(Fundamental) Physics of Elementary Particles

Grand Unification: Pati-Salam, Georgi-Glashow, and other, more complex models

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Fundamental Physics of Elementary Particles PROGRAM

- The Grand Unification Model Building Yarn
- The Pati-Salam Model
 - Unification of quarks and leptons
 - Unification of left- and right-chirality
 - Addressing the mass hierarchy & structure
 - Consequences: proton decay, phases
 - The Georgi-Glashow Model
 - Unification of gauge interactions
 - Consequences: proton decay, phases
- More Complex Models
 - Unification of fundamental fermion families
 - Other gauge symmetry groups
 - Concerns



Grand Unification Model Building Yarn

GENERAL COMMENTS

• The fundamental fermion content of the Standard Model:



• Notice the weak hypercharge values!

The Grand Unification Model Building Yarn

GENERAL COMMENTS

- With respect to the $SU(3)_c \times SU(2)_w \times U(1)_y$ gauge group
 - left-chirality quarks $(u, d)_L$ form a $(3, 2)_{\frac{1}{3}}$
 - left-chirality leptons $(e^{-}, v)_L$ form a $(\mathbf{1}, \mathbf{2})_{-1}$
 - right-chirality quarks form: $u_R = (3, 1)_{\frac{4}{3}} \oplus d_R = (3, 1)_{-\frac{2}{3}}$
 - right-chirality leptons form: $e_R^{-}=(1, 1)_{-2} \oplus v_R^{-}=(1, 1)_0$ "sterile"
 - The gauge group representations determine the interactions
 - $SU(3)_c$ **3**'s interact w/gluons, **1**'s don't
 - $SU(2)_w$ **2**'s interact w/ W^{\pm} & W^3 , **1**'s don't
 - $U(1)_y q_y \neq 0$ interact w/gluons, $q_y = 0$ don't
 - Each gauge group factor has a separate gauge-coupling constant/ parameter/charge and "fine structure parameter": α_c , α_w and α_y

Besides the fundamental fermions, the Standard Model only has gauge interaction bosons and the Higgs boson, (1, 2)+1.

The Grand Unification Model Building Yarn

GENERAL COMMENTS

• Recall the suggestive graph:



UNIFICATION OF QUARKS AND LEPTONS

• The fundamental fermion structure in each family and ignoring chirality is

| $SU(3)_c$ | | | | | $\frac{SU(4)_c}{SU(3)_c}$ | | | |
|----------------|----------------|----------------|----------------|---------------|---------------------------|----------------|----------------|----------------|
| q^{r} | q ^y | q ^b | Ł | \rightarrow | q^{r} | q ^y | q ^b | ℓ^ℓ |
| u ^r | и ^у | u ^b | ν_e | | u ^r | и ^у | u ^b | ν_e^{ℓ} |
| d ^r | d ^y | d^{b} | e ⁻ | | d ^r | d ^y | d ^b | $e^{-\ell}$ |

The lepton-ness (non-quark-ness) of leptons is identified as the 4th color, "*lilac*," of a larger $SU(4)_c$ gauge group ... which then needs to be broken $SU(4)_c \rightarrow SU(3)_c \times U(1)_{y'}$ gauge group ... by means of some Higgs scalar.

UNIFICATION OF LEFT- AND RIGHT-CHIRALITY

• Remembering chirality, we have

 $SU(4)_c$ electroweak hirali $\overline{SU(3)}_{c}$ interaction $+\frac{1}{2}$ $-\frac{1}{2}$ u^b d^b $SU(2)_L$ \mathbb{Z}_2 $+\frac{1}{2}$ $-\frac{1}{2}$ u^o d^b R • The "mirror" weak isospin $SU(2)_R$ gauge group • and also the discrete parity symmetry, \mathbb{Z}_2 .

• Pati-Salam group: $SU(4)_c \times SU(2)_L \times SU(2)_R \times \mathbb{Z}_2$.

