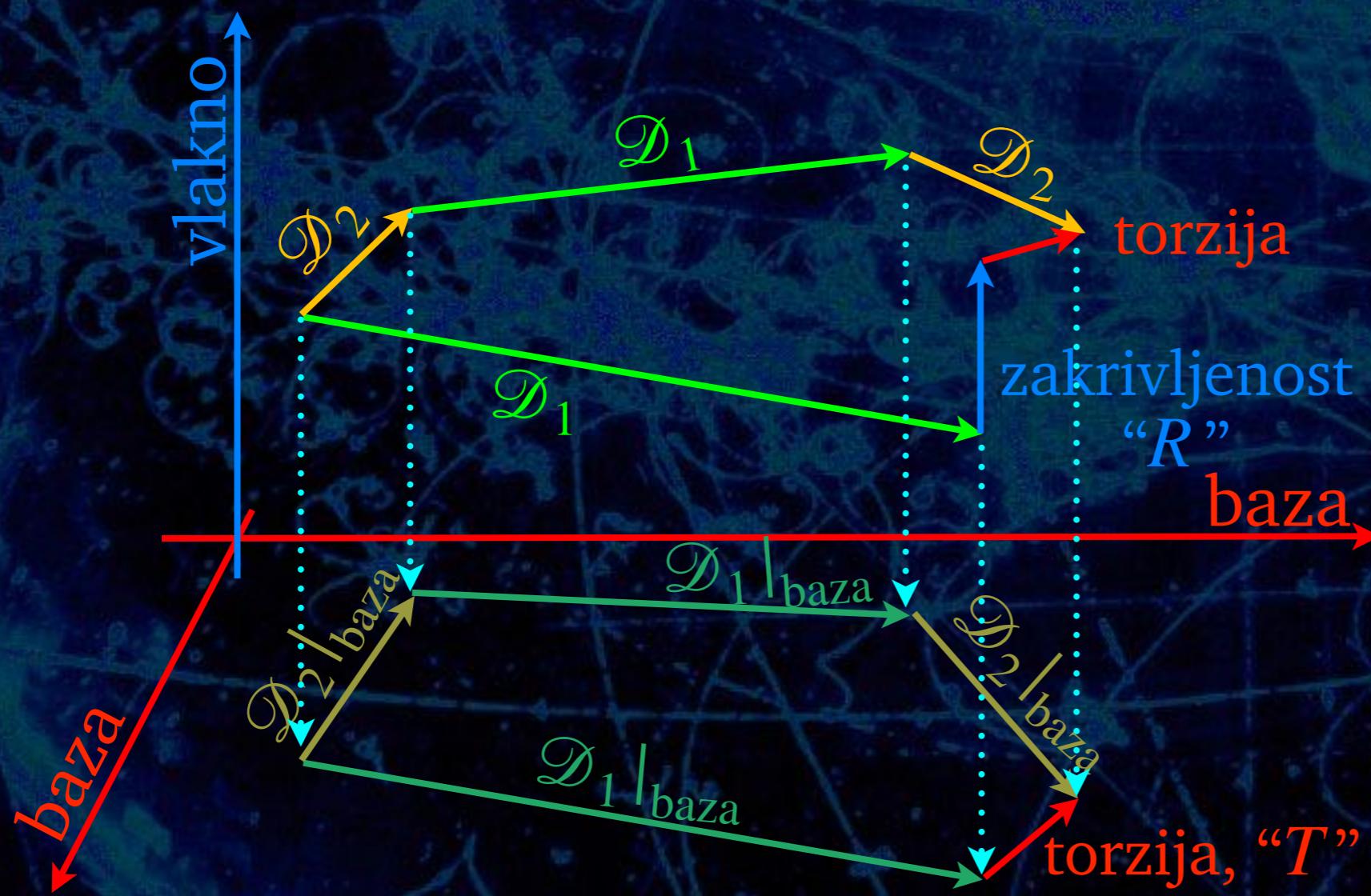
$$\mathcal{D} = \partial + \Gamma, \quad \Gamma = \text{"popravka"}$$

$$[\mathcal{D}, \mathcal{D}] = T \cdot \mathcal{D} + R$$

Digresija

Zakrivljenost i torzija

- Pređočimo rezultat računanja $[\mathcal{D}_1, \mathcal{D}_2] \equiv \mathcal{D}_1 \mathcal{D}_2 - \mathcal{D}_2 \mathcal{D}_1$
- ... u *totalnom* prostoru "vlaknastog/vektorskog svežnja":



U opštem, operatori stepena promene (\mathcal{D}) će omašiti u komutaciji, kako u

- pravcu "vlakana" (\rightarrow zakrivljenot: u istom "mestu", ali sa drugom "vrednošću") tako i u
- pravcu baznog prostora (\rightarrow torzija: na različitom "mestu").

