(Fundamental) Physics of Elementary Particles

Nonperturbative comments: strong CP-violation, Weinberg-Witten theorem, anomaly cancellation.

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Fundamental Physics of Elementary Particles

Program

- *CP*-Violation
  - Weak *CP*-Violation, Revisited
  - Strong *CP*-Violation —why not?
- The Weinberg-Witten Theorem
  - Are There "Quarks" Within Quarks? …all the way down?
  - The Claim of the Theorem and its Assumptions
  - An Epilogue Segue
- Anomalies
  - A pair of classical symmetries
  - A measure of quantum symmetry breaking…
  - …and their consequences.
- An End-of-Semester Review
  - What have we learned?

CP-Violation

Weak CP-Violation, Revisited

- Strange particles:
  (M. Gell-Mann & A. Pais, 1954)
- …created by strong interactions $(\sim 10^{-23}\mathrm{s})$, in pairs

$$\pi^-_{(0)} + p^+_{(0)} \to \Lambda^0_{(-1)} + K^0_{(+1)},$$

$$\pi^+_{(0)} + p^+_{(0)} \to p^+_{(0)} + \overline{K}^0_{(-1)} + K^+_{(+1)}.$$

- …but decay by weak interactions $(\sim 10^{-8-10}\mathrm{s})$. (decays are delayed since strangeness is violated)
- The neutral kaons are pseudo-scalars:

$$\mathsf{CP}|K^0\rangle = -\mathsf{C}|K^0\rangle = -|\overline{K}^0\rangle, \qquad \mathsf{CP}|\overline{K}^0\rangle = -\mathsf{C}|\overline{K}^0\rangle = -|K^0\rangle,$$

- …and the *CP*-eigenstates are:

$$|K^0_+\rangle := \tfrac{1}{\sqrt{2}}\big(|K^0\rangle - |\overline{K}^0\rangle\big), \qquad \text{and} \qquad |K^0_-\rangle := \tfrac{1}{\sqrt{2}}\big(|K^0\rangle + |\overline{K}^0\rangle\big),$$

$$\mathsf{CP}|K^0_\pm\rangle = (\pm 1)|K^0_\pm\rangle.$$

CP-Violation

Weak CP-Violation, Revisited

- Neutral kaons can decay (among other things) into pions.
- Pions are also pseudo-scalars, and π^0 is its own antiparticle:

$$CP|\pi^0\rangle = -C|\pi^0\rangle = -|\pi^0\rangle, \qquad CP|\underbrace{\pi^0\cdots\pi^0}_{n}\rangle = (-1)^n|\pi^0\rangle.$$

- Thus,

$$\boxed{\text{short}} \quad K^0_+ \to 2\pi, \qquad vs. \qquad K^0_- \to 3\pi, \quad \boxed{\text{long}}$$

- ... so the decay constants differ:

$$\Gamma_{K^0_+} \propto \sqrt{1-(2m_{\pi^0}/m_{K^0})^2} \quad vs. \quad \Gamma_{K^0_-} \propto \sqrt{1-(3m_{\pi^0}/m_{K^0})^2}$$

- ... as do the life-times,

$$\tau_{K^0_+} = 0.895\,8\times10^{-10}\,\text{s}, \quad vs. \quad \tau_{K^0_-} = 5.114\times10^{-8}\,\text{s}.$$

- K_{long} lives about 570 times longer than K_{short}.

CP-Violation

Weak CP-Violation, Revisited

- Starting with a 50-50% beam,

$$\frac{N(K^0_+)}{N(K^0_-)} = \frac{e^{-t/\tau^+}}{e^{-t/\tau^-}} = \exp\left\{-\frac{t}{\tau^+}+\frac{t}{\tau^-}\right\} \approx \exp\left\{-569.9\,\frac{t}{\tau^-}\right\}$$

- …approaches 10^{-5} in just 1ns.
- So, in 1964, James Cronin and Val Fitch simply
  - looked for 2-pion decays long after K_{short} must have decayed
  - …and they found, about 1 in 500 3-pion (K_{long}) decays.
- This violates *CP*-symmetry:
  - either if K_{long} itself decays into two pions
  - or if K_{long} transmogrifies into K_{short}.
  - Standard Model weak interactions (and logical consistency)
  - …permit the latter. (K_{long} may be defined as that with a 3π decay.)

