(Fundamental) Physics of Elementary Particles

Unifications in relativistic quantum physics
The theoretical system of relativistic quantum physics
Peeking beyond the Standard Model

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Fundamental Physics of Elementary Particles

Program

- Unifications in relativistic quantum physics
  - Special-relativistic unification
  - General-relativistic unification
  - Quantum unification
- The theoretical system of relativistic quantum physics
  - The 3D quadrant of scientific systems
  - Phase transition *vs*. limiting transitions
  - Three distinct flavors of unification
- Peeking beyond the Standard Model
  - Nine puzzles
  - One more, a bit more technical

Unifications in relativistic quantum physics

Prologue

- The ideas of unification were incorporated into physics…
- (the discipline we now distinguish from natural philosophy)
- …from its very origins
  - Nicolaus **Copernicus** [*De revolutionibus orbium coelestium,* 1543]: heliocentric system—all planets orbit the Sun, alike
  - Johannes **Kepler** w/observations of Tycho **Brahe** [*Astronomia nova,* 1609]: two laws; [*Epitome astronomiae Copernicanae,*1615–1620], third law & physical causes of heavenly motion
  - Galileo **Galileo** [*Il Saggiatore,* 1623]: "Philosophy…is written in the language of mathematics…" & (experiment+math analysis)
  - René **Descartes** [*Discourse on the Method,* 1637]: reason in science
  - Gottfried **Leibnitz** [late 1600's]: KE + PE, space & time relativity
  - Isaac **Newton** ["*Philosophiæ Naturalis Principia Mathematica,*"1687]: universal law of gravity

Unifications in relativistic quantum physics

Special-relativistic unification

- The first example of unifying existing scientific models: Maxwell's equations = Gauss's, Ampère's & Faraday's laws.
- The electric & magnetic fields ↔ electromagnetic field
- Depending on comparisons with speed of light in vacuum

$$\text{(Gauss)}\left\{\begin{array}{ll} \vec{\nabla}\cdot\vec{E} = \dfrac{4\pi\,\rho_e}{4\pi\epsilon_0}, & \vec{\nabla}\times(c\vec{B}) - \dfrac{1}{c}\dfrac{\partial\vec{E}}{\partial t} = \dfrac{1}{4\pi\epsilon_0}\dfrac{4\pi}{c}\vec{\jmath}_e, \quad \text{(Ampère)} \\[2ex] \vec{\nabla}\cdot(c\vec{B}) = 0, & -\vec{\nabla}\times\vec{E} - \dfrac{1}{c}\dfrac{\partial(c\vec{B})}{\partial t} = 0, \quad \text{(Faraday)} \end{array}\right.$$

- with $\mu_0 = 1/\epsilon_0 c^2$, in the $c \to \infty$ limit:

$$\vec{\nabla}\cdot\vec{E} = \frac{4\pi\rho_e}{4\pi\epsilon_0}, \qquad \vec{\nabla}\times\vec{B} = \mu_0\,\vec{\jmath}_e + \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t} \;\to\; \mu_0\,\vec{\jmath}_e,$$

$$\vec{\nabla}\cdot\vec{B} = 0, \qquad \boxed{-\vec{\nabla}\times\vec{E} = \frac{\partial\vec{B}}{\partial t}. \quad \Leftarrow \text{Decoupling condition.}}$$

Unifications in relativistic quantum physics

Special-relativistic unification

- Decoupling in the $c \to \infty$ limit:

$$\vec{\nabla}\cdot\vec{E} = \frac{4\pi\rho_e}{4\pi\epsilon_0}, \qquad \vec{\nabla}\times\vec{B} = \mu_0\,\vec{j}_e + \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t} \;\to\; \mu_0\,\vec{j}_e,$$

$$\vec{\nabla}\cdot\vec{B} = 0, \qquad -\vec{\nabla}\times\vec{E} = \frac{\partial\vec{B}}{\partial t}.$$

  - stationary currents : static magnetic field : electrostatic potential
- In terms of the 4-vector potential,

$$\partial_\mu\partial^\mu A^\nu - \partial^\nu(\partial_\mu A^\mu) = \frac{1}{4\pi\epsilon_0}\frac{4\pi}{c}j_e^\nu \qquad \Box A^\mu = \frac{1}{4\pi\epsilon_0}\frac{4\pi}{c}j_e^\mu,$$

