

(Fundamentalna) Fizika Elementarnih Čestica

Dan 04: Kalibracioni princip: nerelativistički uvod,
elektrodinamika, leptoni i hadroni

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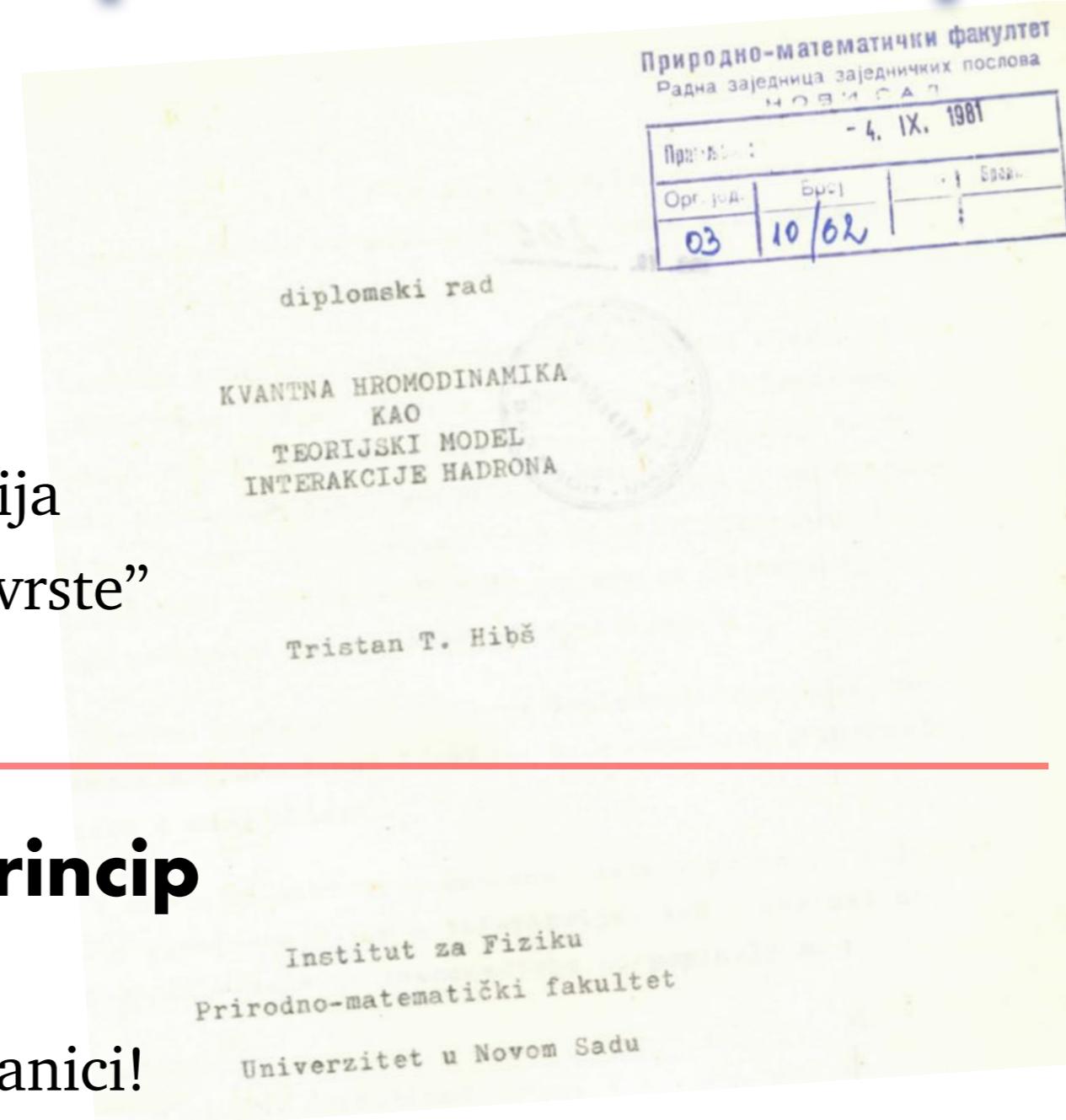
<https://tristan.nfshost.com/>

Kvarkovi, simetrije i interakcije

Program za danas:

• Kvark model

- Kvarkovi = osnovna gradja
- Anti-simetrizacija vezanih stanja
- Izospinska SU(2) približna simetrija
- Približna SU(3) simetrija “ukusa/vrste”
- Egzaktna SU(3) simetrija “boje”



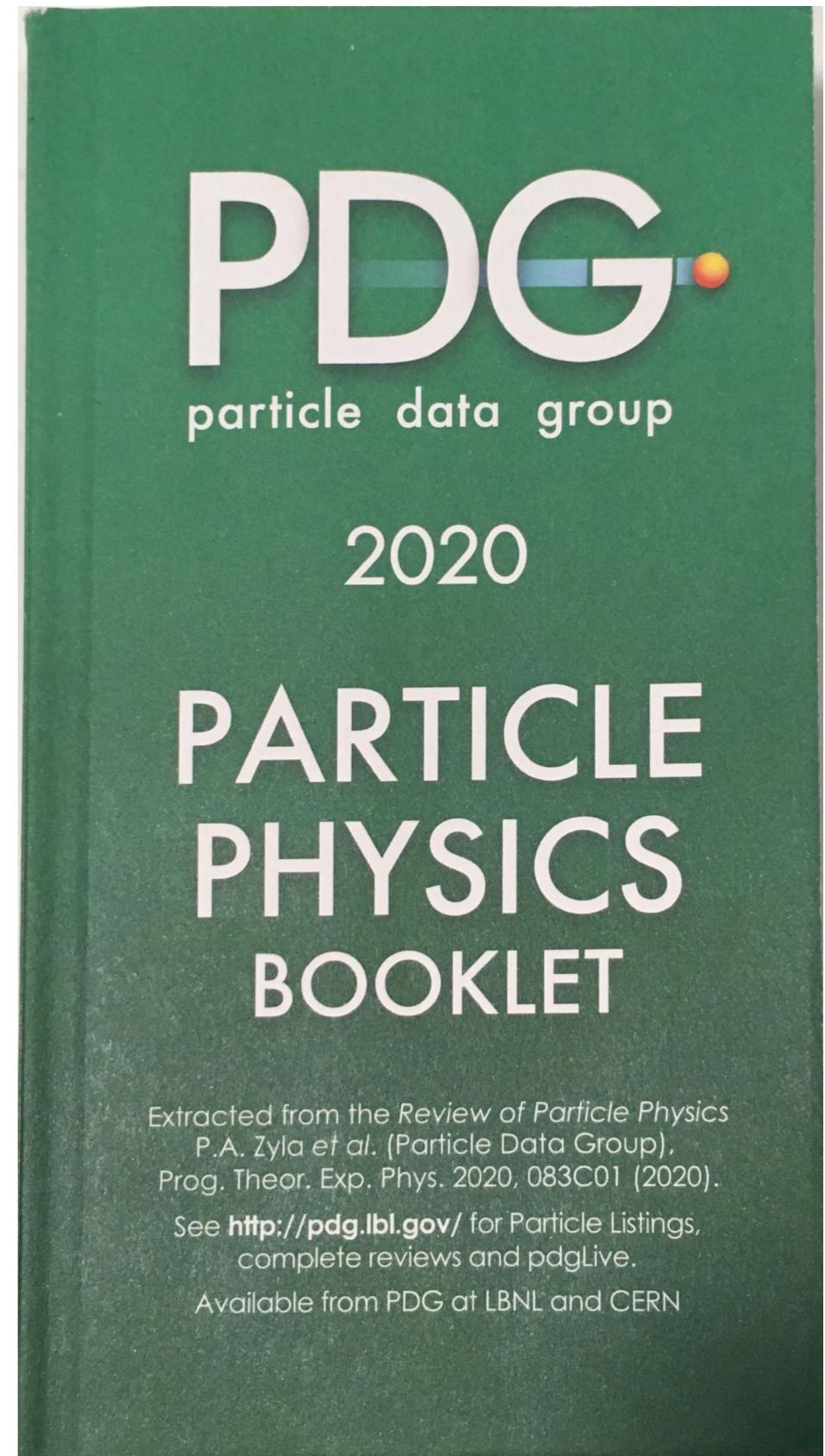
• Kalibracione simetrije i princip

- Elektromagnetno polje...
- ...je neophodno i u kvantnoj mehanici!
- Zakon očuvanja nanelektrisanja