CP-Violation

Weak CP-Violation, Revisited

- Also,

$$K^0_- \begin{cases} \pi^+ + e^- + \bar{\nu}_e, & (a) \\ \pi^- + e^+ + \nu_e, & (b) \end{cases}$$

- where the (b)-mode is the *CP*-image of the (a)-mode.
- If *CP* were an exact & universal symmetry,
  - the two decay modes would occur equally often.
  - But, they don't: there is a ~$3.3{\times}10^{-3}$ relative difference.
- The smallness of this effect agrees with the $K_{\text{long}}/K_{\text{short}}$ one.
- The *CP*-violation is small/weak.
- Explained by (lower) quark mixing: the mass-eigenstates (stationary states) are not the weak interaction eigenstates.

$$\begin{bmatrix} |d_w\rangle \\ |s_w\rangle \\ |b_w\rangle \end{bmatrix} := \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix}$$

complex phase δ_{13}~1.20° Small!

CP-Violation

Strong CP-Violation, a Puzzle

- By contrast, in QCD:

$$\mathscr{L}_{QCD} = -\sum_n \mathrm{Tr}\left[\overline{\mathbf{\Psi}}_n(\mathrm{x})\left[i\hbar c \not{D} - m_n c^2\right]\mathbf{\Psi}_n(\mathrm{x})\right] - \tfrac{1}{4}\,\mathrm{Tr}\left[\mathbb{F}_{\mu\nu}\mathbb{F}^{\mu\nu}\right],$$

- may be generalized (in agreement with $SU(3)_c$ & Lorentz invariance), into

$$\mathscr{L}_{QCD+} = -\sum_n \mathrm{Tr}\left[\overline{\mathbf{\Psi}}_n(\mathrm{x})\left[i\hbar c \not{D} - m_n e^{i\vartheta'\widehat{\gamma}} c^2\right]\mathbf{\Psi}_n(\mathrm{x})\right] - \tfrac{1}{4}\,\mathrm{Tr}\left[\mathbb{F}_{\mu\nu}\mathbb{F}^{\mu\nu}\right] - \frac{n_f\, g_s^2\, \vartheta}{32\pi^2}\,\varepsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\left[\mathbb{F}_{\mu\nu}\mathbb{F}_{\rho\sigma}\right]$$

- where experimentally ϑ & ϑ' are teensy ($<3\times10^{-10}$)!
- This then (10 order of magnitude fine-tuning of ϑ & ϑ') is the strong *CP*-problem (the ϑ_{QCD}-problem).

CP-Violation

Strong CP-Violation, a Puzzle

- The extra term, $\varepsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\left[\mathbb{F}_{\mu\nu}\mathbb{F}_{\rho\sigma}\right]$
- is the 4-divergence of the so-called Loos-Chern-Simons current (density)

$$\mathcal{K}^\mu = \frac{n_f g_s^2}{32\pi^2}\varepsilon^{\mu\nu\rho\sigma}\left(\delta_{ab}\,A^a_\nu\,F^b_{\rho\sigma} - \tfrac{1}{3}g_s f_{abc}\,A^a_\nu\,A^b_\rho\,A^c_\sigma\right).$$

- Then define a charge $\mathcal{Q} := \int \mathrm{d}^3\vec{r}\,\mathcal{K}^0$
- A corresponding ϑ-transformation, $\exp\{i\vartheta\mathcal{Q}\}$,
  - …transforms $|0\rangle$ into $|\vartheta\rangle$, as well as all operators.
  - The Hilbert space has identical "sectors," labeled by ϑ.
- But, QCD equations of motion are non-linear
  - …and have nontrivial solutions
  - …that tunnel from $\vartheta=0$ to $\vartheta\neq0$
  - …the sectors are not independent.

