(Fundamental) Physics of Elementary Particles

Partial compactification; Mirror symmetry
Warped cosmology; Brane-Worlds and localization of matter and interactions

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Fundamental Physics of Elementary Particles

Program

- Anomalies and the critical dimension
  - Worldsheet fields
  - Worldsheet reparametrizations
  - Geometric quantization & loop space
- Partial spacetime compactification
  - Tori & left-right asymmetry
  - Calabi-Yau manifolds & bundles
- Mirror symmetry
- Warped cosmology
- Brane-Worlds
  - The Randall-Sundrum mechanism
  - Exospaces
- Localization of matter and interactions

Anomalies and the Critical Dimension

Worldsheet Fields

- Recall Polyakov's action for the string propagation

$$S[\mathscr{X};\eta_{\mu\nu}] = \frac{1}{4\pi\alpha'\hbar c^2}\int_\Sigma \mathrm{d}^2\xi\; g^{\alpha\beta}(\xi)\,\eta_{\mu\nu}\,(\partial_\alpha X^\mu)(\partial_\beta X^\nu),$$

- Here,
  - the (ξ^0, ξ^1) are local coordinates on the worldsheet
  - the $(X^0, X^1, \dots, X^{D-1})$ are local coordinates on the target spacetime
  - the $g_{\alpha\beta}(\xi)$ is the metric tensor on the worldsheet
  - the $\eta_{\mu\nu}(X)$ is the metric tensor on the target spacetime

$$G_{\alpha\beta}(X) := \eta_{\mu\nu}(\partial_\alpha X^\mu)(\partial_\beta X^\nu)$$

  - is the pull-back of the spacetime metric to the worldsheet
- Polyakov's action thus measures the "distance function" of the spacetime in terms of the "distance function" on the worldsheet.
- 1976: L. Brink, P. di Vecchia & P. Howe and S. Deser & B. Zumino; 1981: A. Polyakov, path-integral quantization of (super)strings

Anomalies and the Critical Dimension

Worldsheet Reparametrizations

- The (*Brink-di Vechia-Howe-Deser-Zumino-*)Polyakov action

$$S[\mathscr{X};\eta_{\mu\nu}] = \frac{1}{4\pi\alpha'\hbar c^2}\int_\Sigma \mathrm{d}^2\xi\; g^{\alpha\beta}(\xi)\,\eta_{\mu\nu}\,(\partial_\alpha X^\mu)(\partial_\beta X^\nu),$$

- is invariant with respect to $g_{\alpha\beta} \to e^{i\phi}\, g_{\alpha\beta}$:
  - $\det[g_{\alpha\beta}] \to e^{2i\phi}\det[g_{\alpha\beta}]$, so $\mathrm{d}^2\xi = (\det[g_{\alpha\beta}])^{1/2}\,\mathrm{d}\xi^0\mathrm{d}\xi^1 \to e^{i\phi}\mathrm{d}^2\xi$,
  - but $g^{\alpha\beta} \to e^{-i\phi}\, g^{\alpha\beta}$ then cancels this.
  - This is unique to 2-dimensional actions of either signature.
- 1981, Polyakov: conformal invariance is anomalous
  - $Z[\Upsilon;\eta] \to Z[\Upsilon;\eta]\cdot\exp\{-(D/96\pi)\int\mathrm{d}^2\xi\;[\partial_\alpha\,\phi(\xi)]^2\}\cdot$(reparametrization)
  - which cancels when $D = 26$.
- Soon: free fermions (instead of X^μ) contribute ½ each.
  - That's $26_B = 10_B + 16_B \simeq 10_B + 32_F = 10_B + 16_F + \overset{(-)}{16}_F$
  - 26D bosonic string or 10D N = 2 superstring (Type IIA or IIB)

quick & dirty!

The full computation is more… …detailed!

Anomalies and the Critical Dimension

Geometric Quantization and Loop Space

- A rigorous, constructive procedure: Classical → Quantum model
- Start with the classical model,
  - The phase space Φ
  - ...w/unique symplectic structure ω
  - in coordinates: $\omega = \mathrm{d}p_\alpha \wedge \mathrm{d}q^\alpha$.
  - Also $\omega^{-1} = \frac{\partial}{\partial p_\alpha} \wedge \frac{\partial}{\partial q^\alpha}$
  - Canonical inner products:

    $\mathrm{d}q^\alpha \cdot \frac{\partial}{\partial q^\beta} = \delta^\alpha_\beta \quad \mathrm{d}p_\alpha \cdot \frac{\partial}{\partial p_\beta} = \delta^\beta_\alpha$
- Observables are functions over Φ, $f = f(p_\alpha, q^\alpha)$.
- Replace $\Phi \to \mathscr{H}$
- ① To each observable f on Φ, assign a Hermitian operator O_f on $\mathscr{H}$.
- Require that ② $O_{\{f,g\}} = -i\hbar\, [O_f, O_g]$
- and that ③ $O_1 = \mathbb{1}$.
- The requirements ①–③ imply the need for a "polarization":
- ...so $\mathscr{H}$ is "holomorphic."

$$O_f := i\hbar\Big(\frac{\partial f}{\partial p_\alpha}\frac{\partial}{\partial q^\alpha} - \frac{\partial f}{\partial q^\alpha}\frac{\partial}{\partial p_\alpha}\Big) + f - p_\alpha \frac{\partial f}{\partial p^\alpha}$$

Anomalies and the Critical Dimension

Geometric Quantization and Loop Space

- 1987, M. Bowick & S. Rajeev, and J. Mickelsson: Geometric Quantization to string theories
- Soon (1987): K. Pilch & N. Warner and D. Harari, D. Hong, P. Ramond & V. Rodgers: generalization to superstrings
- The main result:

> The Loop space of spacetime must be Ricci-flat.

- Loop(X) = space of all immersions of S^1 in X.
- This implies that spacetime itself must be Ricci-flat.
- The result was argued to be non-perturbative & exact.
- It does not include p-branes for $p \neq 1$.
- No luck in extending it to p-branes with $p > 1$.

Partial Spacetime Compactification

Tori & Left-Right Asymmetry

- From the worldsheet perspective, there's no harm in requiring

$$X^i(\xi) \simeq X^i(\xi) + 2\pi R_i, \qquad i = 4, \ldots, 25, \quad \forall \xi \in \Sigma.$$

- This periodicity however implies a *global geometry* change:

$$\mathscr{X} = \mathbb{R}^{1,25} \quad \longrightarrow \quad \mathscr{X}' = \mathbb{R}^{1,3} \times T^{22}, \quad T^{22} := S^1_{(R_4)} \times \cdots \times S^1_{(R_{25})}$$

- where the subscripts denote the circumference radii.
  - The number of periodic coordinates determines the topology, $T^{22} = \mathbb{R}^{22}/\Lambda$, where Λ is a (simple, cubic) lattice.
  - The radii determine (some of) the "shape" of the torus.
  - We could switch to an oblique rectilinear coordinate system…
    - …or more generally, pick a different lattice.
    - The space of lattice choices defines a coarse choice of different tori
    - …the lattice-spacings complement these choices
    - …the choice of "constants" (such as $\eta_{\mu\nu}$) over this lattice = more choices.