Kvark model

Elementarne čestice

- Hiljade hadrona ne mogu svi da budu “elementarni”



Kvark model

Elementarne čestice

Hiljade hadrona ne mogu svi da budu "elementarni"

Meson Summary Table					
					43
42 Meson Summary Table					
Lepton family number (LF), Lepton number (L), $\Delta S = \Delta Q$ (SQ) violating modes, or $\Delta S = 1$ weak neutral current (SI) modes					
$\pi^+ \pi^+ e^- \bar{\nu}_e$	SQ	< 1.3	$\times 10^{-8}$		
$\pi^+ \pi^+ \mu^- \bar{\nu}_\mu$	SQ	< 3.0	$\times 10^{-6}$	CL=90%	
$\pi^+ e^+ e^-$	SI	(3.00 \pm 0.09) $\times 10^{-7}$		CL=90%	
$\pi^+ \mu^+ \mu^-$	SI	(9.4 \pm 0.6) $\times 10^{-8}$			
$\pi^+ \nu \bar{\nu}$	SI	(1.7 \pm 1.1) $\times 10^{-10}$		CL=90%	
$\pi^+ \pi^0 \nu \bar{\nu}$	LF	< 2.1	$\times 10^{-8}$	CL=90%	
$\mu^+ \nu e^+ e^+$	LF	[d] < 4	$\times 10^{-3}$	CL=90%	
$\mu^+ \nu_e$	LF	< 1.3	$\times 10^{-11}$	CL=90%	
$\pi^+ \mu^+ e^-$	LF	< 5.2	$\times 10^{-10}$	CL=90%	
$\pi^+ \mu^- e^+$	L	< 5.0	$\times 10^{-10}$	CL=90%	
$\pi^- \mu^+ e^+$	L	< 2.2	$\times 10^{-10}$	CL=90%	
$\pi^- e^+ e^+$	L	< 4.2	$\times 10^{-11}$	CL=90%	
$\pi^- \mu^- \mu^+$	L	[d] < 3.3	$\times 10^{-3}$	CL=90%	
$\mu^+ \bar{\nu}_e$	L	< 3	$\times 10^{-3}$	CL=90%	
$\pi^0 e^+ \bar{\nu}_e$	[x] <	2.3	$\times 10^{-9}$	CL=90%	
$\pi^+ \gamma$					
K_S^0 DECAY MODES					
Hadronic modes					
$\pi^0 \pi^0$					209
$\pi^+ \pi^-$					206
$\pi^+ \pi^- \pi^0$					133
$\pi^+ \pi^- \gamma$					
$\pi^+ \pi^- e^+ e^-$					
$\pi^0 \gamma \gamma$					
$\gamma \gamma$					
Modes with photons or $\ell \bar{\ell}$ pairs					
$\pi^+ \pi^- \gamma$	[t,aa]	(1.79 \pm 0.05) $\times 10^{-3}$			206
$\pi^+ \pi^- e^+ e^-$	[aa]	(4.79 \pm 0.15) $\times 10^{-5}$			230
$\pi^0 \gamma \gamma$	[aa]	(4.9 \pm 1.8) $\times 10^{-8}$			249
$\gamma \gamma$		(2.63 \pm 0.17) $\times 10^{-6}$	S=3.0		
Semileptonic modes					
$\pi^\pm e^\mp \nu_e$	[bb]	(7.04 \pm 0.08) $\times 10^{-4}$			229
CP-violating (CP) and $\Delta S = 1$ weak neutral current (SI) modes					
$3\pi^0$	CP	< 2.6	$\times 10^{-8}$	CL=90%	139
$\mu^+ \mu^-$	SI	< 8	$\times 10^{-10}$	CL=90%	225
$e^+ e^-$	SI	< 9	$\times 10^{-9}$	CL=90%	249
$\pi^0 e^+ e^-$	SI	[aa] (3.0 \pm 1.5) $\times 10^{-9}$			230
$\pi^0 \mu^+ \mu^-$	SI	(2.9 \pm 1.5) $\times 10^{-9}$			177
K_L^0					
$I(J^P) = \frac{1}{2}(0^-)$					
$m_{K_L} - m_{K_S}$					
$= (0.5293 \pm 0.0009) \times 10^{10} \text{ } \hbar \text{ s}^{-1}$	(S = 1.3)	Assuming CPT			
$= (3.484 \pm 0.006) \times 10^{-12} \text{ MeV}$	Assuming CPT				
$= (0.5289 \pm 0.0010) \times 10^{10} \text{ } \hbar \text{ s}^{-1}$	Not assuming CPT				
Mean life $\tau = (5.116 \pm 0.021) \times 10^{-8} \text{ s}$	(S = 1.1)				
$c\tau = 15.34 \text{ m}$					
Slope parameters [q]					
(See Particle Listings for other linear and quadratic coefficients)					
$K_L^0 \rightarrow \pi^+ \pi^- \pi^0$: $g = 0.678 \pm 0.008$ (S = 1.5)					
$K_L^0 \rightarrow \pi^+ \pi^- \pi^0$: $h = 0.076 \pm 0.006$					
$K_L^0 \rightarrow \pi^+ \pi^- \pi^0$: $k = 0.0099 \pm 0.0015$					
$K_L^0 \rightarrow \pi^0 \pi^0 \pi^0$: $h = (0.6 \pm 1.2) \times 10^{-3}$					
K_L decay form factors [r]					
Linear parametrization assuming μ -e universality					
$\lambda_+(K_{\mu 3}^0) = \lambda_+(K_{e3}^0) = (2.82 \pm 0.04) \times 10^{-2}$ (S = 1.1)					
$\lambda_0(K_{\mu 3}^0) = (1.38 \pm 0.18) \times 10^{-2}$ (S = 2.2)					

Kvark model

Elementarne čestice

- Hiljade hadrona ne mogu svi da budu “elementarni”
- Koristeći Gell-Mann-ovu šemu, moguće ih je sve objasniti kao vezana stanja:
 - Mezon = $[q\bar{q}]$
 - Barion = $[qqq]$
 - ...a tako se i sudaraju!
(Ugaona raspodela rasejanja ukazuje da mezoni imaju dva dominantna centra rasejanja, a barioni tri.)
 - ...a tako se i “ponašaju” u svim procesima

Three Generations of Matter (Fermions)			
	I	II	III
mass →	2.4 MeV	1.27 GeV	171.2 GeV
charge →	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
spin →	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
name →	u up	c charm	t top
Quarks			
mass →	4.8 MeV	104 MeV	4.2 GeV
charge →	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
spin →	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
name →	d down	s strange	b bottom
Leptons			
mass →	<2.2 eV	<0.17 MeV	<15.5 MeV
charge →	0	0	0
spin →	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
name →	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino
Bosons (Forces)			
mass →	0.511 MeV	105.7 MeV	1.777 GeV
charge →	-1	-1	-1
spin →	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
name →	e electron	μ muon	τ tau
mass →	80.4 GeV	80.4 GeV	80.4 GeV
charge →	± 1	± 1	± 1
spin →	1	1	1
name →	W^+ weak force	Z^0 weak force	W^- weak force

<http://www.answers.com/topic/elementary-particle>

Digresija: grupe i notacija

Podsetnik

- Ako su razlike izmedju m_d i m_u zanemarljive, $SU(2)_I$ je (približna) simetrija koja “rotira” u (d, u) prostoru
- $SU(2)$ je množstvo unitarnih 2×2 matrica \mathbb{U} sa jediničnom determinantom.

• Neka je t^α α -ta komponenta 2-komponentnog vektora.