A. Belavin, A. Polyakov
A.S. Schwartz & Y. Tyupkin
in 1975

The Weinberg-Witten Theorem

Are There "Quarks" Within Quarks?

- The explanation of the abundance of hadrons
- … as (anti)quark bound states "caught" on after 1974.
- But, why are there three generations
  - $(u, d; \nu_e, e)$, $(c, s; \nu_\mu, \mu)$ and $(t, b; \nu_\tau, \tau)$??
  - Perhaps the latter are higher excitations of $(u, d; \nu_e, e)$??
  - Which would make sense if these were (orbitally excited) bound states of something else.
  - Call this "something else" ***preons***.
- In which case, (at least some of) the gauge (interaction mediating) particles could be bound states of these preons.
- Such models started being considered…
- … all the way back, in 1974! J.C. Pati & A. Salam

  1st Grand Unification ideas!

The Weinberg-Witten Theorem

Are There "Quarks" Within Quarks?

- Another topic, from Chapter 5, is the Higgs field:
- While some form of symmetry breaking is necessary to accommodate massive $W^{\pm}$- and Z^0-bosons
- …the "agent" need not be an elementary particle
- …but a *collective* state of matter. → I'll be back.
- To that end:

Theorem [Weinberg-Witten] No Quantum field theory in 3+1-dimensional spacetime with a Poincaré-covariant and gauge-invariant 4-vector current J^μ that satisfies a continuity equation may have a massless particle with a helicity bigger than ½ and non-vanishing charge of $\int d^3\boldsymbol{r}\, J^0$.

No quantum field theory in 3+1-dimensional spacetime with a Poincaré-covariant and gauge-invariant rank-2 tensor satisfying a continuity equation may have a massless particle with a helicity bigger than 1.

The Weinberg-Witten Theorem

The Claim of the Theorem and its Assumptions

- Poincaré-covariant = transforms as a tensor with respect to Lorentz & translations in spacetime.
- The general continuity equation is $\partial_\mu T^{\mu\nu\cdots}=0$.
- The proof is non-perturbative and general
- …but the assumptions are stringent.
- In all preonic and "technicolor" models, there exists at least one additional (new) strong & binding gauge interaction
- …which is based on a (new) non-abelian gauge symmetry
- …the conserved current of which is not gauge-invariant
- …and so neither is the "charge" in
- "…no massless particle with a helicity bigger than ½ and non-vanishing charge."

The Weinberg-Witten Theorem

The Claim of the Theorem and its Assumptions

- So,
- …can there be "quarks" within quarks?
- That is, can a subset of
  - $(u, d; \nu_e, e)$, $(c, s; \nu_\mu, \mu)$ and $(t, b; \nu_\tau, \tau)$
  - gauge & the Higgs bosons
- be composite—not elementary—structures?
- Experimentally: no indication. → **Yet.**
- Theoretically: no known model → **Yet.**
  - reproduces the Standard Model fully
  - and simplifies its structure
  - in this literal substructure fashion.
- Quarks (& leptons & gauge bosons) are elementary. Most likely.

The Weinberg-Witten Theorem

A Small Epilogue

- For energies ≫ masses of the *u*,- *d*- and *s*-quarks
- … $SU(3)_f$ is a pretty good approximate symmetry
- … even if considered locally
- … and the lightest vector (spin-1) eight mesons

$$(\rho^{\pm}, \rho^0, K^{*\pm}, K^{*0}, \bar{K}^{*0}, \phi)$$

although demonstrably quark bound states

- … are approximately massless
- … and may be identified as the (approximate) gauge bosons of the approximate $SU(3)_f$ symmetry.
- Thus, the lightness of some hadrons may be interpreted as a signature of approximate symmetries.
- In turn, the lightness of the spin-0 mesons is related to a similarly approximate application of the Goldstone theorem.