Partial Spacetime Compactification

Tori & Left-Right Asymmetry

- So, this string model defines a non-compact $\mathbb{R}^{1,3}$ spacetime
  - with an additional T^{22} at every point of $\mathbb{R}^{1,3}$.
  - Now construct the string modes on such a space, use the expansion

$$X^\mu(\tau,\sigma) = x^\mu + \frac{p_\mu}{p_-} c\tau + i\sqrt{\frac{\alpha'}{2}} \sum_{\substack{n=-\infty \\ n\neq 0}}^{+\infty} \left[\frac{a^\mu_{n,R}}{n} e^{-2\pi i n\varsigma^+} + \frac{a^\mu_{n,L}}{n} e^{2\pi i n\varsigma^-} \right]$$

- where, for $\mu = 4, \ldots, 25$:
  - x^μ is the "center of mass" coordinate, moving on a radius-R_μ circle
  - p_μ is the "center of mass" momentum, quantized in units $\hbar/R_\mu$
  - the a^μ are the various creation ($n < 0$) and annihilation ($n > 0$) operators for the LHO-like oscillators (Fourier modes)
  - ...which act upon a vacuum $|0_L; 0_R\rangle$ to create various (physical) states
- From the Lagrangian, compute the conjugate 26-momentum
  - the square of which gives the mass^2 formula.

Partial Spacetime Compactification

Tori & Left-Right Asymmetry

- When computing the mass^2,
  - Each periodic coordinate contributes $(n_\mu \hbar/R_\mu)^2$ from p_μ,
  - periodicity $X^\mu \simeq X^\mu + w^\mu \cdot 2\pi R_\mu$ contributes $[w^\mu R_\mu/(\alpha' \hbar c^2)]^2$,
  - the oscillators contribute $[2(N_L + N_R - 2)\,/\,(\alpha' \hbar c^2)]^2$.

$$H_L \propto \frac{1}{2}\sum_{n=-\infty}^{\infty} \eta_{\mu\nu}\, a_{-n}^{\mu} a_n^{\nu} = \frac{1}{2}\eta_{\mu\nu}\, a_0^{\mu} a_0^{\nu} + \sum_{n=1}^{\infty} \eta_{\mu\nu}\, a_n^{\dagger\,\mu} a_n^{\nu} + N_{0L}$$

  - ...where the indefinite sums of zero-level energy contributions from both left- and right-movers contribute $N_{0L} + N_{0R} \to -2$.
  - This is independently verified from a ζ-function regularization, from algebraic consistency of Diff(S^1)—the Virasoro algebra, ...
- Additionally, the "level-matching condition" requires that left-movers and right-movers contribute equally to the energy.
  - In effect the left- and right-mover "Fermi levels" must match.
  - BTW: algebraic consistency also requires $D = 26$, related to $N_{0L} + N_{0R}$.

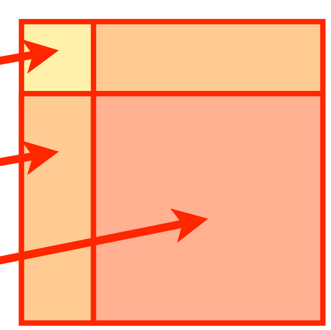
Partial Spacetime Compactification

Tori & Left-Right Asymmetry

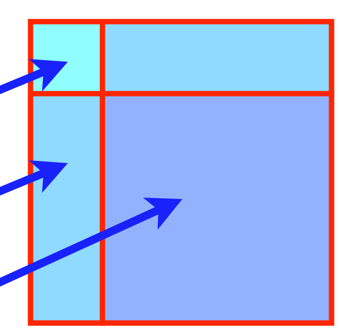
- The lowest-lying state in the bosonic string model
  - propagating in flat $\mathbb{R}^{1,25}$-like spacetime
  - or in any toroidal ($\mathbb{R}^{1,25\text{-}d}\times T^d$) compactification
- …is a tachyon: $m^2c^2 = -4\,/\,\alpha' c^2$.
  - From what we learned w/Higgs field: unstable vacuum.
- Next are massless states ($24^2 = 576$ physical polarizations)
- In $\mathbb{R}^{1,25}$ spacetime:
  - rank-2 symmetric tensor (spin-2) particle — the Pomeron
  - rank-2 antisymmetric tensor particle
  - rank-0 (spin-0) particle

Nordstrøm-Kałuża-Klein

- In $\mathbb{R}^{1,25\text{-}d}\times T^d$ spacetime:
  - rank-2: $G_{\mu\nu}$
  - rank-1: $G_{\mu n}$
  - rank-0: G_{mn}



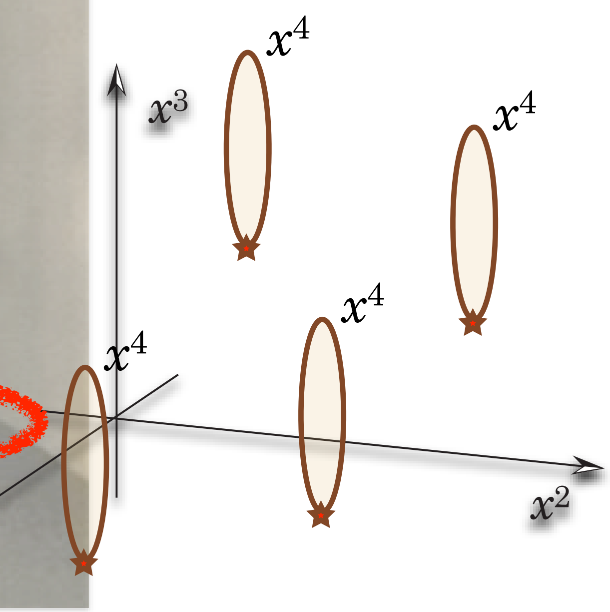
- rank-2: $B_{\mu\nu}$
- rank-1: $B_{\mu n}$
- rank-0: B_{mn}

