(Fundamental) Physics of Elementary Particles

Renormalization, color confinement, anti-screening, asymptotic freedom and "all that"

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Fundamental Physics of Elementary Particles

Program

- Renormalization in QCD as compared to QED
  - Decoupling of unphysical QED gauge potentials
  - Gluon loops and non-commutativity
  - Non-decoupling of unphysical QCD gauge potentials
  - The $SU(n)$ running coupling parameter
- Screening of charge
- Non-abelian anti-screening of charge
  - Non-abelian Gauss's law
  - The Landau pole & dimensional transmutation
- The effective QCD potential revisited
  - asymptotic (ultraviolet) freedom
  - infrared confinement

Renormalization in QCD as compared to QED

Decoupling of unphysical QED gauge potentials

- Remember the photon Lagrangian?

$$F_{\mu\nu}F^{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)$$
$$= (\partial_0 A_i - \partial_i A_0)(\partial^0 A^i - \partial^i A^0) + (\partial_i A_j - \partial_j A_i)(\partial^i A^j - \partial^j A^i)$$

- Remember the 4-vector → (scalar, 3-vector) notation?

$$A_\mu = (\Phi, -c\vec{A}) \;\text{ but }\; A^\mu = (\Phi, c\vec{A})$$
$$\partial_\mu = (\tfrac{1}{c}\partial_t, \vec{\nabla}) \;\text{ but }\; \partial^\mu = (\tfrac{1}{c}\partial_t, -\vec{\nabla})$$

$$[\eta_{\mu\nu}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$F_{\mu\nu}F^{\mu\nu} = \Big((\tfrac{1}{c}\partial_t)(-c\vec{A}) - (\vec{\nabla})(\Phi)\Big)\cdot\Big((\tfrac{1}{c}\partial_t)(c\vec{A}) - (-\vec{\nabla})(\Phi)\Big)$$
$$+ \Big((\vec{\nabla})\times(-c\vec{A})\Big)\cdot\Big((-\vec{\nabla})\times(c\vec{A})\Big)$$
$$= -\vec{E}\cdot\vec{E} + c^2\vec{B}\cdot\vec{B}$$

Renormalization in QCD as compared to QED

Decoupling of unphysical QED gauge potentials

- Re-express the QED Lagrangian in terms of potentials:

$$\begin{aligned} F_{\mu\nu}F^{\mu\nu} &= -(\dot{\vec{A}} + \vec{\nabla}\Phi)^2 + c^2(\vec{\nabla}\times\vec{A})^2 \\ &= -(\dot{A}_1 + \Phi_{,1})^2 - (\dot{A}_2 + \Phi_{,2})^2 - (\dot{A}_3 + \Phi_{,3})^2 \\ &\quad + c^2\left[(A_{2,1} - A_{1,2})^2 + (A_{3,2} - A_{2,3})^2 + (A_{1,3} - A_{3,1})^2\right] \end{aligned}$$

- Now recall that $A_\mu \simeq A_\mu - c(\partial_\mu \lambda)\ldots$
- $\ldots$ and choose $\lambda = -\int \mathrm{d}t\,\Phi$, so that

$$-cA_i \simeq -cA_i + c\int \mathrm{d}t\,\Phi_{,i}$$

$$A_i \simeq A_i - \int \mathrm{d}t\,\Phi_{,i}$$

$$\dot{A}_i \simeq \dot{A}_i - \Phi_{,i}$$

- Use this to simplify the Lagrangian.

Renormalization in QCD as compared to QED

Decoupling of unphysical QED gauge potentials

- So, the QED Lagrangian is gauge-equivalent to

$$F_{\mu\nu}F^{\mu\nu} \simeq -(\dot{A}_1)^2 - (\dot{A}_2)^2 - (\dot{A}_3)^2 + c^2\left[(A_{2,1} - A_{1,2})^2 + (A_{3,2} - A_{2,3})^2 + (A_{1,3} - A_{3,1})^2\right]$$

- … since

$$A_{j,i} - A_{i,j} \simeq (A_{j,i} - \int \mathrm{d}t\, \Phi_{,ji}) - (A_{i,j} - \int \mathrm{d}t\, \Phi_{,ij}) = A_{j,i} - A_{i,j}$$