- in the Lorenz gauge. Then also:

$$\Box\,\vec{B} = \vec{\nabla}\times(\mu_0\vec{j}_e), \qquad \Box\,\vec{E} = -\vec{\nabla}\left(\frac{\rho_e}{\epsilon_0}\right) - \frac{\partial(\mu_0\vec{j}_e)}{\partial t}.$$

Unifications in relativistic quantum physics

Special-relativistic unification

- The Maxwell system of equations has symmetries:
  - Lorentz transformations of spacetime
  - The P, C, T discrete transformations
  - The electric $\leftrightarrow$ magnetic duality,
    - when charges and currents are dually present or absent.
  - Existence of magnetic monopole charges/currents would obstruct the reduction to an "well-defined" gauge 4-vector potential
  - When $c \to \infty$, electrodynamics splits into electro*statics*, magneto*statics* and wave optics.
- But what ***can*** "$c \to \infty$" even mean??
  - The numerical value of c depends on units chosen.
  - Instead, apply the actual $(v_{ij}/c) \to 0$ limit. (math!)
  - …or ***criteria***: "$(v_{ij}/c) \ll 1$?" Approximation! (physics!)

Unifications in relativistic quantum physics

Special-relativistic unification

- **Conclusion**: As c is a natural constant, the formal "$c \to \infty$" qualifier stands in for the qualifiers "$(v_{ij}/c) \to 0$," where…
- … v_{ij} ranges over all relative velocities within the system.
  - Non-relativistic physics is a special, limiting case of relativistic physics. For any given system, in the space of all possible relative speeds $\{v_{12}, v_{13}, \ldots\}$, the strict non- relativistic regime is a point: $v_{ij} = 0$ for all i, j—everything else is relativistic physics.
  - By "non-relativistic systems" one *may* understand only the cases where the relativistic corrections are *negligible*—for which the limits of precision are necessarily subject to *convention*.
  - Since the changes in the electromagnetic field propagate at the speed of light, all systems with moving electromagnetic fields are unavoidably relativistic.
- The *reference speed* is *universal*, and composite of independently measurable quantities:

$$c := \frac{1}{\sqrt{\epsilon_0\,\mu_0}}$$

Unifications in relativistic quantum physics

Special-relativistic unification

- Einstein's unification of electromagnetism and mechanics
- [*"On the electrodynamics of moving bodies"*]
- With the benefit of hindsight:
  - Compare the symmetries of Newtonian and Einsteinian physics:
  - NM has Galilean, while E&M has Lorentz symmetry — patchwork
  - EP: both mechanics and E&M have Lorentz symmetry — unified
  - Two interacting subsystems cannot obey disparate laws — consistent
  - In the "$c \to \infty$" ***limit***, Einstein's mech → Newton's mech.

$$\textbf{Galileo}: \quad \vec{r}\,' = \vec{r} - \vec{v}t, \qquad\qquad t' = t,$$