FERMION MASS HIERARCHY & STRUCTURE

• Recall:



ERMION MASS HIERARCHY & STRUCTURE

- Fundamental fermions: $(4, 2, 1)_L \oplus (4, 1, 2)_R$ irreducible under \mathbb{Z}_2
- Under $SU(4)_c \rightarrow SU(3)_c \times U(1)_{y'}: \mathbf{4} \rightarrow \mathbf{3}_{\frac{1}{3}} \oplus \mathbf{1}_{-1} (``q" \oplus ``\ell")$
- Under $SU(2)_R \rightarrow U(1)_R: \mathbf{2} \rightarrow (+\frac{1}{2}) \oplus (-\frac{1}{2})$ ("spin- \uparrow ", "spin- \downarrow ")
- So, $(4, 2, 1)_L \rightarrow [(3_{\frac{1}{3}}, 2)_0 \oplus (1_{-1}, 2)_0]_L$ and $(4, 1, 2)_R \rightarrow [(3_{\frac{1}{3}}, 1)_{+\frac{1}{2}} \oplus (3_{\frac{1}{3}}, 1)_{-\frac{1}{2}} \oplus (1_{-1}, 1)_{+\frac{1}{2}} \oplus (1_{-1}, 1)_{-\frac{1}{2}}]_R$
- Now, $U(1)_{y'} \times U(1)_R \to U(1)_y$, according to $q_y = q_{y'} + 2I_{3R}$, so • $(4, 2, 1)_L \oplus (4, 1, 2)_R \to [(3, 2)_{\frac{1}{3}} \oplus (1, 2)_{-1}]_L \oplus [(3, 1)_{\frac{4}{3}} \oplus (3, 1)_{-\frac{2}{3}} \oplus (1, 1)_0 \oplus (1, 1)_{-2}]_R$

... the U(1)_y charges reproduce those in the Standard Model
The Higgs field then itself needs to be a (4, 1, 2), so that

... $\langle (4, 1, 2) \rangle = ... + \langle (1, 1)_0 \rangle \neq 0$. This simultaneously breaks $SU(4)_c \times SU(2)_R \times \mathbb{Z}_2 \rightarrow SU(3)_c \times U(1)_y$ & leaves $SU(2)_w$ intact.

FERMION MASS HIERARCHY & STRUCTURE

• What? You did *not* memorize that?!

• Recall: $\overline{\Psi}[i\hbar c \gamma^{\mu} D_{\mu} - \frac{mc}{\hbar} \mathbb{1}] \Psi = \cdots - \frac{mc}{\hbar} [\overline{\Psi}_{-} \Psi_{+} + \overline{\Psi}_{+} \Psi_{-}]$ • So an $SU(4)_{c} \times SU(2)_{L} \times SU(2)_{R} \times \mathbb{Z}_{2}$ -invariant mass-term is

 $\overline{(4,1,2)}\langle \mathbb{H}\rangle(4,2,1) \Rightarrow \mathbb{H} \sim (1,2,2) \rightarrow (1_0,2)_{\pm \frac{1}{2}} \xrightarrow[]{} \rightarrow (1,2)_{\pm 1}$ one of the four gets a vev

FERMION MASS HIERARCHY & STRUCTURE

• So:

- Then, e.g., $-h_u \overline{u_R} (\mathbb{H}^c)^{\dagger} \begin{bmatrix} u \\ d \end{bmatrix}_L$
- stems from the Pati-Salam mass term of the form

 $(\overline{4}, 2, 1) \cdot (1, 2, 2) \cdot (4, 1, 2)$

| | fer | mion fam | ily | charges | | | |
|---|--|--|--|-------------------------------|----------------------------------|------------------------------|--|
| | 1 | 2 | 3 | Q | I_{w} | Y_w | |
| $\Psi = \alpha \Psi$ | $\begin{bmatrix} u \\ d \end{bmatrix}_L$ | $\begin{bmatrix} c \\ s \end{bmatrix}_L$ | $\begin{bmatrix} t \\ b \end{bmatrix}_L$ | $+^{2}/_{3}$ $-^{1}/_{3}$ | $+^{1}/_{2}$ $-^{1}/_{2}$ | $+^{1}/_{3}$ $+^{1}/_{3}$ | |
| $\underbrace{1_{-} = \mathbf{\gamma}_{-} 1}_{\text{left-handed}}$ | $\begin{bmatrix} \nu_e \\ e^- \end{bmatrix}_L$ | $\left[\begin{array}{c}\nu_{\mu}\\\mu^{-}\end{array}\right]_{L}$ | $\left[\begin{array}{c} \nu_{\tau} \\ \tau^{-} \end{array}\right]_{L}$ | 0 -1 | $+\frac{1}{2}$ $-\frac{1}{2}$ | $-1 \\ -1$ | |
| ſ | u_R | ${\cal C}_R$ | t_R | $+^{2}/_{3}$ | 0 | $+4/_{3}$ | |
| w _ o, w) | d_R | s_R | b_R | - ¹ / ₃ | 0 | $-\frac{2}{3}$ | |
| $\underbrace{1_{+}=\boldsymbol{\gamma}_{+}1}_{1}$ | e_R^- | μ_R^- | $	au_{\scriptscriptstyle R}^-$ | -1 | 0 | -2 | |
| right-handed | v_{eR} | $ u_{\mu R}$ | $v_{\tau R}$ | 0 | 0 | 0 | |

so that we must choose $\langle \mathbb{H} \rangle \subset (\mathbf{1}, \mathbf{2})_{+1}$