• Neka je $\mathbf{2} := \{c_1 t^1 + c_2 t^2, c_i \in \mathbb{C}\}$

• Onda “ $\mathbf{2} \otimes \mathbf{2} = \mathbf{3} \oplus \mathbf{1}$ ” znači

$$t^\alpha s^\beta = t^{(\alpha} s^{\beta)} + t^{[\alpha} s^{\beta]}$$

$$\left| \frac{1}{2}, \pm \frac{1}{2} \right\rangle \otimes \left| \frac{1}{2}, \pm \frac{1}{2} \right\rangle =$$

2 2

$$t^{(\alpha} s^{\beta)} \propto (t^\alpha s^\beta + t^\beta s^\alpha)$$

$$t^{[\alpha} s^{\beta]} \propto (t^\alpha s^\beta - t^\beta s^\alpha)$$

$$|1, +1\rangle = |\uparrow\uparrow\rangle,$$

$$|1, 0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle),$$

$$|1, -1\rangle = |\downarrow\downarrow\rangle,$$

$$|0, 0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle),$$

triplet, 3

singlet, 1

Digresija: grupe i notacija

Podsetnik

Notacija:

dim.	formalna ket-oznaka	indeksna*	matrična
1	$\{ 0,0\rangle\}$	t	$[t]$
2	$\left\{ \frac{1}{2}, -\frac{1}{2}\rangle, \frac{1}{2}, +\frac{1}{2}\rangle\right\}$	t^a	$\begin{bmatrix} t^1 \\ t^2 \end{bmatrix}$
3	$\{ 1,-1\rangle, 1,0\rangle, 1,+1\rangle\}$	$t^{(ab)}$	$\begin{bmatrix} x = t^{(11)} \\ y = t^{(12)} \\ z = t^{(22)} \end{bmatrix}$
4	$\left\{ \frac{3}{2}, -\frac{3}{2}\rangle, \frac{3}{2}, -\frac{1}{2}\rangle, \frac{3}{2}, +\frac{1}{2}\rangle, \frac{3}{2}, +\frac{3}{2}\rangle\right\}$	$t^{(abc)}$	$\begin{bmatrix} t^1 = t^{(111)} \\ \vdots \\ t^4 = t^{(222)} \end{bmatrix}$

(2j+1) := $\left\{|\frac{j}{2}, -\frac{j}{2}\rangle, |\frac{j}{2}, -\frac{j-2}{2}\rangle, \dots, |\frac{j}{2}, +\frac{j-2}{2}\rangle, |\frac{j}{2}, +\frac{j}{2}\rangle\right\}$
 $= t^{(a_1 a_2 \cdots a_{2j})}; \quad a_i = 1, 2 \text{ za svako } i = 1, 2, \dots, 2j.$

Digresija: grupe i notacija

Podsetnik

Notacija (još):

$$J_{\pm} = \pm e^{\pm i\phi} \left[\frac{\partial}{\partial\theta} \pm i \cot(\theta) \frac{\partial}{\partial\phi} \right],$$

$$J_3 = -i \frac{\partial}{\partial\phi},$$

$$J^2 = - \left[\frac{1}{\sin(\theta)} \frac{\partial}{\partial\theta} \left(\sin(\theta) \frac{\partial}{\partial\theta} \right) + \frac{1}{\sin^2(\theta)} \frac{\partial^2}{\partial\phi^2} \right].$$

$$\frac{Y_0^0 = \frac{1}{\sqrt{4\pi}}}{}$$

$$= \frac{1}{\sqrt{4\pi}}$$

$$\frac{Y_1^1 = -\sqrt{\frac{3}{8\pi}} \sin\theta e^{i\phi}}{}$$

$$= -\sqrt{\frac{3}{8\pi}} \frac{x+iy}{r}$$

$$\frac{Y_1^0 = +\sqrt{\frac{3}{4\pi}} \cos\theta}{}$$

$$= \sqrt{\frac{3}{4\pi}} \frac{z}{r}$$

No, imajte na umu:
polucelobrojne
reprezentacije
nisu
jednoznačne
funkcije!

$$\begin{aligned} Y_l^m(\vec{r}) &\rightarrow e^{i\alpha J_z} Y_l^m(\vec{r}) \\ &= e^{iam} Y_l^m(\vec{r}) \end{aligned}$$

$$e^{i\alpha \frac{1}{2}} \rightarrow e^{i(2\pi)\frac{1}{2}} = e^{i\pi} = -1$$

Digresija: grupe i notacija

Podsetnik

• Primena:

$$J^2|j, m_j\rangle = j(j+1)|j, m_j\rangle, \quad J_3|j, m_j\rangle = m_j|j, m_j\rangle.$$

$$J_{\pm} := (J_1 \pm iJ_2), \quad J_{\pm}|j, m\rangle = \sqrt{j(j+1) - m(m\pm1)}|j, m\pm1\rangle.$$

• Analogno tome,

$$[J^2, T_{\rho}^{(r)}] = r(r+1)T_{\rho}^{(r)}, \quad [J_3, T_{\rho}^{(r)}] = \rho T_{\rho}^{(r)},$$

$$[J_{\pm}, T_{\rho}^{(r)}] = \sqrt{r(r+1) - \rho(\rho+1)}T_{\rho\pm1}^{(r)},$$

• pa (Wigner-Eckardt-ova teorema):

$$\langle j'm'_j; \alpha' | T_{\rho}^{(r)} | j, m_j; \alpha \rangle = \langle j', m'_j | r, j; \rho, m_j \rangle \langle j'; \alpha' | T^{(r)} | j; \alpha \rangle$$

Kvark Model

Kvarkovi i barioni

Kao što faktorizujemo
 $\Psi(r, \theta, \phi) = \sum_{nlm} R_{nl}(r) Y_l^m(\theta, \phi)$

- Talasne funkcije vezanih stanja možemo faktorizovati
- $\boxed{\Psi(\vec{r}_1, \vec{r}_2, \vec{r}_3) \chi_1(\text{spin}) \chi_2(\text{ukus}) \chi_3(\text{boja})}$
- Osnovno stanje: faktor $\Psi(\vec{r}_1, \vec{r}_2, \vec{r}_3)$ je sferno-simetričan, pa je dakle i simetričan u odnosu na $\vec{r}_i \leftrightarrow \vec{r}_j$ razmenu $\forall i, j$.
- Faktor $\chi_3(\text{boja})$ je antisimetričan u odnosu na razmenu .
 ⇒ faktor $\chi_1(\text{spin}) \chi_2(\text{ukus})$ mora biti simetričan u odnosu na $i \leftrightarrow j$ razmenu $\forall i, j$.
- Ako je faktor $\chi_1(\text{spin}) = \text{totalno simetričan} \Rightarrow \text{spin-}^3/2$,
- onda je i $\chi_2(\text{ukus})$ totalno simetričan ⇒ desetorka

Simetrizacija spina i ukusa

Sabiranje spina

$SU(2)$ & $SU(3)$

- Označimo

$$|\uparrow\rangle := |\frac{1}{2}, +\frac{1}{2}\rangle, \quad |\downarrow\rangle := |\frac{1}{2}, -\frac{1}{2}\rangle; \quad |\uparrow\uparrow\rangle := |\frac{1}{2}, +\frac{1}{2}\rangle \otimes |\frac{1}{2}, +\frac{1}{2}\rangle, \text{ itd.}$$

- Onda imamo da je

$$|\frac{1}{2}, \pm \frac{1}{2}\rangle \otimes |\frac{1}{2}, \pm \frac{1}{2}\rangle = \left\{ \begin{array}{l} |\mathbf{1}, +1\rangle = |\uparrow\uparrow\rangle, \\ |\mathbf{1}, 0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle), \\ |\mathbf{1}, -1\rangle = |\downarrow\downarrow\rangle, \\ |\mathbf{0}, 0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle), \end{array} \right. \begin{array}{l} \text{triplet} \\ \text{singlet} \end{array}$$

- gde je “triplet” simetričan u odnosu na razmenu dva faktora
a “singlet” antisimetričan u odnosu na razmenu dva faktora

Kvark model

$SU(2)$ izospin simetrija

- U izospinskom formalizmu: $|d\rangle := |\downarrow\rangle$ a $|u\rangle := |\uparrow\rangle$.
- Da opišemo izospinske faktore za anti-kvarkove, koristimo osobinu $SU(2)$ grupe da je $\det[U] = 1$.