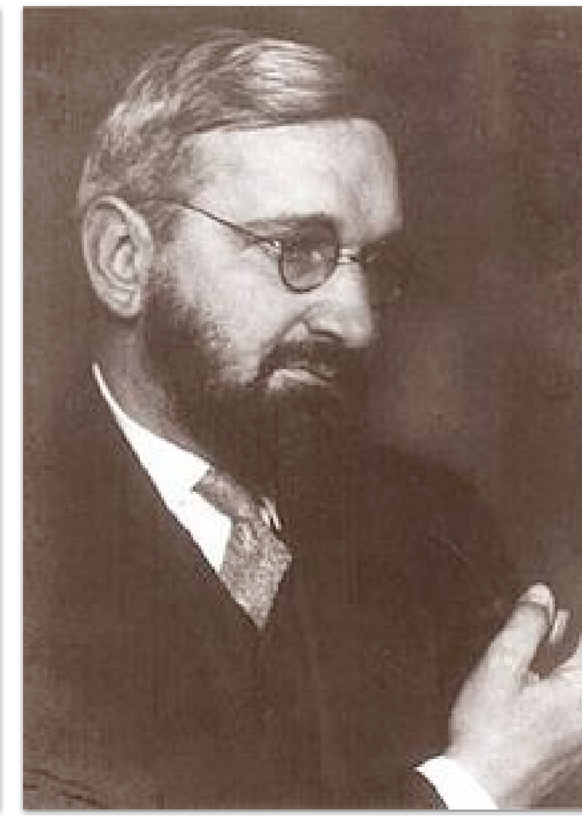


Partial Spacetime Compactification

A Nordstrøm-Kałuża-Klein Digression

- 1914: Gunnar Nordstrøm, (early competitor of Einstein's)
- 1919: Theodor F.E. Kałuża
- 1926: Oscar Klein
- Our spacetime is a sub-spacetime of a bigger (5-dimensional!) one:
- The metric tensor $\mathrm{d}s^2 = g_{ij}\, \mathrm{d}x^i \mathrm{d}x^j$, where $i, j = 0,$—$,4$
  - decomposes: ($g_{\mu\nu}$, $g_{\mu 4}$, g_{44})
  - g_{04} plays the role of Φ,
  - g_{i4} play the role of A_i,
  - ...of electromagnetism!
- Einstein's eq's ***beget*** Maxwell's
- ...provided x^4 curls up in a circle:





Partial Spacetime Compactification

Tori & Left-Right Asymmetry

- Unlike in any other theory, left- and right-movers can be compactified differently!
- 1984, D. Gross, J. Harvey, E. Martinec & R. Rohm:
  - $X_L{}^{\mu}$, for $\mu = 0, \ldots, 9$ — leave "flat & indefinite"
  - $X_L{}^{\mu}$, for $\mu = 10, \ldots, 25$ — torus with an $SO(32)$ or $E_8{\times}E_8$ root lattice
  - $X_R{}^{\mu}$, for $\mu = 0, \ldots, 9$ — leave "flat & indefinite"
  - $X_R{}^{\mu}$, for $\mu = 10, \ldots, 25$ — replace with 32 fermions
- No tachyon: eliminated by target spacetime supersymmetry.
- Massless states:
  - $\{G_{\mu\nu}, B_{\mu\nu}, \Phi \mid \Psi_{\mu}{}^{\alpha}, \chi^{\alpha}\}$ & $\{A_{\mu}{}^{a} \mid \lambda^{a}\}$
  - D. Friedan's quantum stability insures that these fields generate
    - N = 1 super*gravity* in 10-dimensional spacetime
    - Supersymmetric $SO(32)$ or $E_8{\times}E_8$ gauge symmetry

Partial Spacetime Compactification

Calabi-Yau Manifolds & Bundles

- 1984, P. Candelas
  - w/D. Reine in 11-dimensional supergravity
  - w/G. Horowitz, A. Strominger & E. Witten in 10-d superstrings
  - Know that $\delta_{SS}\{G_{\mu\nu},\ B_{\mu\nu},\ \phi,\ A_\mu{}^a\} = f(G_{\mu\nu},B_{\mu\nu},\phi,A_\mu{}^a \mid \Psi_\mu{}^\alpha,\chi^\alpha,\lambda^a)$
  - Since $\langle 0|\text{fermionic field}|0\rangle = 0$,
  - ...set $\langle 0|\ f(G_{\mu\nu},B_{\mu\nu},\phi,A_\mu{}^a \mid \Psi_\mu{}^\alpha,\chi^\alpha,\lambda^a)\ |0\rangle = 0$
  - ...which constrains the coefficients in $\delta_{SS}\{G_{\mu\nu},\ B_{\mu\nu},\ \Phi,\ A_\mu{}^a\}$ $= f^\mu{}_\alpha(G_{\mu\nu},B_{\mu\nu},\phi,A_\mu{}^a)\Psi_\mu{}^\alpha + f_\alpha(G_{\mu\nu},B_{\mu\nu},\phi,A_\mu{}^a)\chi^\alpha + f_a(G_{\mu\nu},B_{\mu\nu},\phi,A_\mu{}^a)\lambda^a$:
- Minkowski $\mathbb{R}^{1,3}$ spacetime w/N=1 supersymmetry **$(h^{2,1}, h^{1,1})$**
- $SO(26)\times U(1)$ or $E_6\times E_8$ gauge symmetry ⇒ *chiral fermions!*
- ...if $\mathbb{R}^{1,9}$ from the heterotic string is replaced by $\mathbb{R}^{1,3}\times\mathscr{Y}$ **$(\mathbf{27}, \mathbf{27}^*)$**
- ...where $\mathscr{Y}$ is a Calabi-Yau (Ricci-flat) manifold.
- Worldsheet: Ricci-flatness of $\mathscr{Y}$ = trace anomaly cancellation
  = Weyl (conformal) symmetry

Partial Spacetime Compactification

Calabi-Yau Manifolds & Bundles

- When compactifying the $E_8{\times}E_8$ heterotic superstring on a compact Calabi-Yau manifold ("3-fold") $\mathscr{Y}$, of $\sim 10^{-33}$ m
  - all massive modes have $n{\cdot}(\sim 10^{17}\ \text{GeV}/c^2)$ masses, $n=1,2,3,\ldots$
  - the effective target space is the Minkowski $\mathbb{R}^{1,3}$ with $N=1$ SuSy
  - the gauge group is broken to $E_6{\times}E_8$; the 2nd factor a "shadow"
  - there are $h^{2,1}$ Standard Model fundamental fermion (s)families

$$h^{2,1} := \dim H^1(\mathscr{Y}, \mathscr{T}_{\mathscr{Y}}), \quad e := e_{\bar\mu}{}^{\nu}\,\mathrm{d}\bar z^{\bar\mu}\partial_\nu \quad \bar\partial_{[\bar\rho}\, e_{\bar\mu]}{}^{\nu} = 0 \quad \delta e_{\bar\rho}{}^{\nu} = \bar\partial_{\bar\rho}\lambda^{\nu}$$

  - there are $h^{1,1}$ Standard Model fundamental mirror (s)families

$$h^{1,1} := \dim H^1(\mathscr{Y}, \mathscr{T}^*_{\mathscr{Y}}), \quad \omega := \omega_{\bar\mu\nu}\,\mathrm{d}\bar z^{\bar\mu}\mathrm{d}z^{\nu} \quad \bar\partial_{[\bar\rho|}\,\omega_{\bar\mu]\nu} = 0 \quad \delta\omega_{\bar\mu\nu} = \bar\partial_{\bar\mu}\lambda_{\nu}$$