The Weinberg-Witten Theorem

A Small Epilogue

- The QCD Lagrangian (and in fact all of the Standard Model)

$$\mathscr{L}_{QCD} = -\sum_n \mathrm{Tr}\left[\overline{\boldsymbol{\Psi}}_n(\mathrm{x})\left[i\hbar c\not{D} - m_n c^2\right]\boldsymbol{\Psi}_n(\mathrm{x})\right] - \tfrac{1}{4}\mathrm{Tr}\left[\mathbb{F}_{\mu\nu}\mathbb{F}^{\mu\nu}\right],$$

n counts flavors

- …couples Ψ_{n+} with Ψ_{n-} only through the mass term.
- Thus, in the regime where the masses can be neglected
- …there is a global $SU(n_f)_+ \times U(1)_+ \times SU(n_f)_- \times U(1)_-$ symmetry.
  acting only on the left-handed right-handed fermions
- However, in the quantum theory, $\langle \overline{\Psi_{n+}}\, \Psi_{n-} \rangle \neq 0$.
- This breaks the symmetry to $SU(n_f) \times U(1)_B$. "diagonal"
  - The $U(1)_A = [U(1)_+ \times U(1)_-]/U(1)_B$ complement, generated by the difference of the + and – generators, is *anomalous*.

Anomalies

… borrowing also from Ch.5

A pair of classical symmetries

- In a toy model with just two flavors of quarks, compute

$$
\begin{aligned}
i\hbar\partial_\mu[\overline{\Psi}_1\boldsymbol{\gamma}^\mu\Psi_2] &= (i\hbar\partial_\mu\overline{\Psi}_1\boldsymbol{\gamma}^\mu)\Psi_2 + \overline{\Psi}_1\boldsymbol{\gamma}^\mu(i\hbar\partial_\mu\Psi_2),\\
&= -\overline{(i\hbar\partial\!\!\!/\Psi_1)}\Psi_2 + \overline{\Psi}_1(i\hbar\partial\!\!\!/\Psi_2),\\
&= -\overline{(m_1c\Psi_1)}\Psi_2 + \overline{\Psi}_1(m_2c\Psi_2),\\
&= (m_2-m_1)c\overline{\Psi}_1\Psi_2,
\end{aligned}
$$

EoM

$$
\begin{aligned}
i\hbar\partial_\mu[\overline{\Psi}_1\widehat{\boldsymbol{\gamma}}\boldsymbol{\gamma}^\mu\Psi_2] &= (i\hbar\partial_\mu\overline{\Psi}_1(-\boldsymbol{\gamma}^\mu\widehat{\boldsymbol{\gamma}}))\Psi_2 + \overline{\Psi}_1\widehat{\boldsymbol{\gamma}}\boldsymbol{\gamma}^\mu(i\hbar\partial_\mu\Psi_2),\\
&= \overline{(i\hbar\partial\!\!\!/\Psi_1)}\widehat{\boldsymbol{\gamma}}\Psi_2 + \overline{\Psi}_1\widehat{\boldsymbol{\gamma}}(i\hbar\partial\!\!\!/\Psi_2),\\
&= \overline{(m_1c\Psi_1)}\widehat{\boldsymbol{\gamma}}\Psi_2 + \overline{\Psi}_1\widehat{\boldsymbol{\gamma}}(m_2c\Psi_2),\\
&= (m_1+m_2)c\overline{\Psi}_1\widehat{\boldsymbol{\gamma}}\Psi_2.
\end{aligned}
$$

EoM

- Classically: $(m_1 = m_2) \Rightarrow$ 1st current is conserved.
- Classically: $(m_1 = m_2 = 0) \Rightarrow$ both currents are conserved.