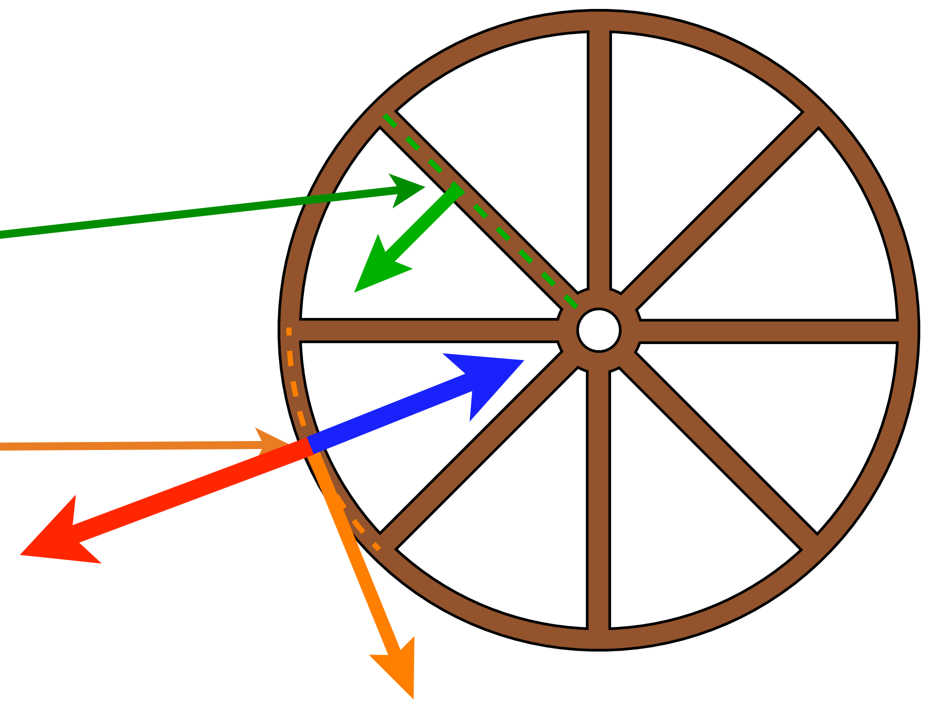
$$\textbf{Lorentz}: \quad \vec{r}\,' = \vec{r} - \gamma\vec{v}t + (\gamma-1)(\hat{v}\cdot\vec{r})\,\hat{v}, \qquad t' = \gamma\Big(t - \frac{\vec{v}\cdot\vec{r}}{c^2}\Big).$$

  - "Galilean E&M" with broken Lorentz symmetry ≠ Nature.
  - Einstein's mechanics ***extends*** Newton's mechanics

Unifications in relativistic quantum physics

General-relativistic unification

- General theory of relativity applies to all coordinate systems
- Spinning wheel of fortune
  - spokes move perpendicular to their length: no contraction
  - wheel segments move along their length, and contract
  - The resulting geometry of the wheel is curved,
  - where circumference < $2\pi R$!!!
  - and there exists a centripetal acceleration, and a "centrifugal force."



- In general, non-inertial systems systems
  - have acceleration, "feel" a force, and have curved spacetime.
  - Inertial systems are a limiting case of non-inertial ones.

Unifications in relativistic quantum physics

General-relativistic unification

- How to quantify the "non-inertial → inertial" limit?
- Velocities, accelerations = differential coordinate expressions
- Spacetime coordinates (positions) are not measurable
- Distances (duration, extends) are.

$$s(\mathrm{x}_i, \mathrm{x}_f) := \int_{\mathrm{x}_i}^{\mathrm{x}_f} \mathrm{d}s, \qquad \text{where} \quad \mathrm{d}s^2 \;:=\; g_{\mu\nu}(\mathrm{x})\,\mathrm{d}x^\mu \mathrm{d}x^\nu,$$

- ... and $g_{\mu\nu}(\mathrm{x})$ must be known with every coordinate system.
  - Flat spacetime: $g_{\mu\nu}(\mathrm{y}) \approx -\eta_{\mu\nu}(\mathrm{x})$,
  - Curved spacetime: $g_{\mu\nu}(\mathrm{y}) \not\approx -\eta_{\mu\nu}(\mathrm{x})$.

$$g_{\mu\nu}(\mathrm{y}) = \frac{\partial x^\rho}{\partial y^\mu}\frac{\partial x^\sigma}{\partial y^\nu}\, g_{\rho\sigma}(\mathrm{x})$$

  - No clear-cut criterion ...

Unifications in relativistic quantum physics

General-relativistic unification

- The formal "no gravitation" limit: "$G_N \to 0$"
  - Note:
  - nothing convenient to compare with …

$$[G_N] = \frac{L^3}{M\,T^2}$$

- Riemann tensor $[R_{\mu\nu\rho}{}^{\sigma}] = L^{-2}$: reciprocal of $(\text{length})^2$
- Curvature invariants $|\boldsymbol{R}_i|$, normalized so that $[\boldsymbol{R}_i]{=}L^{-1}$.
- Then could use Planck length: $\ell_P := \sqrt{\hbar G_N / c^3}$
- … and by "$\ell_P \to 0$" agree to mean "$|\boldsymbol{R}_i|\,\ell_P \ll 1$,"
- where "$\ll$" means "less than a agreed upon tolerance."
  - Strictly, among all coordinate systems, inertial systems are a point.
  - By "inertial systems" one *may* only understand those where curvature corrections are *negligible*—for which the limits of precision are necessarily subject to *convention*.