And then, in fact, the vev is in the lower component, so that
⟨H⟩=(1,-1/2)+1 = 1₀, neutral with respect to Q = I₃+1/2Y.
The mass-term with (1, 2, 2) includes all fermions of a family! 12 makes h_µ=h_d=h_e=h_y

FERMION MASS HIERARCHY & STRUCTURE

- Since the "electroweak" Higgs (1, 2, 2) gives all fundamental fermions (4, 2, 1)_L⊕(4, 1, 2)_R of the same family the same mass
 - ... the mass-hierarchy within a family must be obtained via
 - coupling with another Higgs field, that couples differently to the various components of $(4, 2, 1)_L$ and $(4, 1, 2)_R$
 - such as, e.g., (15, 1, 2) and/or (15, 2, 2) and/or (1, 1, 2).
 - When decomposed in the $SU(3)_c \times SU(2)_L \times U(1)_y$ basis, these introduce (e.g., three) new coupling parameters,
 - ... which then afford a degree of structure in the mass hierarchy.
- This showcases a GUT rephrasing of the mass hierarchy issue.
 - The mass hierarchy structure is restricted by the GUT symmetry
 - ... even if the number of parameters turns out to be the same.
 - It does require a proliferation of Higgs fields.

CONSEQUENCES: PROTON DECAY, PHASES

• The $SU(4)_c$ gauge bosons



... besides the gluons and the *L*-*R* "hyper-photon," $SU(4)_c$ also contains "*lepto-quarks*."

Just as $q^r \rightarrow q^b + g^r_b$, so $q^r \rightarrow \ell^{\ell} + X^r_{\ell}$ allows for quarks to turn into leptons, but conserving the "B+3L" number.

• Thus, the simplest proton decays are

• $p^+ \rightarrow 3\nu_e + (\text{mesons}^+, \text{photons})$

• $p^+ \rightarrow 4v_e + e^+ + (\text{mesons}^0, \text{photons}).$ (must involve a Higgs field) No confirmed p⁺-decay yet...

CONSEQUENCES: PROTON DECAY, PHASES

• The phase space of the model is quite complicated ...



The Georgi-Glashow Model

UNIFICATION OF GAUGE INTERACTIONS

• The smallest Lie group containing $SU(3)_c \times SU(2)_w \times U(1)_y$ is SU(5).



Since the generators of SU(5), $SU(3)_c$ and $SU(2)_w$ are all traceless, SU(5) contains also one diagonal element: $U(1)_y$.

• Since all generators are united into a single algebra, there is a single gauge coupling, α_5 .



The Georgi-Glashow Model

JNIFICATION OF GAUGE INTERACTIONS

• The fermions however fit into:



The Georgi-Glashow Model

CONSEQUENCES: PROTON DECAY, PHASES

• The SU(5) lepto-quarks now directly mediate proton decay:

$$p^{+} = (u + u + d) \rightarrow (u + u + (\overline{X} + e^{+})) \rightarrow (u + (u + \overline{X}) + e^{+})$$

$$\rightarrow (u + \overline{u} + e^{+}) \rightarrow \pi^{0} + e^{+} \rightarrow 2\gamma + e^{+}.$$

- ... conserving the "B–L" number.
 With M_X, M_Y~10¹⁵ GeV/c², τ_p ~ 10²⁸–10²⁹ years
 Waiting experiments: τ_p > 6.6 ×10³³ years
 Higgs: 24 of SU(5) does have an SU(3)_c × SU(2)_w × U(1)_y-invariant component: the diagonal generator of U(1)_y.
 - But, **24** also has an $SU(4) \times U(1)$ -invariant component...
 - ... so there are at least those three phases.
 - Alternatively, a **75** of SU(5) does an $SU(3)_c \times SU(2)_w \times U(1)_{y^-}$ invariant component, no $SU(4) \times U(1)$ -invariant one, but also an SO(5)-invariant one...