$SU(3)_c$ -invarijantni Lagranžijan

Tenzor zakrivljenosti i Bianchi-ev identitet

- Po definiciji, imamo da je

$$\begin{aligned}\mathbb{F}_{\mu\nu} \rightarrow \mathbb{F}'_{\mu\nu} := \frac{\hbar c}{ig_c} [\mathcal{D}'_\mu, \mathcal{D}'_\nu] &= \frac{\hbar c}{ig_c} [U_\varphi \mathcal{D}_\mu U_\varphi^{-1}, U_\varphi \mathcal{D}_\nu U_\varphi^{-1}] \\ &= \frac{\hbar c}{ig_c} U_\varphi [\mathcal{D}_\mu, \mathcal{D}'_\nu] U_\varphi^{-1} \\ &= U_\varphi \mathbb{F}_{\mu\nu} U_\varphi^{-1}\end{aligned}$$

Nije invarijantno
nega kovarijantno!

- Nezavisno,

$$\begin{aligned}\mathcal{D}_\mu \mathbb{F}_{\nu\rho} f(\mathbf{x}) &= [\mathcal{D}_\mu, \mathbb{F}_{\nu\rho}] f(\mathbf{x}) = \frac{\hbar c}{ig_c} [\mathcal{D}_\mu, [\mathcal{D}_\nu, \mathcal{D}_\rho]] f(\mathbf{x}) \\ \text{gde važi } [A, [B, C]] + [B, [C, A]] + [C, [A, B]] &= 0 \\ \text{pa onda } \varepsilon^{\mu\nu\rho\sigma} \mathcal{D}_\mu \mathbb{F}_{\nu\rho} f(\mathbf{x}) &= \frac{\hbar c}{ig_c} \varepsilon^{\mu\nu\rho\sigma} [\mathcal{D}_\mu, [\mathcal{D}_\nu, \mathcal{D}_\rho]] f(\mathbf{x}) = 0\end{aligned}$$

- Za sve $SU(n)$ grupe: $\text{Tr}[\mathbb{F}_{\mu\nu}] = F_{\mu\nu}^a \text{Tr}[Q_a] = 0$

- Važi za sve poluproste Lie-jeve grupe (= bez abelovskih faktora).

$SU(3)_c$ -invarijantni Lagranžijan

Jednačine kretanja

- Pošto se matrična zakrivljenost transformiše transformacijom sličnosti, $\mathbb{F}'_{\mu\nu} = U_\varphi \mathbb{F}_{\mu\nu} U_\varphi^{-1}$
 - ...na šta je funkcija traga invarijantna, $\text{Tr}[UXU^{-1}] = \text{Tr}[X]$,
 - $\text{Tr}[\mathbb{F}_{\mu\nu} \mathbb{F}^{\mu\nu}] \rightarrow \text{Tr}[\mathbb{F}'_{\mu\nu} \mathbb{F}'^{\mu\nu}] = \text{Tr}[U_\varphi \mathbb{F}_{\mu\nu} U_\varphi^{-1} U_\varphi \mathbb{F}^{\mu\nu} U_\varphi^{-1}]$
 $= \text{Tr}[\mathbb{F}_{\mu\nu} \mathbb{F}^{\mu\nu} U_\varphi^{-1} U_\varphi]$
 $= \text{Tr}[\mathbb{F}_{\mu\nu} \mathbb{F}^{\mu\nu}]$
- pa biramo:
 - $\mathcal{L}_{\text{QCD}} = \sum_n \text{Tr}[\bar{\Psi}_n [i\hbar c \not{D} - m_n c^2] \Psi_n] - \frac{1}{4g_c^2} \text{Tr}[\mathbb{F}_{\mu\nu} \mathbb{F}^{\mu\nu}]$
 - $= \sum_n \text{Tr}[\bar{\Psi}_{an} [i\gamma^\mu (\hbar c \delta_\beta^\alpha \partial_\mu + ig_c A_\mu^\alpha \frac{1}{2} [\lambda_a]^\alpha_\beta) - m_n c^2 \delta_\beta^\alpha] \Psi_n^\beta] - \frac{1}{4g_c^2} F_{\mu\nu}^a F_a^{\mu\nu}$
- gde drag ostaje još samo po komponentama Dirac-ovih spinora i matrice, $\text{Tr}[\bar{\Psi}_{an} \dots \gamma^\mu \dots \Psi_n^\beta] = \bar{\Psi}_{anA} \dots [\gamma^\mu]^A_B \dots \Psi_n^{\beta B}$