Unifications in relativistic quantum physics

General-relativistic unification

- Physics laws must be stated as
  - systems of (differential) equations
  - covariant with respect to all (invertible) coordinate changes.
  - (= "Physics laws are absolutely democratic,
    in that they apply equally to all conceivable observers")
  - (= general relativity)
- General relativity thus unifies
  - all observers (in the sense that they are all treated on par)
  - acceleration & gravitation (as indistinguishable)
  - force & spacetime curvature (as interchangeable)
  - …

I'll be back

Unifications in relativistic quantum physics

Quantum unification

- Oft-stated as "sufficiently small systems are quantum"
- but, this has nothing to do with (spatial) size.
- Notion of "particle" is "well localized in position space."
- Notion of "wave" is "well localized in momentum space."
- But these cannot be perfect:

$$\triangle x \, \triangle p_x \geqslant \tfrac{1}{2}\hbar$$

- Still, quantum physics ≠ granular phase-space classical physics
- Nevertheless, $\hbar$ is indeed the "reference quantity."
- So, "$\hbar \to 0$" should be understood to mean $(\hbar/S_i) \to 0$,
  - where S_i ranges over all characteristic quantities w/units ML^2/T
  - angular momenta, (Hamilton) action functionals, …

Unifications in relativistic quantum physics

Quantum unification

- A repeated digression on Heisenberg indeterminacy.
- Given to Hermitian operators, A and B, define:

$$C := -i\big[\,A\,,\,B\,\big], \qquad C^{\dagger} = C.$$

- Then

$$\big[\,A_0\,,\,B_0\,\big] \;:=\; \big[\,(A - \langle A\rangle)\,,\,(B - \langle B\rangle)\,\big] \;=\; iC,$$

$$\begin{aligned} 0 \leqslant \langle\big|A_0 - i\omega B_0\big|^2\rangle &= \langle A_0^2\rangle - i\omega\langle\big[\,A_0\,,\,B_0\,\big]\rangle + \omega^2\langle A_0^2\rangle, \\ &= \Delta_A^2 + \omega\langle C\rangle + \omega^2\Delta_B^2. \end{aligned}$$

- So, this is also true for the minimal value of ω:

$$\Delta_A^2\,\Delta_B^2 \;\geqslant\; \tfrac{1}{4}\langle C\rangle^2, \qquad \text{that is,} \qquad \Delta_A\,\Delta_B \;\geqslant\; \tfrac{1}{2}\,|\langle C\rangle| = \tfrac{1}{2}\,|\langle[A,B]\rangle|$$

- This inequality is state-dependent!!!

$$\big[\,J_x\,,\,J_y\,\big] = i\hbar\,J_z, \qquad \Rightarrow \qquad \Delta_{J_x}\,\Delta_{J_y} \;\geqslant\; \tfrac{1}{2}\big|\langle J_z\rangle\big|.$$