$$\begin{aligned}\tilde{\epsilon}^{ab} &= U_c{}^a U_d{}^b \epsilon^{cd} = U_c{}^a U_d{}^b \frac{1}{2}(\epsilon^{cd} - \epsilon^{dc}) = \frac{1}{2}(U_c{}^a U_d{}^b \epsilon^{cd} - U_c{}^a U_d{}^b \epsilon^{dc}) \\ &= \frac{1}{2}(U_c{}^a U_d{}^b \epsilon^{cd} - \underbrace{U_d{}^b U_c{}^a \epsilon^{dc}}_{d:c}) = \frac{1}{2}(U_c{}^a U_d{}^b \epsilon^{cd} - U_c{}^b U_d{}^a \epsilon^{cd}) \\ &= \frac{1}{2}(U_c{}^a U_d{}^b - U_c{}^b U_d{}^a) \epsilon^{cd}, \quad \Rightarrow \quad \tilde{\epsilon}^{ab} = -\tilde{\epsilon}^{ba}.\end{aligned}$$

$$\begin{aligned}&= \frac{1}{2}(\delta_e^a \delta_f^b - \delta_e^b \delta_f^a) U_c{}^e U_d{}^f \epsilon^{cd} = \frac{1}{2} \epsilon^{ab} \epsilon_{ef} U_c{}^e U_d{}^f \epsilon^{cd} \\ &= \epsilon^{ab} \left[\frac{1}{2} \epsilon_{ef} U_c{}^e U_d{}^f \epsilon^{cd} \right] = \epsilon^{ab} \det[U] = \epsilon^{ab} \quad \text{QED}\end{aligned}$$

- Onda $(t^\alpha)^\dagger = t_\alpha = \epsilon_{\alpha\beta} t^\beta$, t.j. $t_1 = \epsilon_{12} t^2 = +t^2$ ali $t_2 = \epsilon_{21} t^1 = -t^1$.
- Odnosno, $|\bar{u}\rangle = |\downarrow\rangle$ ali je $|\bar{d}\rangle = \star | \uparrow \rangle$.

dokaz

Kvark model

$SU(2)$ izospin simetrija

- Dakle: $|d\rangle := |\downarrow\rangle$, $|u\rangle := |\uparrow\rangle$ pa $|\bar{u}\rangle = |\downarrow\rangle$, ali $|\bar{d}\rangle = \star - |\uparrow\rangle$.
- Onda su moguća $[q\bar{q}]$ stanja:

$$\begin{aligned} |u\rangle \otimes |\bar{u}\rangle &= |\uparrow\rangle \otimes |\downarrow\rangle &= \frac{1}{\sqrt{2}} |\pi^0\rangle + \frac{1}{\sqrt{2}} |\eta^0\rangle \\ |d\rangle \otimes |\bar{u}\rangle &= |\downarrow\rangle \otimes |\downarrow\rangle &= |1, -1\rangle = \pi^- \\ |u\rangle \otimes |\bar{d}\rangle &= |\uparrow\rangle \otimes (-|\uparrow\rangle) &= -|1, +1\rangle = \pi^+ \\ |d\rangle \otimes |\bar{d}\rangle &= |\downarrow\rangle \otimes |\uparrow\rangle &= -\frac{1}{\sqrt{2}} |\pi^0\rangle + \frac{1}{\sqrt{2}} |\eta^0\rangle \end{aligned}$$

- Mada je triplet $\{|1,\pm\rangle, |1,0\rangle\}$ simetričan kao reprezentacija $SU(2)$ a singlet $|0,0\rangle$ je antisimetričan, razmena kvarka i antikvarka se očigledno može smatrati simetrijom.

Kvark model

$SU(2)$ izospin simetrija

• I, obratno,

$$|1, +1\rangle = |\uparrow\uparrow\rangle = -|u\rangle \otimes |\bar{d}\rangle$$

$$|1, 0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) = \frac{1}{\sqrt{2}}(|u\rangle \otimes |\bar{u}\rangle - |d\rangle \otimes |\bar{d}\rangle)$$

$$|1, -1\rangle = |\downarrow\downarrow\rangle = |d\rangle \otimes |\bar{u}\rangle$$

$$|0, 0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) = \frac{1}{\sqrt{2}}(|u\rangle \otimes |\bar{u}\rangle + |d\rangle \otimes |\bar{d}\rangle)$$

• Mada 1., 3. i 4. stanje izgleda simetrično and 2. antisimetrično, razmena kvarka i antikvarka nije simetrija.

• Operatori J_1, J_2, J_3 generišu $SU(2)_I$ transformacije d, u para

• Npr.: $|\uparrow\rangle \mapsto \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ & $|\downarrow\rangle \mapsto \begin{bmatrix} 0 \\ 1 \end{bmatrix}$: $J_i \mapsto \frac{1}{2}\sigma_i$ (Pauli-eve matrice)

Kvark model

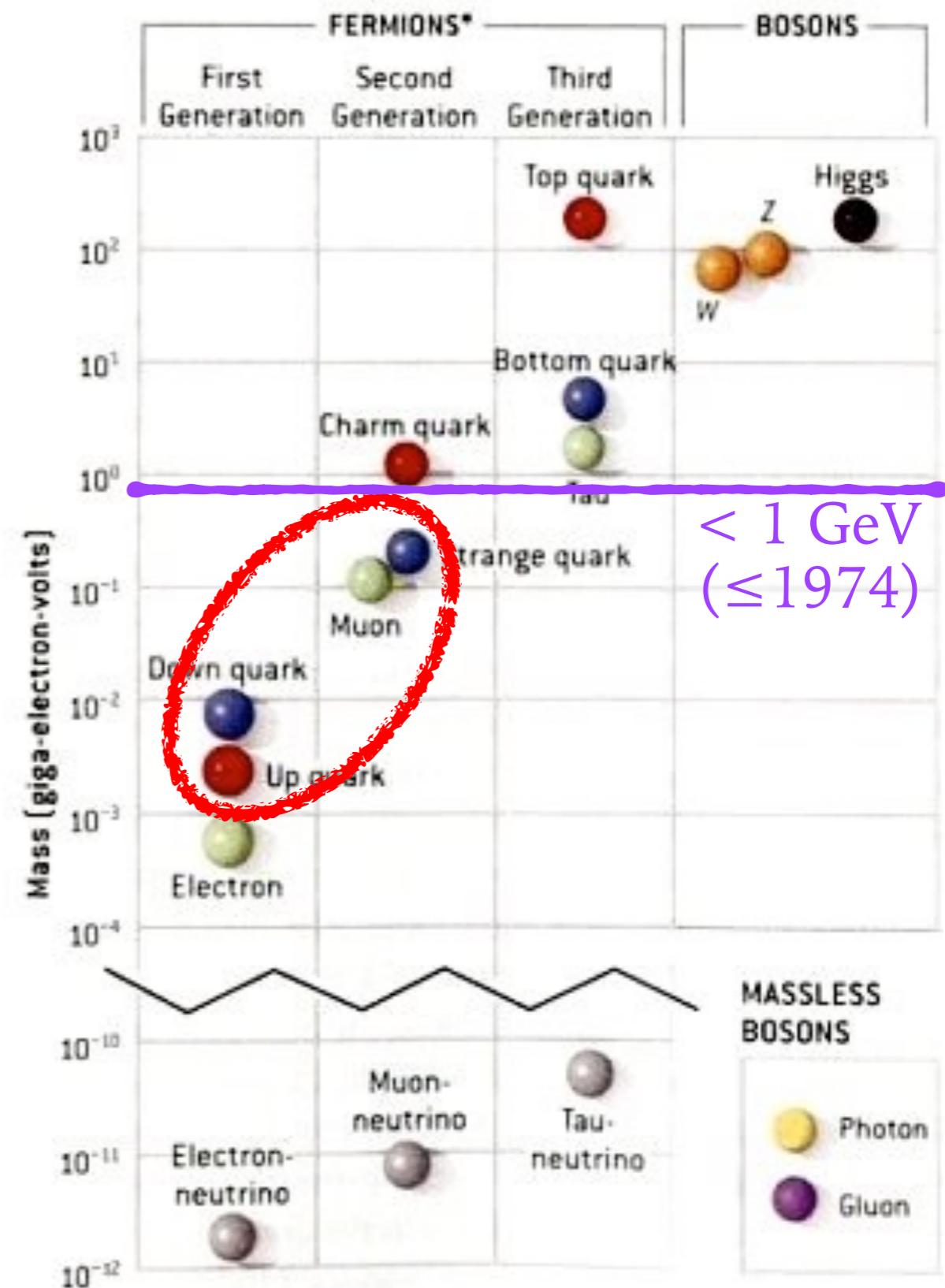
$SU(2)$ izospin simetrija

- $SU(2)_I$ transformacije “rotiraju” unutar 3D prostora $\{ \pi^-, \pi^0, \pi^+ \}$, što je onda irreducibilna reprezentacija (irrep) “3” (=“izospin-1”) grupe $SU(2)_I$ — tj. ne postoji potprostor u kome su $SU(2)_I$ -transformacije zavorene.
- $SU(2)_I$ transformacije ne menjaju 1D prostor $\{ \eta^0 \}$, te je to invarijanta, “1” (=“izospin-0”) irrep grupe $SU(2)_I$.
- Slično tome, $\{ d, u \}$ čine “2” (=“izospin- $1/2$ ”) irrep, a $\{ \bar{d}, \bar{u} \}$ čine ekvivalentnu “2” (=“izospin- $1/2$ ”) irrep.
- U “čestičarskoj” notaciji, $2 \otimes 2 = 3 \oplus 1$
izospin- $(1/2 \otimes 1/2) = 1 \oplus 0$
- Za 3-kvark stanja, izospin- $(2 \otimes 2 \otimes 2 = (3 \oplus 1) \otimes 2 = 4 \oplus 2 \oplus 2)$.
- Isto kao $spin-(1/2 \otimes 1/2 \otimes 1/2) = spin-(3/2 \oplus 1/2 \oplus 1/2)$ u kv. mehanici.