Anomalies

… borrowing also from Ch.5

A pair of classical symmetries

- If $(m_i = m_j) \approx 0$,

density	definition	continuity	conserved charge
4-vector:	$\mathfrak{J}^\mu_{ij} := [\overline{\Psi}_i \boldsymbol{\gamma}^\mu \Psi_j]$	$\partial_\mu \mathfrak{J}^\mu_{ij} \overset{\text{cl.}}{=} 0$	$Q_{ij} := \int \mathrm{d}^3\vec{r}\, \mathfrak{J}^0_{ij}$
pseudo-4-vector:	$\widehat{\mathfrak{J}}^\mu_{ij} := [\overline{\Psi}_i \widehat{\boldsymbol{\gamma}} \boldsymbol{\gamma}^\mu \Psi_j]$	$\partial_\mu \widehat{\mathfrak{J}}^\mu_{ij} \overset{\text{cl.}}{\approx} 0$	$\widehat{Q}_{ij} := \int \mathrm{d}^3\vec{r}\, \widehat{\mathfrak{J}}^0_{ij}$

- For example, baryon number $= 3\Sigma_{j=i}\, Q_{ij}$.
- ***Conversely*** to Noether's theorem,
  - each current density that satisfies a continuity equation defines a conserved charge
  - … which generates a (global) symmetry.
- Thus these conserved charges define the classical symmetries of the quark system.

Anomalies

…borrowing also from Ch.5

A measure of quantum symmetry breaking…

- However, quantum dynamics
  need not preserve all classical symmetries!
- In general, then, we may ***define***:

$$\partial_\mu \mathfrak{J}^\mu_{ij} = \mathfrak{A}_{ij}, \qquad \text{and} \qquad \partial_\mu \widehat{\mathfrak{J}}^\mu_{ij} = \widehat{\mathfrak{A}}_{ij},$$

- where the r.h.s. terms, computed as a contribution of some quantum processes, provide
- … a measure of quantum breaking of classical symmetries
- … and are called ***anomalies***.
- There is no *a priori* reason for perturbative computability.
- Anomalies are characteristics of the quantum dynamics.
- Curiously, most known anomalies are detected in 1st order perturbative computation, and are stable thereafter.

Anomalies

... borrowing also from Ch.5

...AND THEIR CONSEQUENCES

- Approximate symmetries
  - Even classically, charge is only approximately conserved
  - "Gauge" & "Goldstone" modes have masses ~ approx. tolerance
- Global symmetries
  - Anomalies break the symmetry
  - Baryon number: $\mathfrak{A} \propto \vartheta \varepsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}[\mathbb{F}_{\mu\nu}\mathbb{F}_{\rho\sigma}]$ $\quad \vartheta < 3\times 10^{-10}$ **Why?**
- Gauge symmetries
  - Anomalies render gauge interactions inconsistent
  - ... and so must cancel from various contributions. **!**
- 1969, S. Adler + J.S. Bell & R. Jackiw:
  - The EM continuity equation is anomalous, proportional to

$$\textstyle\sum_i Q_i^{(EM)} = 3\big[(+\tfrac{2}{3}) + (-\tfrac{1}{3})\big] + (0) + (-1) = 0$$

Ch.5

  - ... canceling only for complete generations, $(u, d; \nu_e, e^-)$, etc.

An End-of-Semester Review

What Have You Learned?

- As for myself: I shall not seek gainful employment as a typist.
- Also, please gauge your opinion of the contents (0="no opinion," 1="too little," 2="OK" and 3="too much"):
  - The first "half" (Ch.–1, 0, 1, 2, 3 & 4) of the text
  - Ch.–1 (fundamental physics, units & dim's, Quantum limits on info)
  - Ch.0 (el. particles, historical inventory, logic & conservation laws)
  - Ch.1 (Lorentz transf., relativistic kinematics, Feynman diagrams)
  - Ch.2 (bound states, finite symmetries, isospin, eightfold way & $SU(3)_f$)
  - Ch.3 (local (gauge) symmetry, electrodynamics, QED Feynman calculus)
  - Ch.4 (local color symmetry, color factor computations, non-perturbative)
  - App. A (Groups)
  - App. B (Nobel prizes, data, some homework answers)
  - App. C (Jargon)
  - Overall structure
  - Digressions & comments
  - End-of-section problems
  - Worked-out examples
  - Breadth of the material
  - Unifying all fundamentals

Thanks!

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