Unifications in relativistic quantum physics

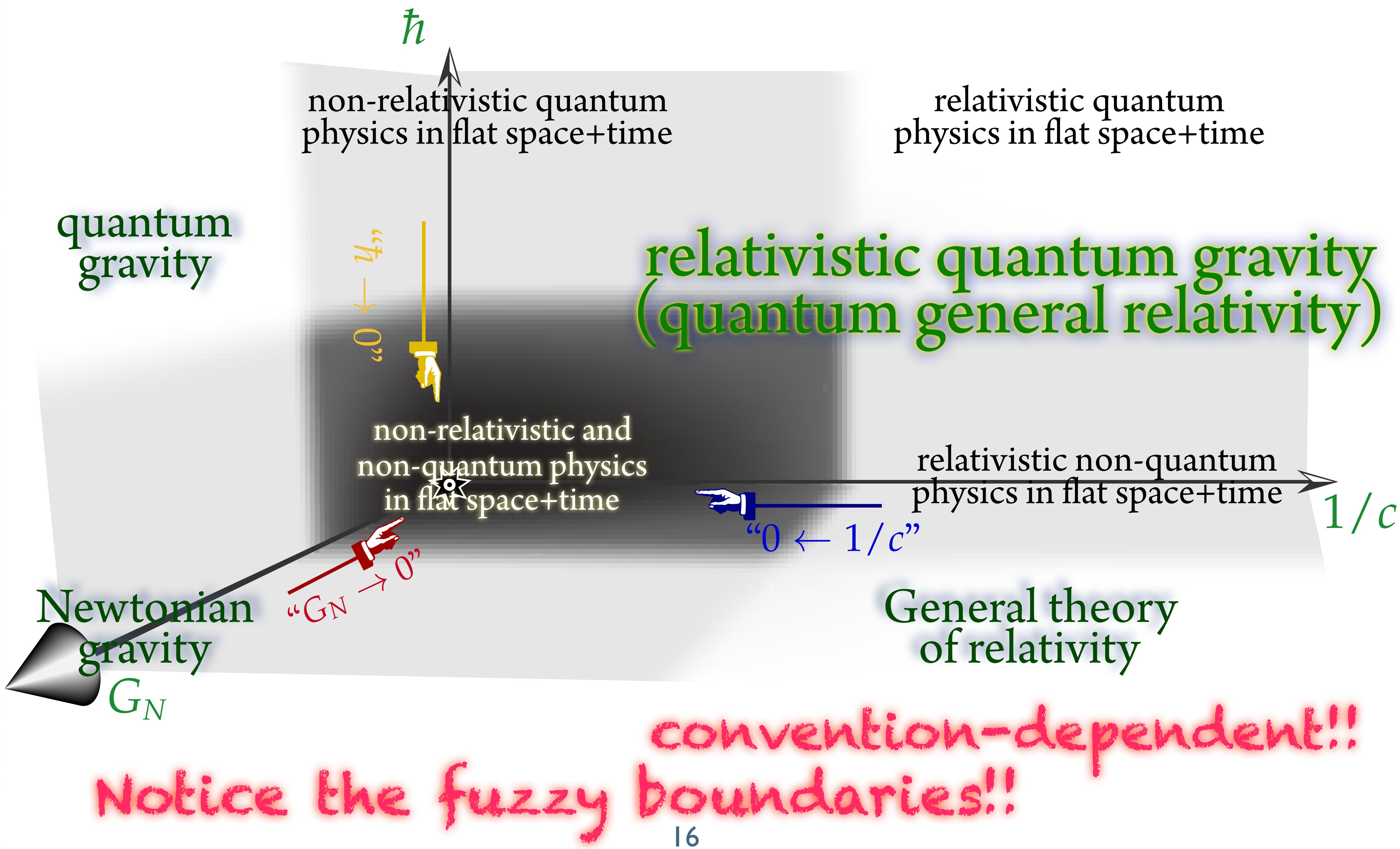
Quantum unification

- **Conclusion**: As $\hbar$ is a natural constant, the formal "$\hbar \to 0$" qualifier stands in for the qualifiers "$(\hbar/S_i) \to 0$," where…
- … S_i ranges over all angular momenta within the system and its (Hamilton) action functional.
    - Non-quantum physics is a special, limiting case of quantum physics. For any given system, in the space of all possible angular momenta and (Hamilton) action functionals $\{S_1, S_2, \dots\}$, the strict non-quantum regime is a point: $S_i = 0$ for all i—everything else is quantum physics.
    - By "non-quantum systems" one *may* understand only the cases where the quantum corrections are *negligible*—for which the limits of precision are necessarily subject to *convention*.
- The *reference action* ($\hbar$) is universal, and occurs in many different phenomena; it is a hallmark of "quantumness."