  - Yukawa couplings:

$$\int_{\mathscr{Y}} \mathrm{d}^3z\,\mathrm{d}^3\bar z\; \varepsilon^{\bar\mu\bar\nu\bar\rho}\varepsilon^{\mu\nu\rho}\; \omega_{\bar\mu\mu}\,\omega_{\bar\nu\nu}\,\omega_{\bar\rho\rho}$$

Classic result; "corrected" by worldsheet instantons

and

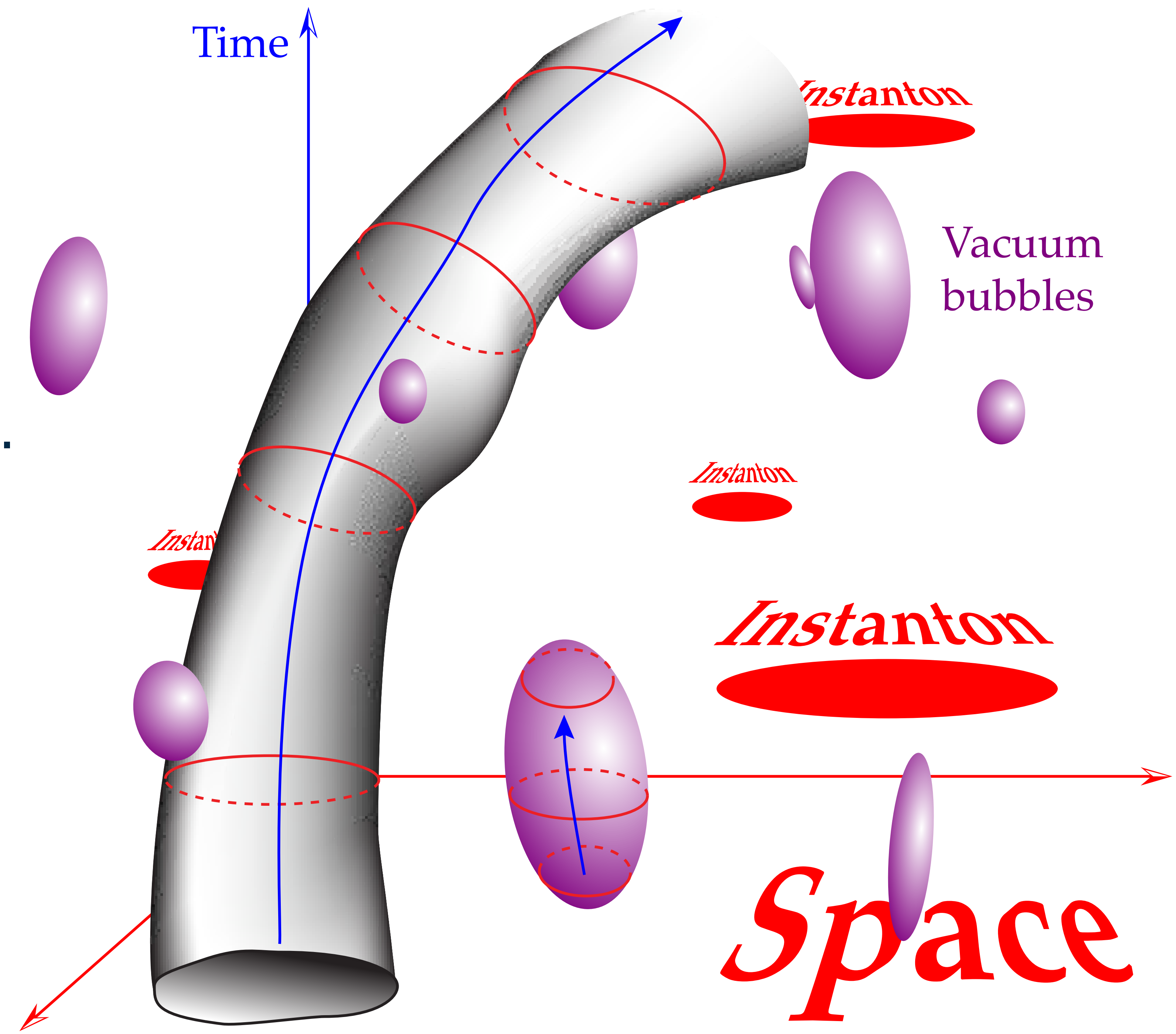
$$\int_{\mathscr{Y}} \mathrm{d}^3z\,\mathrm{d}^3\bar z\; \varepsilon^{\bar\mu\bar\nu\bar\rho}\varepsilon_{\mu\nu\rho}\; e_{\bar\mu}{}^{\mu}\,e_{\bar\nu}{}^{\nu}\,e_{\bar\rho}{}^{\rho}$$

New result; ***exact***, no q. corrections

Partial Spacetime Compactification

Calabi-Yau Manifolds & Bundles

- String instantons?
- Vacuum bubbles
  = virtual strings
  - pop out of the vacuum,
  - propagate,
  - vanish into the vacuum.
  - Must be bosons!
- Bose-Einstein
  distribution!
- May well be
  non-contractible



Partial Spacetime Compactification

Calabi-Yau Manifolds & Bundles

- In fact, left- and right-movers may be compactified differently.
- In many cases, this corresponds to replacing
  - $(\mathscr{Y}, \mathscr{T}_{\mathscr{Y}})$ — Calabi-Yau manifold & its holomorphic tangent bundle
    - while bosons provide coordinates for a manifold
    - fermions (supersymmetry variations of the bosons) are tangential
    - in a general Lagrangian,
      - fermion kinetic terms have derivatives gauge-covariant w/Christoffel symbols
      - 4-fermion term has the coefficient = the Riemann tensor
  - with $(\mathscr{Y}, \mathscr{F}_{\mathscr{Y}})$, a stable holomorphic bundle over $\mathscr{Y}$
    - fibers are holomorphic "functions" over $\mathscr{Y}$,
    - the structure group is $SU(n)$, with $n \geq 3$.
    - ($\mathscr{T}_{\mathscr{Y}}$ has the structure group $SU(3)$ = the holonomy of $\mathscr{Y}$.)
- Since $E_6 \times E_8$ is the centralizer of $SU(3) \subset E_8 \times E_8$,
  - $C[SU(4) \subset E_8 \times E_8] = SO(10) \times E_8$, and $C[SU(5) \subset E_8 \times E_8] = SU(5) \times E_8$!!!

Precisely the GUT gauge groups!!!