$SU(3)_c$ -invarijantni Lagranžijan

Jednačine kretanja

$$\mathcal{L}_{\text{QCD}} = \sum_n \text{Tr} [\bar{\Psi}_n [i\hbar c \not{D} - m_n c^2] \Psi_n] - \frac{1}{4g_c^2} \text{Tr} [\mathbb{F}_{\mu\nu} \mathbb{F}^{\mu\nu}]$$

- Varijacija po A_μ^a daje:
 - $\mathcal{D}_\mu F^{a\mu\nu} = g_c \sum_n \bar{\Psi}_{n\alpha A} [\gamma^\nu]^A_B \frac{1}{2} [\lambda^a]^\alpha_\beta \Psi_n^{\beta B}$
 - $J_{(q)}^{a\mu} = g_c \sum_n \bar{\Psi}_{n\alpha A} [\gamma^\nu]^A_B \frac{1}{2} [\lambda^a]^\alpha_\beta \Psi_n^{\beta B}$ — kvarkovska struja boje
- U elektrodinamici je
 - $(\mathcal{D}_\mu F^{\mu\nu} = \partial_\mu F^{\mu\nu}) = g_e \bar{\Psi}_A [\gamma^\nu]^A_B \Psi^B =: J_e^\nu$ — struja nanelektrisanja
 - Pa je $\partial_\nu J_e^\nu = \frac{4\pi\epsilon_0 c}{4\pi} \partial_\mu \partial_\nu F^{\mu\nu} \equiv 0$ pošto je $F_{\mu\nu} \equiv -F_{\nu\mu}$
- Isto ne sledi za $\mathcal{D}_\mu F^{a\mu\nu} \neq \partial_\mu F^{a\mu\nu}$ za ne-abelovske sile
- Umesto toga: $\mathcal{D}_\mu F^{a\mu\nu} = J_{(q)}^{a\nu}$,
 - pa je $\mathcal{D}_\nu J_{(q)}^{a\nu} = \mathcal{D}_\nu \mathcal{D}_\mu F^{a\mu\nu} = -\frac{1}{2} [\mathcal{D}_\mu, \mathcal{D}_\nu] F^{a\mu\nu} = -\frac{1}{2} f_{bc}^a F_{\mu\nu}^b F^{c\mu\nu} \equiv 0$
 - pošto je $f_{bc}^a = -f_{cb}^a$

$SU(3)_c$ -invarijantni Lagranžijan

Očuvanje boje i jednačina kontinuiteta

- Neabelovski kovarijantni izvod nije jednačina kontinuiteta:

- $0 = \mathcal{D}_\mu J_{(q)}^{a\mu} = \partial_\mu J_{(q)}^{a\mu} - \frac{g_c}{\hbar c} f_{bc}^{a} A_\mu^b J_{(q)}^{c\mu}$

- daje $\frac{d}{dt} \int_V d^3 \vec{r} \partial_\mu J_{(q)}^{a0} = - \oint_{\partial V} d^2 \vec{r} \partial_\mu \vec{J}_{(q)}^a - \frac{g_c}{\hbar c} f_{bc}^{a} \int_V d^3 \vec{r} (A_\mu^b J_{(q)}^{c\mu})$

- ...pošto dodatni član na desnoj strani ne isčezava u opštem.