Relativistic quantum physics

The 3D quadrant of scientific systems

- Putting it all together:



Relativistic quantum physics

Phase transition vs. limiting transitions

- The transition between $(v_{ij} \ll c) \leftrightarrow (v_{ij} \approx c)$ regimes
- The transition between $(S_i \gg \hbar) \leftrightarrow (S_i \approx \hbar)$ regimes
- The transition between $(\varkappa = +1) \leftrightarrow (\varkappa = -1)$ EW regimes
  - have some similarities, but also some differences
- The former two are examples of transitions where
  - unification/separation happens at a "place" which is determined by observational precision/tolerance
- The latter is an example of a phase transition, where
  - unification/separation happens at a "place" which is determined by a characteristic (order) parameter of the system
- But, both have a higher symmetry in the "unified" regime
  - and a "symmetry breaking" effect and consequence

Relativistic quantum physics

Phase transition vs. limiting transitions

	United regime	Separated regime
Electromagnetism	The relative speed between at least two subsystems is not negligibly small, $v_{ij}/c \not\ll 1$.	The relative speed between at least two subsystems is negligibly small, $v_{ij}/c \ll 1$.
	The transition demarcation is ***specified by a convention*** in resolution.	
	Separation and differentiation between the $\vec{E}$- and the $\vec{B}$-fields depends on the choice of the coordinate system; see example 3.1, p. 191, and relations (3.75) and (3.76).	In a system where the free charges are static and the idealized currents stationary, the electric and the magnetic fields are static and perfectly separated.
	The symmetries of the Maxwell equations form the Lorentz group, together with space-time translations, *i.e.*, the Poincaré group, $Po(1,3)$.	The symmetries of electro- and magneto-static systems are limited to rotations in space, Galilean boosts and translations in space and time, $Ga(1,3) \subsetneq Po(1,3)$.
Electroweak int.	Particles in a process have energies $E_i > \hbar c\sqrt{\lambda}\langle\mathbb{H}\rangle\|_{\varkappa<0} \sim M_{W^\pm}c^2$.	particles in a process have energies $E_i < \hbar c\sqrt{\lambda}\langle\mathbb{H}\rangle\|_{\varkappa<0} \sim M_{W^\pm}c^2$.
	The transition demarcation (the order parameter critical value) is ***determined by the system***.	
	$W^\pm, W^3_\mu$ and B_μ are normal modes, all massless.	B_μ and W^3_μ are not normal modes; A_μ (massless) and Z_μ (massive) are; see rel. (5.84)–(5.85).
	Local (gauge) symmetries of electroweak interactions form the $SU(2)_w \times U(1)_y$ group.	Local (gauge) symmetries of electroweak interactions reduce $U(1)_Q \subset SU(2)_w \times U(1)_y$.

Relativistic quantum physics

Three distinct flavors of unification

- ***Conclusion***: Since Newton's Principia (1687) and through the unification of electroweak interactions (Glashow, Weinberg and Salam, 1979 Nobel prize), three distinct notions of unification grew into fundamental physics:
  - ***conceptual***: in the sense that Nature is one and that its scientific descriptions (models) better be conceptually uniform, and not a patchwork (hodgepodge) of diverse and disparate ideas;
  - ***limiting***, in the sense that one marked "regime" of behavior of a system is, strictly speaking, merely a special limiting case (*i.e.*, approximation) of another, more general and/or more exact description;
  - ***phase/regime***, where the description of a system contains a definition of an order parameter and its critical value that divides two phases, *i.e.*, regimes of a system.

Peeking beyond the Standard Model

Nine puzzles

- The Standard Model unifies & explains a lot. *However, …*
  - **Spacetime**: one assumes 4 dimensions and 1+3 signature
  - **Interaction hierarchy**: The relative magnitudes of the coupling parameters of the $SU(3)_c \times SU(2)_w \times U(1)_y$ gauge intractions, i.e., the $\alpha_c : \alpha_w : \alpha_y$ relative ratios (at any particular energy)
  - **Mass scales, hierarchy and structure**: In the Standard Model, all masses of the fundamental fermions arise being proportional to the $\langle \mathbb{H} \rangle$, but multiplied by undetermined coefficients:

$$h_u, h_d \sim 10^{-5}, \quad h_s \sim 10^{-3}, \quad h_c, h_b \sim 10^{-2}, \quad h_t \sim 1.$$