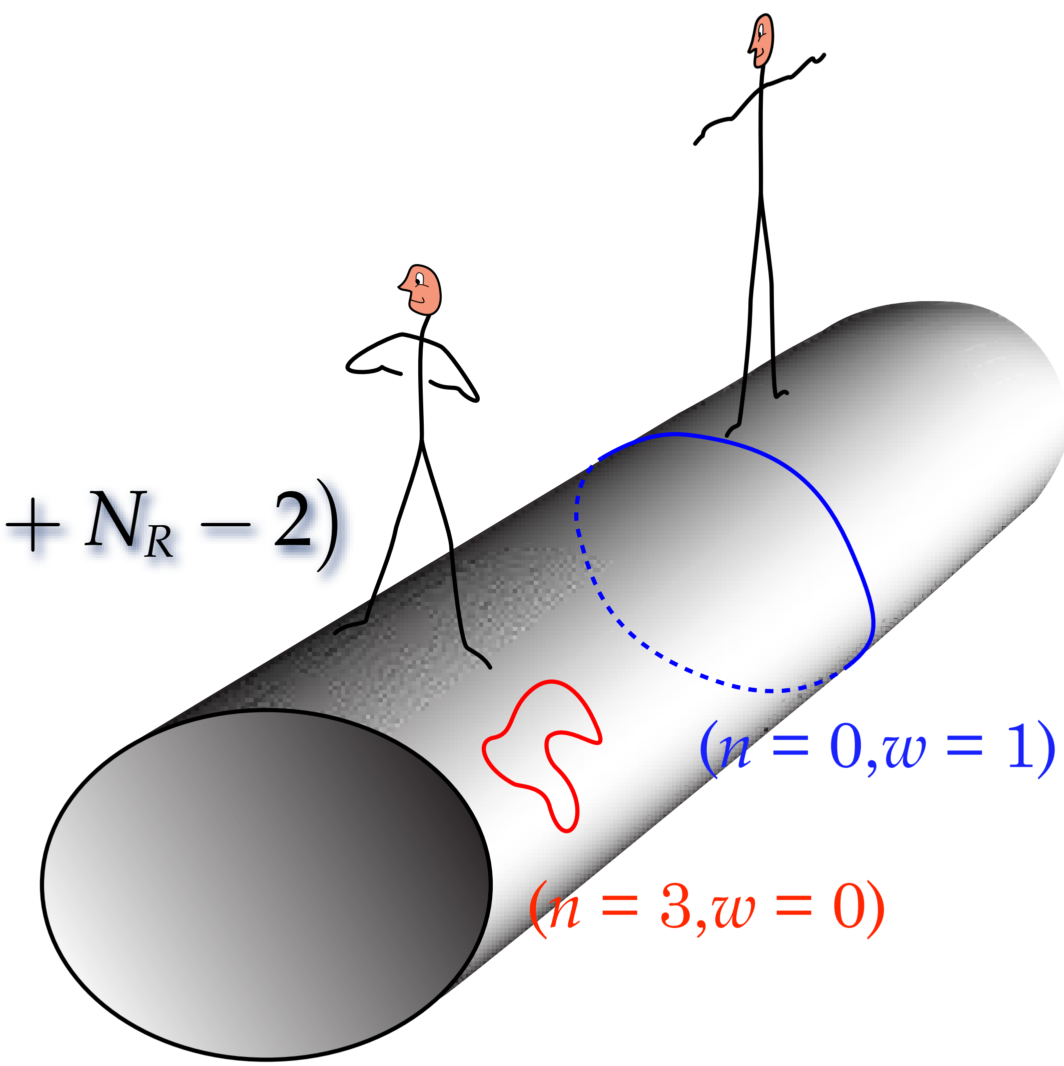
Mirror Symmetry

Through the Looking Glass

- And then, something really, Really, REALLY funny turned up…
- Not "funny – ha-ha," but "funny – **weird**."
- Compactified on a CY 3-fold $\mathscr{Y}$, the superstring model has
  - $h^{2,1}$ Standard Model families
  - $h^{1,1}$ Standard Model mirrors
  - plus dim $H^1(\mathscr{Y}, \mathrm{End}\,\mathscr{T}_{\mathscr{Y}})$ "junk"
- Recall

$$(mc^2)^2 = \frac{n^2\hbar^2c^2}{R^2} + \frac{w^2R^2}{\alpha'^2\hbar^2c^2} + \frac{2}{\alpha'}(N_L + N_R - 2)$$

- $(n, w; R) \leftrightarrow (w, n; \alpha'\hbar^2c^2/R)$ **duality**
- With six real compact dimensions, there's many more ways to dualize.



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- With six real compact dimensions, there’s many more ways to dualize.

B. Greene &
R. Plesser

Mirror Symmetry

Through the Looking Glass

- Long story short, physicist compute.
- P. Candelas, X. de la Ossa, P. Green & L. Parkes have found that the "hard" Yukawa coupling on a 1-parameter (t-)family of CY 3-folds $\mathscr{Y}$ may be expressed as a power series

Recognize this?

$$5+\sum_{k=1}^{\infty}\frac{n_k k^3 e^{2\pi ikt}}{1-e^{2\pi ikt}} = 5+2875\,e^{2\pi it}+\ldots$$