Kvark model

Elementarne čestice

- Po masama su kvarkovi drugačije grupisani:
 - u i d kvarkovi su vrlo slični, $300\text{--}350 \text{ MeV}/c^2$
 - s je malo teži, $\sim 500 \text{ MeV}/c^2$.
- $m_c \sim 1.5 \text{ GeV}/c^2$
- $m_b \sim 4.7 \text{ GeV}/c^2$
- $m_t \sim 173.1 \text{ GeV}/c^2$
- Stoga su u , d i s "sličniji"
- Razmena u , d i s kvarka čini *aproksimativnu* simetriju



Kvark model

$SU(3)_f$ simetrija

< 1 GeV
(≤ 1974)

- U eksperimentima gde su razlike izmedju m_u , m_d i m_s zanemarljive, $SU(3)_f$ je (približna) simetrija koja “rotira” u { d , u , s } prostoru:
- Tu je prostor kompleksnih linearnih kombinacija d , u i s kvarkova (tj. njihovih talasnih funkcija) “3” u $SU(3)_f$.
- Mezoni se transformišu kao $3 \otimes \bar{3} = 1 \oplus 8$ u $SU(3)_f$
 - otud oktet i singlet u šemi “osmostrukog puta”.
- Barioni kao $3 \otimes 3 \otimes 3 = (6 \oplus \bar{3}) \otimes 3 = 10 \oplus 8 \oplus 1 \oplus 8$
 - otud dekuplet, dva različita okteta, i singlet;
 - Spin, pa i Pauli-ev princip, se analizira dodatno.
- Evidentno: $SU(2)_I \subsetneq SU(3)_f$, tako da $3_{SU(3)} = (2 \oplus 1)_{SU(2)}$
- Regularna podgrupa: $\{d, u, s\} = \{d, u\} \oplus \{s\}$

Kvark model

$SU(3)_f$ simetrija

- Za $SU(2)$ znamo da je, u proizvodu “ $2 \otimes 2 = 3 \oplus 1$, “3” simetrično a “1” antisimetrično u odnosu na razmenu prve i druge “2”.
- Kod $SU(3)$ proizvoda, situacija je komplikovanija
- U razvoju “ $3 \otimes 3 \otimes 3 = 10 + 8 + 8 + 1$,” “10” je totalno simetrično, “1” totalno antisimetrično...
- A obe “8” irrep su delimično simetrične: možemo odabratи bazis u kome je jedan oktet antisimetričan u odnosu na razmenu prva dva kvarka, $\mathbf{8}_{[12]3}$, a drugi antisimetričan u odnosu na razmenu poslednja dva, $\mathbf{8}_{1[23]}$.
- Antisimetrizacija po trećem paru, $\mathbf{8}_{[1|2|3]} = \mathbf{8}_{[12]3} + \mathbf{8}_{1[23]}$, nije linearno nezavisna od prve dve.
- Ili, možemo odabratи bazis koji ima $\mathbf{8}_{[12]3}$ i $\mathbf{8}_{(12)3}$.

Kvark model

$SU(3)_f$ simetrija

- (Anti)simetrizacija: $3 \otimes 3 = \mathbf{6} \oplus \bar{\mathbf{3}}$ $\bar{\mathbf{3}} \simeq (t \cdot s)_\alpha := \frac{1}{2} \epsilon_{\alpha\beta\gamma} t^{[\beta} s^{\gamma]}$

$$t^\alpha s^\beta = t^{(\alpha} s^{\beta)} + t^{[\alpha} s^{\beta]}, \quad \begin{cases} t^{(\alpha} s^{\beta)} &:= \frac{1}{2} (t^\alpha s^\beta + t^\beta s^\alpha), \\ t^{[\alpha} s^{\beta]} &:= \frac{1}{2} (t^\alpha s^\beta - t^\beta s^\alpha); \end{cases}$$

- ... pošto je Levi-Civita simbol $\epsilon_{\alpha\beta\gamma}$ totalno antisimetričan.

- Pa onda: $\mathbf{6} \otimes \mathbf{3} = \mathbf{10} \oplus \mathbf{8}$ $t^{(\alpha\beta} s^\gamma = t^{(\alpha\beta} s^\gamma + \frac{4}{3} t^{(\alpha[b)} s^{\gamma]},$

$$\begin{aligned} t^{(\alpha\beta} s^\gamma &:= \frac{1}{3} (t^{(\alpha\beta} s^\gamma + t^{(\beta\gamma} s^\alpha + t^{(\gamma\alpha} s^\beta), \\ t^{(\alpha[b)} s^{\gamma]} &:= \frac{1}{4} ((t^{(\alpha\beta} s^\gamma - t^{(\alpha\gamma} s^\beta) + (t^{(\beta\alpha} s^\gamma - t^{(\beta\gamma} s^\alpha)), \\ &= \frac{1}{4} (2t^{(\alpha\beta} s^\gamma - t^{(\alpha\gamma} s^\beta - t^{(\beta\gamma} s^\alpha) \end{aligned}$$

- Matematičari: $\mathbf{10} = \text{Sym}(\mathbf{6} \otimes \mathbf{3})$ (što je projekcija),
 $\mathbf{8} = \ker[(\mathbf{6} \otimes \mathbf{3}) \rightarrow \text{Sym}(\mathbf{6} \otimes \mathbf{3})]$ (što je “ostatak”)

Digresija

3-čestična stanja

		123	132	312	321	231	213
a	[1 2 3]	+	-	+	-	+	-
b	(1 2 3)	+	+	+	+	+	+
c	[1 2] 3	+2	+	-	+	-	-2
d	(1 2) 3	+2	-	-	-	-	+2
e	1 [2 3]	+2	-2	-	+	-	+
f	1 (2 3)	+2	+2	-	-	-	-

$\{a, b, (\text{bilo koja dva od } c-f)\}$ čine kompletan (lin. nez.) bazis;

$\{a, b, c, d\}$ kao i $\{a, b, e, f\}$ su i "komplementarni" bazisi.

$$|a\rangle + |b\rangle + |c\rangle + |d\rangle = |123\rangle \quad \& \quad |a\rangle + |b\rangle + |e\rangle + |f\rangle = |123\rangle$$

Kvark model

$SU(3)_f \times SU(2)_s$ simetrija

- Ali, $SU(3)_f \times SU(2)_s$ nije dosta:
 - $\mathbf{10}_f$ je simetričan pod razmenom bilo koja dva kvarka
 - ima spin- $3/2$ — dakle, sva tri kvarka imaju isti (paralelni) spin, $1/2$
 - ili Pauli-ev princip za kvarkove ne važi (nisu pravi fermioni), ili mora da postoji još jedan kvantni broj po kome su antisimetrični, pa da tri kvarka mogu da budu totalno antisimetrični.
- Taj treći kvantni “broj” je boja: $SU(3)_c$.
- Onda je “dekuplet” = $\mathbf{4}_s \otimes \mathbf{10}_f \otimes \mathbf{1}_c = \chi_{(123)s}^{\mathbf{4}} \chi_{(123)f}^{\mathbf{10}} \chi_{[123]c}^{\mathbf{1}}$ ima talasnu funkciju koja je antisimetrična pri razmeni bilo koja dva kvarka.