  - **CKM quark mixing**: there is no *a priori* reason for weak interaction eigenstates to coincide with the free propagation ones. But the CKM angles are undetermined.

$$\begin{bmatrix} |d_w\rangle \\ |s_w\rangle \\ |b_w\rangle \end{bmatrix} := \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix}$$

Peeking beyond the Standard Model

Nine puzzles

- **Neutrino mixing**: again, there is no reason for weak eigenstates to coincide with the free propagation ones, but the concrete values of the matrix are undetermined,
- as is the parameter M_ν for the see-saw mechanism.
- **Fermion family profusion**: why are there three?
- ***CP*-violation**: why is δ_{12} so small, and why is there no strong CP-violation, *i.e.*, why is $\vartheta = 0$ in $\mathscr{L} = \mathrm{Tr}\big[\mathbb{F}_{\mu\nu}\mathbb{F}^{\mu\nu} + \vartheta\varepsilon^{\mu\nu\rho\sigma}\mathbb{F}_{\mu\nu}\mathbb{F}_{\rho\sigma}\big]\big)$?
- **Cosmological constant**: phase transitions have latent heat; since the Universe has no external reservoir to siphon it away, latent heat drives the expansion of the Universe. More generally, "*dark energy*" is anything that provides for an accelerated expansion. Why is Λ today so small—but nonzero? What other "dark energy" is there?
- **Dark matter**: visible matter rotates in big systems (galaxies) at rates that imply vast amounts of invisible matter (for which there are no Standard Model candidates) distributed surpassing the visible system. What is it?

Peeking beyond the Standard Model

One more puzzle, a bit more technical

- The Standard Model: a tool for systematizing questions about the "particle physics"-fundamental physics, & delving beyond.
- Recall renormalization?

$$\frac{1}{\alpha_{1,R}(|\mathrm{q}^2|)} \approx \frac{1}{\alpha_{1,R}(\mu^2 c^2)} - \frac{4}{12\pi}\ln\left(\frac{|\mathrm{q}^2|}{\mu^2 c^2}\right)$$

$$|\mathrm{q}^2| \gg \mu^2 c^2$$

$$\frac{1}{\alpha_{n,R}(|\mathrm{q}^2|)} \approx \frac{1}{\alpha_{n,R}(\mu^2 c^2)} + \frac{11n-2n_f}{12\pi}\ln\left(\frac{|\mathrm{q}^2|}{\mu^2 c^2}\right)$$

- In the Standard Model:

$$SU(3)_c: \quad n_f = 3\times 2_{(w)}, \quad 11n - 2n_f = +21,$$

$$SU(2)_w: \quad n_f = 3\times(3_{(c)}+1), \quad 11n - 2n_f = -2,$$

- so $\alpha_y{}^{-1}$ and $\alpha_w{}^{-1}$ have a negative slope, while $\alpha_c{}^{-1}$ rises.

Peeking beyond the Standard Model

One more puzzle, a bit more technical

- The system the becomes

$$U(1)_y: \quad \frac{1}{\alpha_{y,R}(|\mathrm{q}^2|)} \approx \frac{1}{\alpha_{y,R}(\mu^2 c^2)} - \frac{4}{12\pi} \ln\left(\frac{|\mathrm{q}^2|}{\mu^2 c^2}\right),$$

$$SU(2)_w: \quad \frac{1}{\alpha_{w,R}(|\mathrm{q}^2|)} \approx \frac{1}{\alpha_{w,R}(\mu^2 c^2)} - \frac{2}{12\pi} \ln\left(\frac{|\mathrm{q}^2|}{\mu^2 c^2}\right),$$