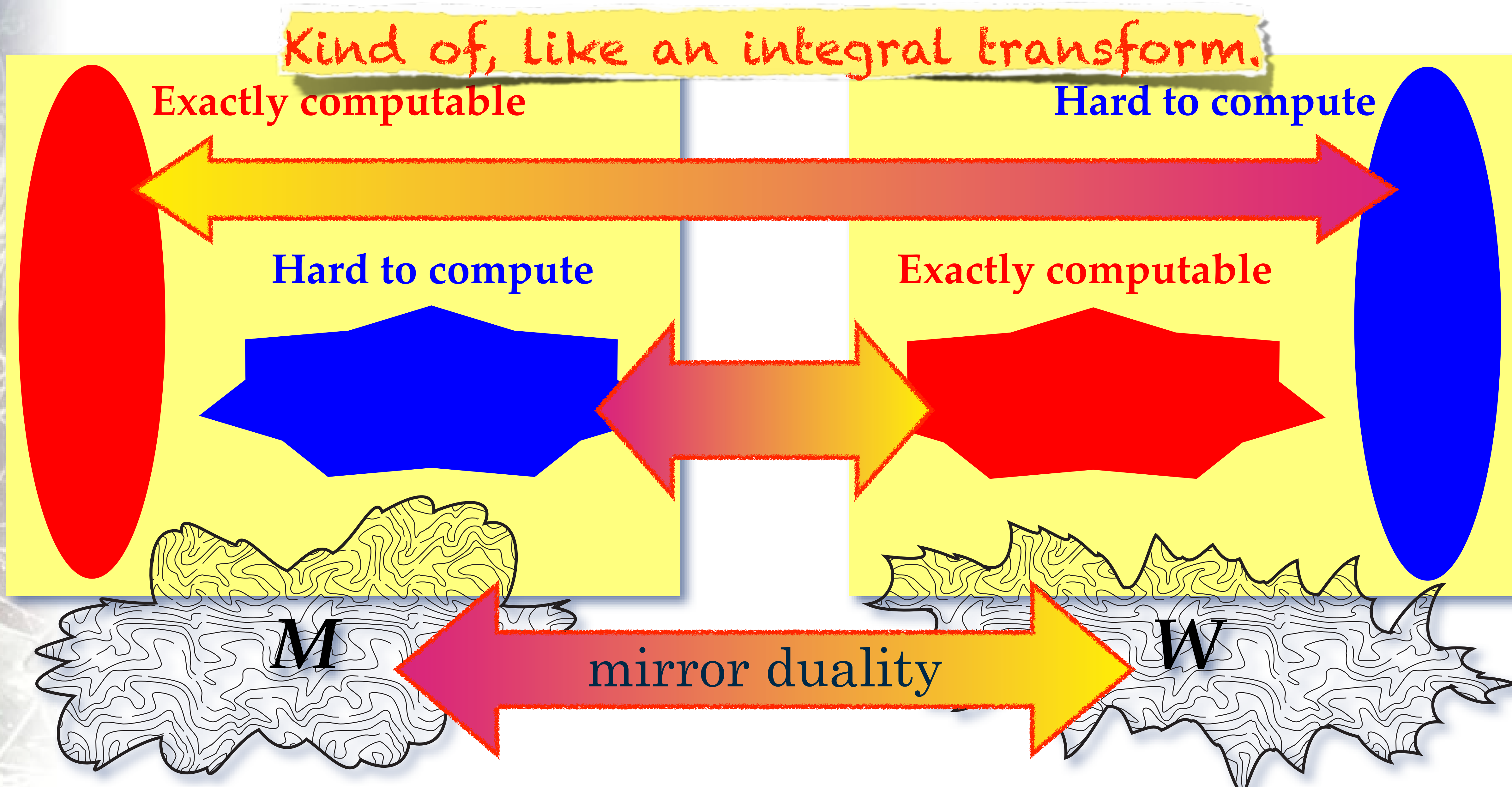
$$t := -\frac{5}{2\pi i}\left\{\log(5\psi)-\frac{1}{\varpi_0(\psi)}\sum_{m=1}^{\infty}\frac{(5m)!}{(m!)^5(5\psi)^{5m}}\left(\psi^{(0)}(1+5m)-\psi^{(0)}(1+m)\right)\right\}$$

- This tells the interaction strength for ***every*** model in this 1-parameter family.
- The n_k give the number of degree-k embeddings of $\mathbb{C}P^1$'s (S^2's – string instantons) in $\mathscr{Y}$.
- The k=2 result of known "classic enumer. algebraic geometry"

Mirror Symmetry

Through the Looking Glass …A Motivational Map

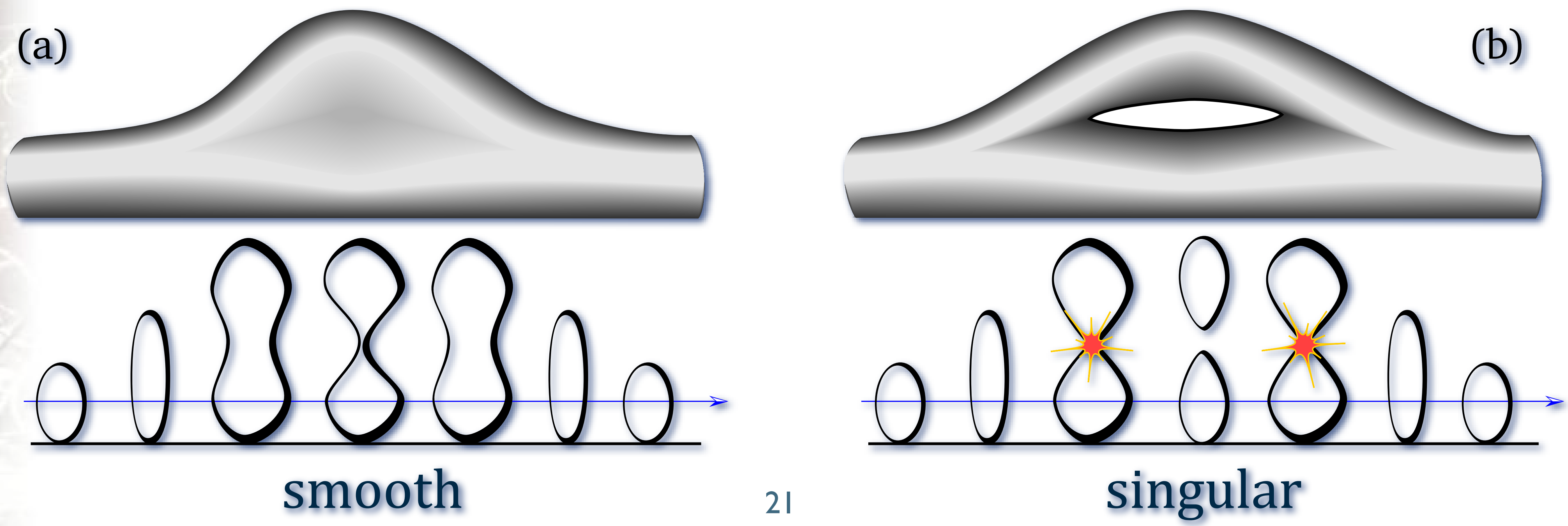
- $H^1(\mathscr{Y}, \mathscr{T}_\mathscr{Y})$ with a “matrix” product generates a ring
- $H^1(\mathscr{Y}, \mathscr{T}_\mathscr{Y}^*)$ needs a quantum product for the mirror ring



Warped Cosmology

Warping Internal Space through Spacetime Weft

- But, that's not all....by faaaaaaaaaaaaaaaaaaarrrrrrrr
- In all Nordstrøm-Kałuża-Klein compactifications, the global geometry of the spacetime is $\mathbb{R}^{1,3} \times \mathscr{Y}$.
- That is the copy of $\mathscr{Y}$ over every point in $\mathbb{R}^{1,3}$ is identical.
- There's no reason for this: $\mathscr{Y}$ is not directly observable, so it may well vary over (along) $\mathbb{R}^{1,3}$.
- Fibrations



Warped Cosmology

Warping Internal Space through Spacetime Weft

- If one assumes that $\mathscr{Y}$ varies complex-analytically over $\mathbb{R}^{1,3}$,
  - $\mathscr{Y}$ must become singular at certain locations in $\mathbb{R}^{1,3}$
  - These locations span (cosmic) filamentary structures
  - The spacetime metric near these cosmic strings acquires the contribution $\delta g_{\mu\nu} = (\partial_\mu \phi^a) R_{ab}(\phi)(\partial_\nu \phi^b)$, the pull-back of the Ricci tensor of the moduli (parameter) space of $\mathscr{Y}$
  - …*i.e.*, a mass determined by the geometry of the space of shapes of $\mathscr{Y}$ *i.e.*, the "moduli space of $\mathscr{Y}$."
  - The total number of such stringy cosmic strings is a topological characteristic of the moduli space of $\mathscr{Y}$.
  - These cosmic strings attract "regular" matter w/a gravitational force that is ~1/r, ***stronger*** than point-attraction
  - …so affects early galaxy formation
  - Relaxing complex analyticity perturbatively ⇒ dynamics