Notacija: $\chi_{(123)}$ je totalno simetrični a $\chi_{[123]}$ je totalno antisimetrični faktor

Kvark model

$$SU(3): 3 \otimes 3 \otimes 3 = 10_S \oplus 8_M \oplus 8_M \oplus 1_A$$

$$SU(2): 2 \otimes 2 \otimes 2 = 4_S \oplus 2_M \oplus 2_M$$

$SU(3)_f \times SU(2)_s$ simetrija

(BEZ BOJE)

- Pokušajmo: $\Psi(\vec{r}_1, \vec{r}_2, \vec{r}_3) [\chi_{123} \text{ spin } \chi_{123f}]_A$
- Pošto spin- $\frac{1}{2}$ faktori imaju samo dve komponente (\uparrow i \downarrow), ne može da postoji totalno antisimetrična trojka, pa onda:
 - Plan A: $[\chi_{123} \text{ spin}]_S [\chi_{123f}]_A$
 - $[\chi_{(123)\text{spin}}]_S = [2 \otimes 2 \otimes 2]_S = [(2 \otimes 2)_S \otimes 2]_S = [3 \otimes 2]_S = 4$ daje spin- $\frac{3}{2}$,
 - Pošto $[\chi_{[123]f}]_A = [3 \otimes 3 \otimes 3]_A = [(3 \otimes 3)_A \otimes 3]_A = [\bar{3} \otimes 3]_A = 1$
 $\chi_{[123]f} \propto (|dus\rangle - |dsu\rangle + |sdu\rangle - |sud\rangle + |usd\rangle - |uds\rangle)$
 - “Plan A” predviđa spin- $\frac{3}{2}$ singlet (a ne dekuplet) \times
a spin- $\frac{3}{2}$ dekuplet?
 - Plan B: $(\chi_{(12)3s} \chi_{[12]3f} + \chi_{(1|2|3)s} \chi_{[1|2|3]f} + \chi_{1(23)s} \chi_{1[23]f})$
 - Totalno simetrizovani proizvod sa komplementarno mešovitom simetrijom može da dà samo spin- $\frac{1}{2}$ oktet; $(3 \otimes 3 \otimes 3)_M = 8_M$. $\smile ?$

Simetrizacija spina i ukusa

Dekuplet (10)

(BEZ BOJE)

- Totalno simetrični spinski faktor, $\chi_{(123)s}$, je

$$|\frac{3}{2}, m_s\rangle = \left\{ |\downarrow\downarrow\downarrow\rangle, |\downarrow\downarrow\uparrow\rangle_{(123)}, |\downarrow\uparrow\uparrow\rangle_{(123)}, |\uparrow\uparrow\uparrow\rangle \right\}$$
$$m_s = \begin{matrix} -\frac{3}{2} \\ -\frac{1}{2}, \\ +\frac{1}{2}, \\ +\frac{3}{2} \end{matrix}$$

• gde je

$$|\uparrow\downarrow\downarrow\rangle_{(123)} := \frac{1}{\sqrt{3}} [|\uparrow\downarrow\downarrow\rangle + |\downarrow\uparrow\downarrow\rangle + |\downarrow\downarrow\uparrow\rangle].$$
$$m_s = \begin{matrix} -\frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \end{matrix}$$

- Sve linearne kombinacije ovih stanja su totalno simetrične.
- Faktor ukusa/vrste je $\chi_{(123)f} := \{ |ddd\rangle, |ddu\rangle_{(123)}, |duu\rangle_{(123)}, |uuu\rangle,$
 $|dds\rangle_{(123)}, |dus\rangle_{(123)}, |uus\rangle_{(123)},$
 $|dss\rangle_{(123)}, |uss\rangle_{(123)},$
 $|sss\rangle \}$
- \Rightarrow “dekuplet” (desetorka, 10 bazisnih stanja)
=totalno simetrična
- Stoga je $\chi_{(1,2,3)\text{spin}}$ $\chi_{(1,2,3)f}$ totalno simetričan



Simetrizacija spina i ukusa

Dekuplet (10)



- Totalno simetrični spinski faktor, $\chi_{(123)s}$, je

$$|\frac{3}{2}, m_s\rangle = \left\{ |\downarrow\downarrow\downarrow\rangle, |\downarrow\downarrow\uparrow\rangle_{(123)}, |\downarrow\uparrow\uparrow\rangle_{(123)}, |\uparrow\uparrow\uparrow\rangle \right\}$$
$$m_s = \begin{matrix} -3/2 \\ -1/2, \\ +1/2, \\ +3/2 \end{matrix}$$

- gde je

$$|\uparrow\downarrow\downarrow\rangle_{(123)} := \frac{1}{\sqrt{3}} [|\uparrow\downarrow\downarrow\rangle + |\downarrow\uparrow\downarrow\rangle + |\downarrow\downarrow\uparrow\rangle].$$
$$m_s = \begin{matrix} -1/2 \\ -1/2 \\ -1/2 \end{matrix}$$

- Sve linearne kombinacije ovih stanja su totalno simetrične.

- Faktor ukusa/vrste je $\chi_{(123)f} := \{ |ddd\rangle, |ddu\rangle_{(123)}, |duu\rangle_{(123)}, |uuu\rangle,$

- \Rightarrow “dekuplet” (desetorka, 10 bazisnih stanja)
=totalno simetrična

$$\begin{aligned} &|dds\rangle_{(123)}, |dus\rangle_{(123)}, |uus\rangle_{(123)}, \\ &|dss\rangle_{(123)}, |uss\rangle_{(123)}, \\ &|sss\rangle \} \end{aligned}$$

- Stoga je $\chi_{(1,2,3)\text{spin}} \chi_{(1,2,3)f}$ totalno simetričan



Kvark model

$$SU(3)_f \times SU(2)_s \times SU(3)_c$$

$$SU(3): 3 \otimes 3 \otimes 3 = 10_S \oplus 8_M \oplus 8_M \oplus 1_A$$

$$SU(2): 2 \otimes 2 \otimes 2 = 4_S \oplus 2_M \oplus 2_M$$

(SA BOJOM)

- Za $SU(3)_f$ oktet, uzmimo $\mathbf{8}_{[12]3f}$.
- Treba nam stanje totalnog spina $\frac{1}{2}$, a antisimetrično pod razmenom $1 \leftrightarrow 2$: $2_{[12]3} = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\bullet\rangle - |\downarrow\uparrow\bullet\rangle)$, $\bullet = \uparrow, \downarrow$
- $(\chi_{[12]3f}^8 \otimes \chi_{[12]3s}^2 + \chi_{[1|2|3]f}^8 \otimes \chi_{[1|2|3]s}^2 + \chi_{1[23]f}^8 \otimes \chi_{1[23]s}^2) \otimes \chi_{[123]c}^1$ je talasna funkcija koja je totalno antisimetrična pri razmeni bilo koja dva kvarka.
- Bez $SU(3)_c$, spin- $\frac{1}{2}$ oktet bi mogao da se konstruiše, ali ne i spin- $\frac{3}{2}$ dekuplet.
- Dakle, mora da postoji dodatni stepen slobode po kojem tri inače jednaka kvarka mogu da se antisimetrizuju.
- Taj dodatni stepen slobode mora da ima tačno tri “vrednosti” kako bi “[]” bilo jedino totalno antisimetrično stanje.

$SU(3)_c$

QED

dokaz

Kvarkovi

Da ponovimo:

Particle Data Group: <http://pdg.lbl.gov>

Ime / Energy	Spin	Q	$I_3^{(W)}$
ν_e $< 3 \text{ eV}$	ν_μ $< 0.19 \text{ MeV}$	ν_τ $< 18.2 \text{ MeV}$	$\pm \frac{1}{2}$
e .511 MeV	μ 106 MeV	τ 1.78 GeV	$\pm \frac{1}{2}$
u, u, u 1.5–4.5 MeV	c, c, c 1.0–1.4 GeV	t, t, t .17–.18 TeV	$\pm \frac{1}{2}$
d, d, d 5.0–8.5 MeV	s, s, s .08–.15 GeV	b, b, b 4.0–4.5 GeV	$\pm \frac{1}{2}$

Zašto 3 ?!