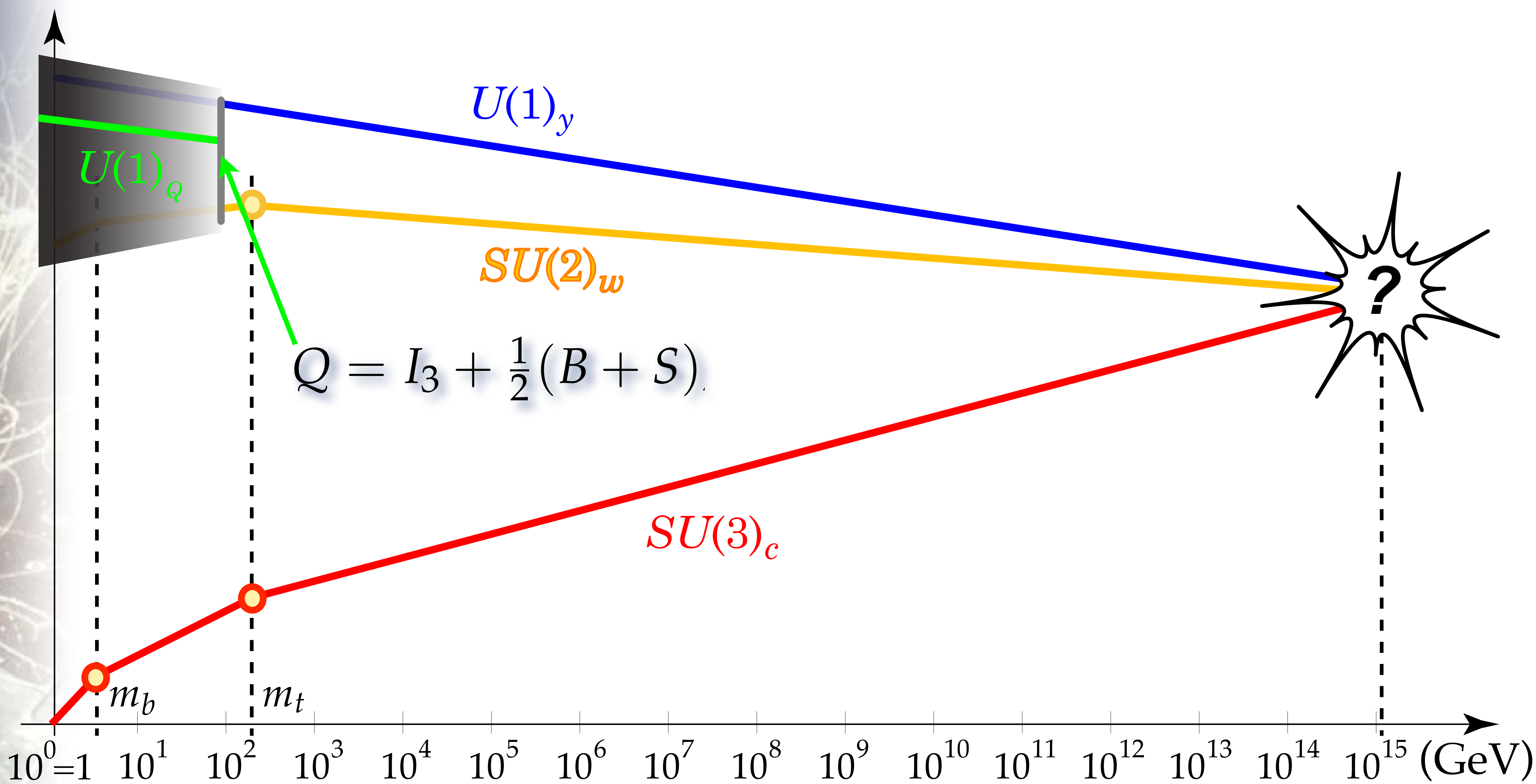
$$SU(3)_c: \quad \frac{1}{\alpha_{s,R}(|\mathrm{q}^2|)} \approx \frac{1}{\alpha_{s,R}(\mu^2 c^2)} + \frac{21}{12\pi} \ln\left(\frac{|\mathrm{q}^2|}{\mu^2 c^2}\right).$$

- where μ is an experimentally convenient mass scale, where the coupling parameters $\alpha_y(\mu^2 c^2)$, $\alpha_w(\mu^2 c^2)$ and $\alpha_c(\mu^2 c^2)$ can be measured reliably.
- This permits extrapolation across the "grand desert" (no new fundamental fermions after the 3^{rd} family).

Peeking beyond the Standard Model

One more puzzle, a bit more technical

- On a log-scale then:



Thanks!

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