Brane-Worlds

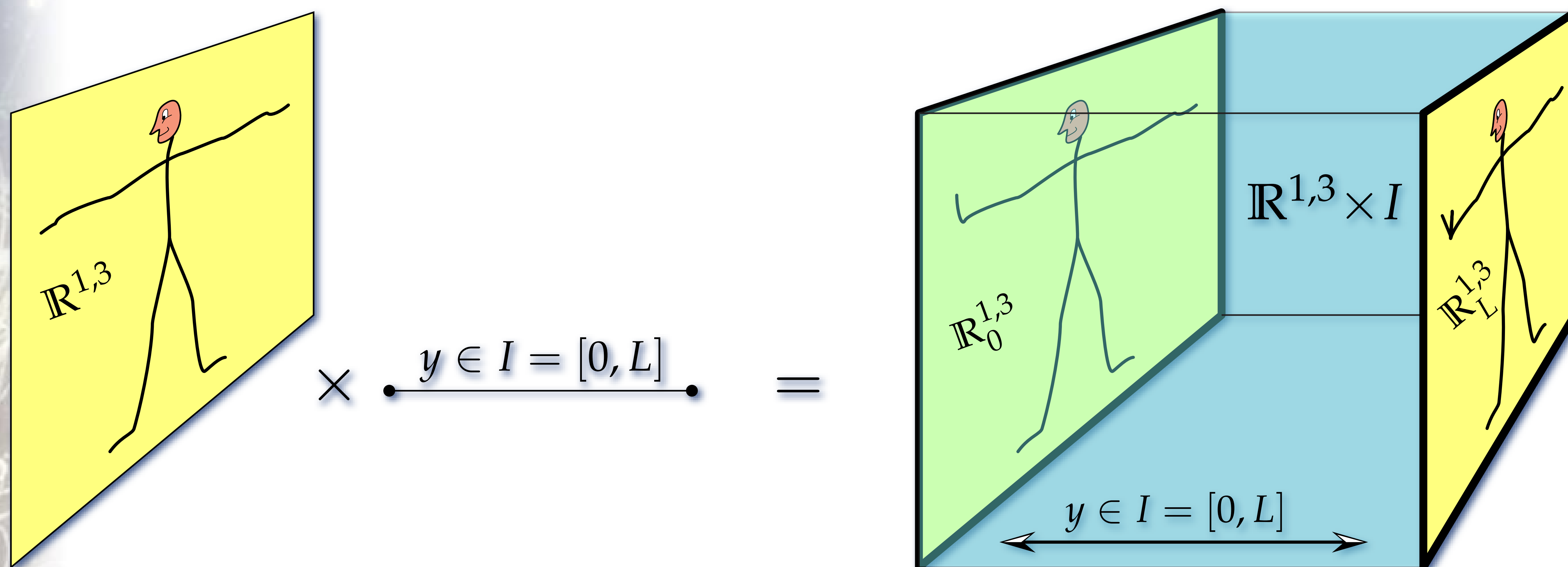
Sub-Spacetimes

- And then, there's yet more…
- Theorem: CY 3-folds have isolated S^2's
  - Isolated = no local deformations
  - …*i.e.*, all vibrations are localized within this S^2.
  - Then CY 5-folds have isolated 4-dimensional subspaces
  - …with all its vibrations localized within the 4-dimensional subspace.
- General worldsheet anomaly cancellation implies Ricci-flatness of the ***total*** spacetime (*incl*. gauge, flat & compactified directions)
- The $t \to it$ analytic continuation of spacetime must be Ricci-flat *i.e.*, a non-compact Calabi-Yau 5-fold
- Surgery: Take compact $\mathscr{Z}$ w/Ric > 0. Locate a Ricci-flat subspace $\mathscr{Y} \subset \mathscr{Z}$. Then [Tian & Yau] $(\mathscr{Z} - \mathscr{Y})$ is Ricci-flat & non-compact.
- And gravity?

Brane-Worlds

Localization of Gravity

- 1999, Lisa Randall & Raman Sundrum



$$\mathrm{d}s^2 = -e^{-2k|y|}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}y^2, \quad i.e., \quad [\boldsymbol{g}(\mathrm{x}, y)] = \begin{bmatrix} -e^{-2k|y|} & 0 & 0 & 0 & 0 \\ 0 & e^{-2k|y|} & 0 & 0 & 0 \\ 0 & 0 & e^{-2k|y|} & 0 & 0 \\ 0 & 0 & 0 & e^{-2k|y|} & 0 \\ 0 & 0 & 0 & & 1 \end{bmatrix}$$

"warping"

Brane-Worlds

Localization of Gravity

- RS-1: exponential hierarchy
  - $M_P \sim 10^{19}$ GeV (gravity becomes confiningly strong)
  - $M_W \sim 10^2$ GeV (masses of $W^\pm$, Z^0)
  - ...exponentially related through x^4 ***curvature*** ☺
- RS-2: localized gravity
  - bulk gravity ~ $1/r^3$ force
  - "boundary" gravity ~ $1/r^2$ force ☺
  - But, not in one and the same end-of-the-world World! ☹

...but, we'd like 'em both, and in the same World! (Ours!)

- Gravity localization

$$ds^2 = -e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \quad i.e., \quad [\boldsymbol{g}(x,y)] = \begin{bmatrix} -e^{-2k|y|} & 0 & 0 & 0 & 0 \\ 0 & e^{-2k|y|} & 0 & 0 & 0 \\ 0 & 0 & e^{-2k|y|} & 0 & 0 \\ 0 & 0 & 0 & e^{-2k|y|} & 0 \\ 0 & 0 & 0 & & 1 \end{bmatrix}$$

$$[\boldsymbol{R}] = \left[\begin{array}{c|c} -\eta_{\mu\nu}\, e^{-2k|y|} f(y) & 0 \\ \hline 0 & g(y) \end{array}\right], \qquad \begin{cases} f(y) &= 2k\big[\delta(y) - 2k\ \mathrm{sig}^2(y)\big], \\ g(y) &= 4k\big[2\,\delta(y) - k\ \mathrm{sig}^2(y)\big], \end{cases}$$

Brane-Worlds

Localization of Gravity

- The appearance of $f(y) = 2k\big[\delta(y) - 2k\ \mathrm{sig}^2(y)\big]$
- ...induces a "Schrödinger-like" equation for transversal modes:

$$\Big[-\frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}z^2} + \widetilde{V}_\pm(z)\Big]\hat{\psi}(z) = 0, \quad \widetilde{V}_\pm(z) = \frac{15\,k^2}{8(k\,|z|+1)^2} \pm \tfrac{3}{2}\,k\,\delta(z) - \frac{m^2c^2}{\hbar^2}$$