Grand Unification Model Building Yarn ... *zoom-out*

CONSEQUENCES

- Unification of quarks and leptons leads to proton-decay
 - ... preserving some " α B+ β L" number.
 - "B+3L" number for the Pati-Salam model
 - Particulars of unifying quarks and leptons • "B–L" number for the Georgi-Glashow model
- Phases/regimes of the model depend on
 - the grand-unification (Lie) group
 - Subalgebra chains of Lie groups classic results, used recursively
 - the Higgs fields employed
 - Representation decomposition w.r.t. subgroup tables trick-or-treat
- Mass hierarchy:
 - Typically a hierarchy of Higgs fields
 - One w/ $\langle H_1 \rangle \sim 10^{15}$ GeV, the GUT scale — beyond Standard Model
 - one w/ $\langle \mathbb{H}_2 \rangle \sim 10^2$ GeV, the electroweak scale — Standard Model masses

More Complex Models

UNIFICATION OF FUNDAMENTAL FERMION FAMILIES

As the P-S model fully unifies fermions but not gauge fields
And the G-G model fully unifies gauge fields but not fermions
... seek a bigger model that unifies them all.



No "extra" (junk) fermions (except v_{eR}), but extra gauge bosons: 45 = ((8+3+1)+6+3)+24 = ((8+3+1)+12)+20+1
Still does not unify the three families of fermions.

More Complex Models

UNIFICATION OF FUNDAMENTAL FERMION FAMILIES

- The simplest idea: add an SU(3) gauge group that transforms any one family into a linear combination of all three
- Breaking the symmetry in stages
 - All 48 fermions \rightarrow three families of 16
 - Left-right breaking
 - electroweak \rightarrow weak + electromagnetism
 - Needs a Higgs hierarchy, which could reproduce the fermion hierarchy *SU*(3) is *ad hoc* ["for this"], and there are no exp. signs of any such family gauge interactions — no family gauge group.
- "Familial symmetry" may well be
 - a broken global symmetry
 - a symmetry that is broken at a higher yet scale
 - an "artifact" of the structure of a higher GUT symmetry

More Complex Models

OTHER GAUGE SYMMETRY GROUPS

• Trinification (Georgi, Glashow and de Rujula)

- Gauge group: $SU(3)_c \times SU(3)_L \times SU(3)_R$,
- with fermions $(3, 3^*, 1) \oplus (3^*, 1, 3) \oplus (1, 3, 3^*) = 27$
- with a Higgs (1, 3, 3*) and perhaps its conjugate
- a maximal subgroup of E_6 , with fermions in **27** (*string theory*)

Really big: $SO(18) \supset SO(10) \times SO(8)$, the latter – "familial"

- Fundamental fermions: $256 = (16, 8_s) \oplus (16^*, 8_c)$
- Georgi-Bagger "no-go theorem": no way to remain with 3 families
 w.r.t. orthogonal subgroups, fermions always split in powers of 2
- D.Chang, T.H., R.Mohapatra: SO(8) has a triality: $\langle \mathbf{8}_{\nu}, \mathbf{8}_{s}, \mathbf{8}_{c} \rangle$
 - $\mathbf{SO}(8) \supset SO(5) \times SO(3): \mathbf{8}_s \rightarrow \mathbf{8}_v = (\mathbf{5}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3}), \mathbf{8}_c \rightarrow \mathbf{8}_s = (\mathbf{4}, \mathbf{2})$
 - Found a Higgs: $(16, 8_s) \cdot \langle \mathbb{H} \rangle \cdot (16^*, 8_c) \neq 0$ for (16, (5, 1)), -massive!

• ... leaving (16, (1, 3)) massless, along with 3 of $(16^*, (4, 2))$ — *mirrors*!

• ... thus breaking the familial $SO(5) \times SO(3)$ gauge symmetry to an SU(2)

The Grand Unification Model Building Yarn

- To unify both all fundamental fermions (a single family) and also all (Yang-Mills) gauge interactions: *SO*(10)
 - 33 XS gauge bosons! typically, extra unbroken U(1)
 - Higgs fields typically in the hundreds!
- To unify the three families with a single group, need SO(18)
 - 141 XS gauge bosons!!
 - mirror fermions!! both massive and massless
 - Higgs fields typically in the (tens of) thousands!!
- The structures that are being introduced (Higgs, XS gauge)
 - are *ad hoc* ["for this"]
 - turn out to be more complicated and plentiful
 - ... than the structure they supposedly "explain": Standard Model

Thanks!

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http://homepage.mac.com/thubsch/