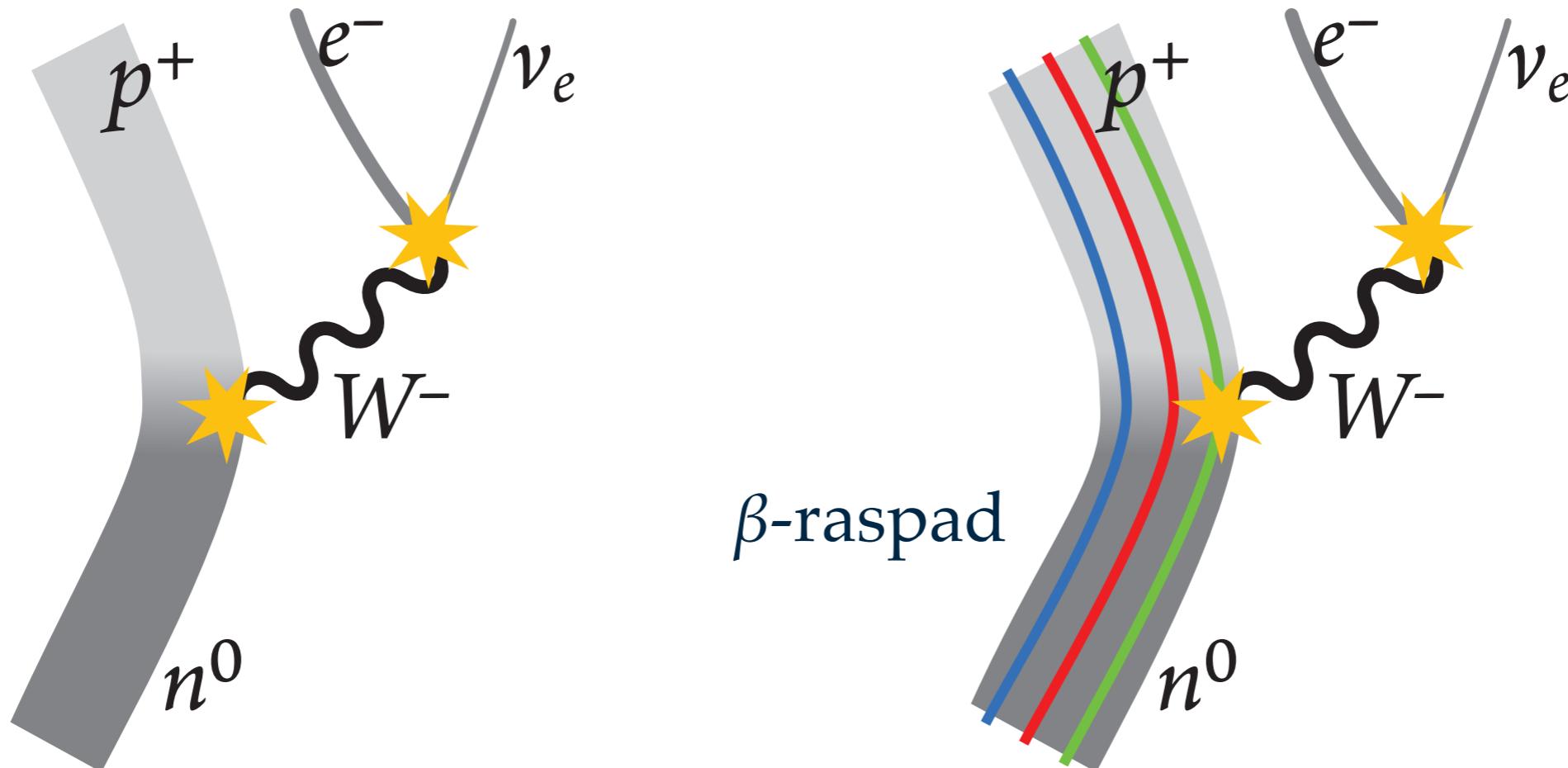
Plus posrednici interakcija:
foton, W^\pm , Z^0 , gluoni i gravitoni.

i Higgs-ova čestica.

Kvark model

Hadronski procesi

- Procesi vezanih stanja kvarkova (i anti-kvarkova) se svode na procese samih kvarkova:



Kalibracioni princip

Kvantna mehanika predviđa EM polje

- Smenom $\Psi(\vec{r}, t) \rightarrow e^{i\varphi(\vec{r}, t)} \Psi(\vec{r}, t)$ dobijemo:

$$i\hbar \frac{\partial}{\partial t} \left(e^{i\varphi} \Psi \right) = \left[-\frac{\hbar^2}{2m} \vec{\nabla}^2 + V(\vec{r}) \right] \left(e^{i\varphi} \Psi \right)$$

$$i\hbar \left(e^{i\varphi} i \frac{\partial \varphi}{\partial t} \right) \Psi + i\hbar e^{i\varphi} \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \vec{\nabla} \cdot \left(e^{i\varphi} (i \vec{\nabla} \varphi) \Psi + e^{i\varphi} \vec{\nabla} \Psi \right) + V(\vec{r}) e^{i\varphi} \Psi$$

$$-\hbar e^{i\varphi} \left(\frac{\partial \varphi}{\partial t} \right) \Psi + e^{i\varphi} \left(i\hbar \frac{\partial \Psi}{\partial t} \right)$$

Izvorna Schrödinger-ova jednačina

$$= -\frac{\hbar^2}{2m} \left(e^{i\varphi} (i \vec{\nabla} \varphi)^2 \Psi + e^{i\varphi} (i \vec{\nabla}^2 \varphi) \Psi + 2e^{i\varphi} (i \vec{\nabla} \varphi) \cdot (\vec{\nabla} \Psi) \right) - \frac{\hbar^2}{2m} e^{i\varphi} \vec{\nabla}^2 \Psi + V(\vec{r}) e^{i\varphi} \Psi$$

$$\frac{\partial \varphi}{\partial t} = \frac{\hbar}{2m} \left((i \vec{\nabla} \varphi)^2 + i \vec{\nabla}^2 \varphi + 2(i \vec{\nabla} \varphi) \cdot (\vec{\nabla} \ln(\Psi)) \right)$$

- što nameće uslov na fazu—da je zavisna od stanja $\Psi(\vec{r}, t)$!
- A to nikako ne bi smelo (!?): promena faze talasne funkcije mora da bude potpuno proizvoljna i nemerljiva (neprimetna)!

Kalibracioni princip

Kvantna mehanika predvidja EM polje

- Ostaje nam samo da “popravimo” parcijalne izvode:

$$\frac{\partial}{\partial t} \rightarrow \mathcal{D}_t := \frac{\partial}{\partial t} + X \quad \vec{\nabla} \rightarrow \vec{\mathcal{D}} := \vec{\nabla} + \vec{Y}$$

i zahtevamo da se “korekcije” X i \vec{Y} transformišu simultano sa Ψ , i to tako da njihove transformacije potiru “suvišne članove”.

- To će se dogoditi tačno onda ako za $\widetilde{\Psi} = e^{i\varphi}\Psi$ sledi da je

$$\widetilde{\mathcal{D}}_t(e^{i\varphi}\Psi) = e^{i\varphi}(\mathcal{D}_t\Psi) \quad \widetilde{\vec{\mathcal{D}}}(e^{i\varphi}\Psi) = e^{i\varphi}(\vec{\mathcal{D}}\Psi)$$

to jest, ako

$$\widetilde{\mathcal{D}}_t = e^{i\varphi} \mathcal{D}_t e^{-i\varphi}$$

$$\widetilde{\vec{\mathcal{D}}} = e^{i\varphi} \vec{\mathcal{D}} e^{-i\varphi}$$

- pa su \mathcal{D}_t i $\vec{\mathcal{D}}$ tzv. **kovarijantni izvodi** za faznu promenu.

Kalibracioni princip

Kvantna mehanika predvidja EM polje

$$\begin{aligned}\widetilde{\mathcal{D}}_t &= e^{i\varphi} \mathcal{D}_t e^{-i\varphi} \\ \widetilde{\vec{\mathcal{D}}} &= e^{i\varphi} \vec{\mathcal{D}} e^{-i\varphi}\end{aligned}$$

- Iz čega sledi:

$$\frac{\partial}{\partial t} + \widetilde{X} = e^{i\varphi} \left[\frac{\partial}{\partial t} + X \right] e^{-i\varphi}$$

$$\frac{\partial}{\partial t} + \widetilde{X} = \frac{\partial}{\partial t} - i \frac{\partial \varphi}{\partial t} + X$$

$$\vec{\nabla} + \widetilde{\vec{Y}} = e^{i\varphi} [\vec{\nabla} + \vec{Y}] e^{-i\varphi}$$

$$\vec{\nabla} + \widetilde{\vec{Y}} = \vec{\nabla} - i(\vec{\nabla} \varphi) + \vec{Y}$$

- pa se X i \vec{Y} transformišu nehomogeno (što je tipično za koneksiju!):

$$\widetilde{X} = X - i \frac{\partial \varphi}{\partial t}$$

$$\widetilde{\vec{Y}} = \vec{Y} - i(\vec{\nabla} \varphi)$$

- Da li vas to potseća na nešto što ste već videli?