  - The δ-function (w/– sign) guarantees one localized mode of $g_{\mu\nu}$
  - well isolated from the continuum bulk by the "barrier" term
- The mechanism hinges on the non-analytic dependence of the global geometry on a "transversal" coordinate
  - and in particular, $g_{\mu\nu}$ must depend on $|z|$.
- Back to the "($\mathscr{Z}$–$\mathscr{Y}$) is Ricci-flat & non-compact" generic case
  - if the metric on $\mathscr{Z}$ depends on $|z-z_0|$, where z_0 locates $\mathscr{X} \subset (\mathscr{Z}-\mathscr{Y})$,
  - gravity can be localized to $\mathscr{X}$,
    which already has localized matter and Yang-Mills gauge interactions

Localization of Matter and Interactions

All Together, Now

- The continuum of "bulk" modes: $V(r) = G_N \frac{M_1\, M_2}{r}\Big(1 + \frac{1}{(kr)^2}\Big)$

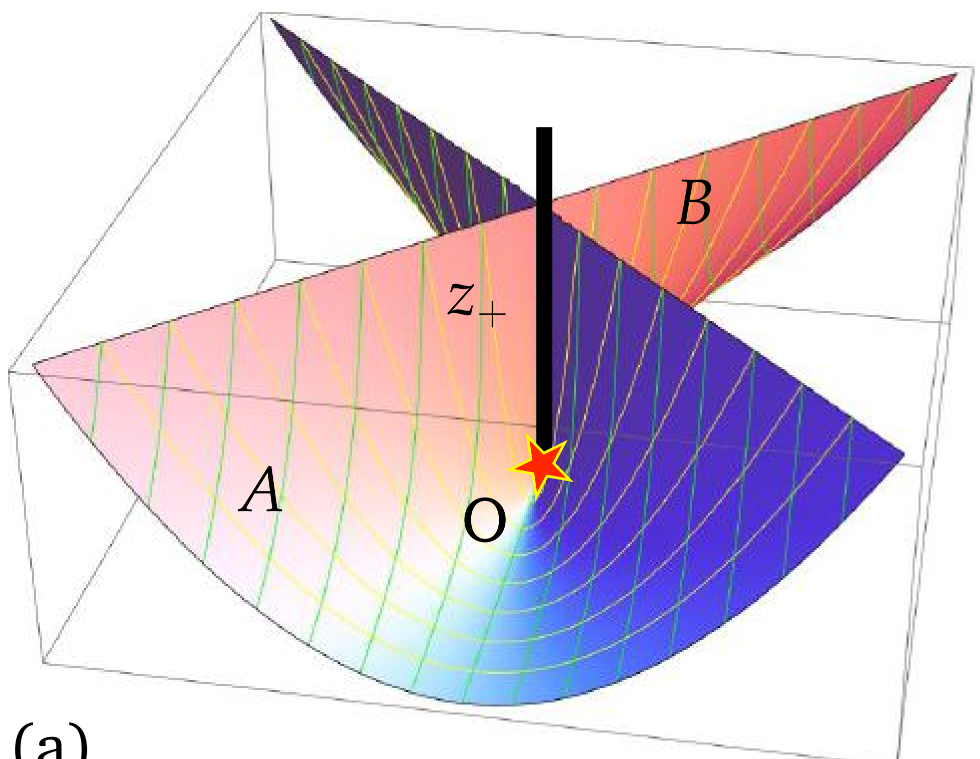
  Extrinsic curvature of $\mathscr{X} \subset (\mathscr{Z}-\mathscr{Y})$

- It is possible to find:
  - a codimension-2 brane-World with
  - de Sitter geometry & Λ (cosmological constant)
  - gravity localized to the brane-World
  - exponential M_P:M_W ratio
  - the brane-geometry is induced by a modulus
  - …and $SL(2,\mathbb{Z})$ symmetry
  - Λ is related to supersymmetry breaking
  - …in a phenomenologically acceptable way.

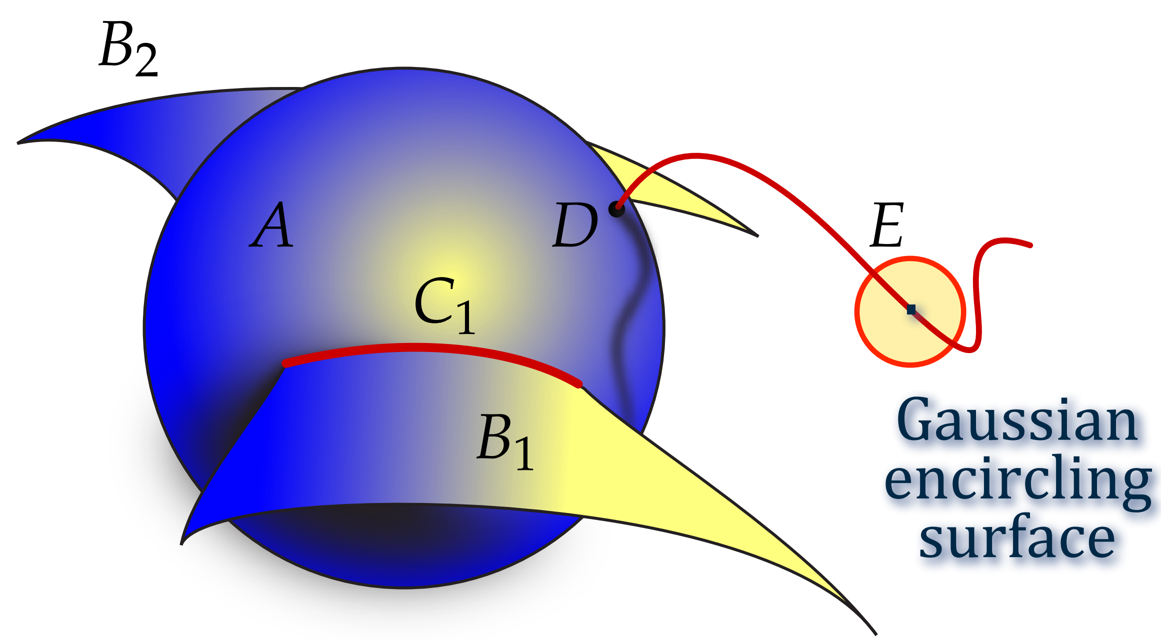
Localization of Matter and Interactions

Exospaces

- There are even stranger stringy cosmologies…
- (Super)strings have a minimal length ~10^{-35} m
- …whence singular points are not really singular
- …so superstrings consistently thread *stratified pseudomanifolds:*



(a)



(b)

Thanks!

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