- Medjutim, $J_{(q)}^{a\nu} = \mathcal{D}_\mu F^{a\mu\nu} = \partial_\mu F^{a\mu\nu} - \frac{g_c}{\hbar c} f_{bc}^{a} A_\mu^b F^{c\mu\nu}$

- pa $\partial_\mu F^{a\mu\nu} = J_{(c)}^{a\nu} := J_{(q)}^{a\nu} + \frac{g_c}{\hbar c} f_{bc}^{a} A_\mu^b F^{c\mu\nu}$

- daje $\partial_\nu J_{(c)}^{a\nu} = \partial_\nu \partial_\mu F^{a\mu\nu} \equiv 0$

- ...pa i kvarkovi i gluoni doprinose očuvanom naboju boje:

- $Q_{(c)}^a := \int d^3 \vec{r} J_{(c)}^{a0} := g_c \int d^3 \vec{r} \left(\sum_n \text{Tr}[\bar{\Psi}_n \gamma^0 \lambda^a \Psi_n] + \frac{1}{\hbar c} f_{bc}^{a} A_\mu^b F^{c\mu 0} \right)$

- gde je trag i po bojama kvarkova i po Dirac-ovim komponentama

stacionarna struja boje
 $J_{(c)}^{a\nu} := J_{(q)}^{a\nu} + \frac{g_c}{\hbar c} f_{bc}^{a} A_\mu^b F^{c\mu\nu}$

$SU(3)_c$ -invarijantni Lagranžijan

Očuvanje boje i jednačina kontinuiteta

- Ovo menja analogon Gauss-Ampère-ovih zakona.
- Pogledajmo $\nu=0$ slučaj jednačine $\mathcal{D}_\mu F^{a\mu\nu} = J_{(q)}^{a\nu}$:
 - $\partial_\mu F^{a\mu 0} - \frac{g_c}{\hbar c} f_{bc}^{a} A_\mu^b F^{c\mu 0} = J_{(q)}^{a 0}$
 - ...i definišemo: $\vec{E}^a := \hat{e}_i F^{ai 0}$, $\vec{A}^a := -\hat{e}^i A_i^a$, $\rho_{(q)}^a := J_{(q)}^{a 0}$
 - Onda je $\vec{\nabla} \cdot \vec{E}^a = \rho_{(q)}^a - \frac{g_c}{\hbar c} f_{bc}^{a} \vec{A}^b \cdot \vec{E}^c$ Gauss-ov zakon za boju
- Valja primetiti:
 - Nemoguće je napisati neabelovske analogone Maxwell-ovih jednačina bez korišćenja kalibracionih potencijala
 - I kvarkovi i gluoni služe kao “izvori” za polje sile boje
 - Analogoni Maxwell-ovih jednačina su nelinearne po poljima

$SU(3)_c$ -invarijantni Lagranžijan

Očuvanje boje i jednačina kontinuiteta

- Da sumiramo:
 - Maxwell-ove jednačine $\mathcal{D}_\mu \mathbb{F}^{\mu\nu} = \mathbb{J}_{(q)}^\nu$ i $\epsilon^{\mu\nu\rho\sigma} (\mathcal{D}_\mu \mathbb{F}_{\nu\rho}) = 0$
 - kvarkovska struja: $\mathbb{J}_{(q)}^\nu := g_c \sum_n \bar{\Psi}_{n\alpha A} [\gamma^\mu]_B^A \frac{1}{2} [\lambda^a]^\alpha_\beta \Psi_n^{\beta B} Q_a$
 - Kompletna struja: $\mathbb{J}_{(c)}^\nu := \mathbb{J}_{(q)}^\nu + \frac{ig_c}{\hbar c} [\mathbb{A}_\mu, \mathbb{F}^{\mu\nu}] \quad \partial_\mu \mathbb{F}^{\mu\nu} = \mathbb{J}_{(c)}^\nu$
 - Jednačina kontinuiteta: $\partial_\nu \mathbb{J}_{(c)}^\nu = 0$
 - Očuvani naboј boje: $\frac{d}{dt} \int_V d^3 \vec{r} \mathbb{J}_{(c)}^0 = - \oint_{\partial V} d^2 \vec{r} \cdot \vec{J}_{(c)}$

Kafica?

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