Kalibracioni princip

Kvantna mehanika predviđa EM polje

- Nova Schrödinger-ova jednačina onda treba da bude:

$$i\hbar \left[\frac{\partial}{\partial t} + X \right] \Psi(\vec{r}, t) = \left[-\frac{\hbar^2}{2m} \left[\vec{\nabla} + \vec{Y} \right]^2 + V(\vec{r}) \right] \Psi(\vec{r}, t)$$

- Za razbibrigu (nije domaći zadatak, pošto ima u knjizi):

Naći α, β, γ u zamenama $X = \alpha\Phi$, $\vec{Y} = \beta\vec{A}$, i $\varphi = \gamma\lambda$, tako da Φ i \vec{A} budu skalarni i vektorski potencijal elektromagnetskog polja, a λ parametar kalibracione transformacije elektromagnetizma.

Kalibracioni princip

Kvantna mehanika predvidja EM polje

• Prepoznatljive definicije:

$$\Phi := \frac{\hbar}{iq} X, \quad \vec{A} := \frac{i\hbar}{q} \vec{Y}, \quad \lambda := \frac{\hbar}{q} \varphi,$$

$$D_t := \frac{\partial}{\partial t} + i \frac{q}{\hbar} \Phi, \quad \vec{D} := \vec{\nabla} - i \frac{q}{\hbar} \vec{A},$$

$$\Phi \rightarrow \Phi' = \Phi - \frac{\partial \lambda}{\partial t}, \quad \vec{A} \rightarrow \vec{A}' = \vec{A} + (\vec{\nabla} \lambda),$$

$$\Psi(\vec{r}, t) \rightarrow \Psi'(\vec{r}, t) = e^{iq\lambda(\vec{r}, t)/\hbar} \Psi(\vec{r}, t),$$

$$\vec{B} := \vec{\nabla} \times \vec{A} \quad \text{i} \quad \vec{E} := -\vec{\nabla} \Phi - \frac{\partial}{\partial t} \vec{A}$$

Invarijante kalibracionih transformacija!

Kalibracioni princip

Kvantna mehanika predvidja EM polje

• Prepoznatljivi rezultati:

$$i\hbar \left[\frac{\partial}{\partial t} + X \right] \Psi = \left[-\frac{\hbar^2}{2m} (\vec{\nabla} + \vec{Y})^2 + V(\vec{r}, t) \right] \Psi,$$

$$i\hbar \frac{\partial}{\partial t} \Psi(\vec{r}, t) = H_{\text{EM}} \Psi(\vec{r}, t),$$

$$H_{\text{EM}} = \frac{1}{2m} \left(\frac{\hbar}{i} \vec{\nabla} - q \vec{A}(\vec{r}, t) \right)^2 + \left[V(\vec{r}, t) + q\Phi(\vec{r}, t) \right]$$

• Takođe:

$$\int dt d^3\vec{r} \left\{ c_1 (\epsilon_0 \vec{E}^2) + c_2 \left(\frac{1}{\mu_0} \vec{B}^2 \right) + c_3 \left(\sqrt{\frac{\epsilon_0}{\mu_0}} \vec{E} \cdot \vec{B} \right) - (\rho \Phi + \vec{j} \cdot \vec{A}) \right\}$$

• gde c_1, c_2, c_3 odredimo tako da su jednačine kretanja
= Maxwell-ovim jednačinama.

Lorentz-invarijantnost: $c_1 = -c_2$.

Kalibracioni princip

Kvantna mehanika predvidja EM polje

- Maxwell-ove jednačine:

$$\left\{ \begin{array}{l} (\text{Gauss}) \quad \vec{\nabla} \cdot \vec{E} = \frac{1}{4\pi\epsilon_0} 4\pi \rho_e, \quad \vec{\nabla} \times (c\vec{B}) - \frac{1}{c} \frac{\partial \vec{E}}{\partial t} = \frac{1}{4\pi\epsilon_0} \frac{4\pi}{c} \vec{j}_e, \\ (\text{Ampère}) \quad \vec{\nabla} \cdot (c\vec{B}) = \frac{\mu_0}{4\pi} 4\pi \rho_m, \quad -\vec{\nabla} \times \vec{E} - \frac{1}{c} \frac{\partial (c\vec{B})}{\partial t} = \frac{\mu_0}{4\pi} \frac{4\pi}{c} \vec{j}_m, \\ (\text{Faraday}) \end{array} \right.$$

- ako dozvolimo postojanje i magnetnih naboja i struja.

- Izvorno, Maxwell:

$$\vec{B} := \vec{\nabla} \times \vec{A} \quad \text{i} \quad \vec{E} := - \left(\vec{\nabla} \Phi + \frac{\partial \vec{A}}{\partial t} \right)$$

$$\rho_m = 0$$

$$j_m = 0$$

Magnetni naboji i
struje su prepreka
za potencijale

Kalibracioni princip

Kvantna mehanika predvidja EM polje

- Maxwell-ove jednačine impliciraju zakon održanja nanelektrisanja:

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{4\pi\epsilon_0} 4\pi \rho_e, \Rightarrow \frac{\partial(\vec{\nabla} \cdot \vec{E})}{\partial t} = \frac{1}{4\pi\epsilon_0} 4\pi \frac{\partial \rho_e}{\partial t},$$
$$\vec{\nabla} \times (\cancel{c} \vec{B}) - \frac{1}{\cancel{c}} \frac{\partial \vec{E}}{\partial t} = \frac{1}{4\pi\epsilon_0} \frac{4\pi}{\cancel{c}} \vec{j}_e, \Rightarrow \vec{\nabla} \cdot (\vec{\nabla} \times (\cancel{c}^2 \vec{B})) - \frac{\partial(\vec{\nabla} \cdot \vec{E})}{\partial t} = \frac{1}{4\pi\epsilon_0} 4\pi \vec{\nabla} \cdot \vec{j}_e,$$

$$\Rightarrow 0 = \frac{\partial \rho_e}{\partial t} + \vec{\nabla} \cdot \vec{j}_e.$$

$$\frac{\partial \rho_e}{\partial t} = -\vec{\nabla} \cdot \vec{j}_e,$$

\Rightarrow

$$\frac{dQ_{e,V}}{dt} = - \oint_{\partial V} d^2 \vec{r} \cdot \vec{j}_e.$$

- Dakle, elektrodinamika bi bila (logički) nekonzistentna da je zakon očuvanja nanelektrisanja narušen.
- Pažnja: “narušenje” (breaking) je sistemsko ometanje simetrije, dok je “kršenje” (violation) jednokratni događaj.

Kalibracioni princip

Kvantna mehanika predvidja EM polje

- U 4-dimenzionaloj notaciji:

$$A_\mu := (\Phi, -c\vec{A}), \quad (\text{4-vektor})$$

$$F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu, \quad (\begin{smallmatrix} \text{antisimetrični} \\ \text{tenzor ranga 2} \end{smallmatrix})$$

$$[F_{\mu\nu}] = \begin{bmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & -cB_3 & cB_2 \\ -E_2 & cB_3 & 0 & -cB_1 \\ -E_3 & -cB_2 & cB_1 & 0 \end{bmatrix}$$

$$\partial_\mu F^{\mu\nu} = \frac{1}{4\pi\epsilon_0} \frac{4\pi}{c} j_e^\nu \quad \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} \partial_\mu F_{\nu\rho} = \frac{\mu_0}{4\pi} \frac{4\pi}{c} j_m^\sigma,$$

$$\frac{1}{2} F_{\mu\nu} F^{\mu\nu} = \vec{E}^2 - c^2 \vec{B}^2, \quad \text{i} \quad \frac{1}{4} \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} = -c \vec{E} \cdot \vec{B},$$

$$\mathcal{L}_{EM} = C_1 F_{\mu\nu} F^{\mu\nu} + C_2 \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} - j^\mu A_\mu$$

sa 3 na 2 proizvoljne konstante

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