- Expanding:

$$F_{\mu\nu}F^{\mu\nu} \simeq -(\dot{A}_1)^2 - (\dot{A}_2)^2 - (\dot{A}_3)^2 + c^2\left[(A_{2,1})^2 + (A_{1,2})^2 + (A_{3,2})^2 + (A_{2,3})^2 + (A_{1,3})^2 + (A_{3,1})^2 - 2A_{2,1}A_{1,2} - 2A_{3,2}A_{2,3} - 2A_{1,3}A_{3,1}\right]$$

Renormalization in QCD as compared to QED

Decoupling of unphysical QED gauge potentials

- For a photon traveling at the speed of light along $\hat{e}_3$,
  - … A_1 and A_2 are physical (transversal) polarizations,
  - … A_3 is not.

$$F_{\mu\nu}F^{\mu\nu} \simeq -(\dot{A}_1)^2 - (\dot{A}_2)^2 - (\dot{A}_3)^2 + c^2\big[(A_{2,1} - A_{1,2})^2 + (A_{3,2})^2 + (A_{3,1})^2 + (A_{2,3})^2 + (A_{1,3})^2 - 2A_{3,2}A_{2,3} - 2A_{1,3}A_{3,1}\big]$$

  - Additionally, A_1 and A_2 do not vary in the 3rd direction,
  - … so also $|A_{1,3}| = 0 = |A_{2,3}|$. ($\approx$ FitzGerald-Lorentz contraction.)
- Thus:

$$F_{\mu\nu}F^{\mu\nu} \simeq -(\dot{A}_1)^2 - (\dot{A}_2)^2 - (\dot{A}_3)^2 + c^2\big[(A_{2,1} - A_{1,2})^2 + (A_{3,2})^2 + (A_{3,1})^2\big]$$

decouple

Renormalization in QCD as compared to QED

Decoupling of unphysical QED gauge potentials

- Why does classical decoupling also indicate a decoupling in the full quantum theory?
- Because of the Feynman-Hibbs construction.
- The partition functional

The classical Lagrangian

$$\mathcal{Z}[\vartheta] := \int \mathbf{D}[\phi]\, e^{i\hbar^{-1}\int d^4x\,(\mathscr{L}(\phi)+\vartheta\cdot\phi)}$$

$$\left[\frac{\delta}{\delta\vartheta(x_1)}\cdots\frac{\delta}{\delta\vartheta(x_n)}\mathcal{Z}[\vartheta]\right]_{\vartheta=0} = \left[\int \mathbf{D}[\phi]\,\phi(x_1)\cdots\phi(x_n)\, e^{i\hbar^{-1}\int d^4x\,(\mathscr{L}(\phi)+\vartheta\cdot\phi)}\right]_{\vartheta=0}$$

$$= G(x_1,\cdots,x_n)$$

computed by Feynman calculus

- …are correlation functions, stating the correlation of perturbations in the ϕ-field at the spacetime points $x_1 \ldots x_n$.
- $G(x_1, x_2)$ is the well-known Green's function.

Renormalization in QCD as compared to QED

Gluon loops and non-commutativity

- Non-abelian (non-commutative) QCD: a similar Lagrangian

$$
\begin{aligned}
\mathrm{Tr}\left[\mathbb{F}_{\mu\nu}\mathbb{F}^{\mu\nu}\right] &= \mathrm{Tr}\Big[\big(\partial_\mu \mathbb{A}_\nu - \partial_\nu \mathbb{A}_\mu + \tfrac{i g_c}{\hbar c}[\mathbb{A}_\mu, \mathbb{A}_\nu]\big) \\
&\qquad \big(\partial^\mu \mathbb{A}^\nu - \partial^\nu \mathbb{A}^\mu + \tfrac{i g_c}{\hbar c}[\mathbb{A}^\mu, \mathbb{A}^\nu]\big)\Big] \\
&= \big(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \tfrac{g_c}{\hbar c} f^a{}_{bc} A^b_\mu A^c_\nu\big) \\
&\qquad \big(\partial^\mu A^\nu_a - \partial^\nu A^\mu_a - \tfrac{g_c}{\hbar c} f_a{}^{eh} A^\mu_e A^\nu_h\big) \\
&= \big(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\big)\big(\partial^\mu A^\nu_a - \partial^\nu A^\mu_a\big) \\
&\qquad - \tfrac{2 g_c}{\hbar c}\big(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\big)\big(f_a{}^{bc} A^\mu_b A^\nu_c\big) + \tfrac{g_c{}^2}{\hbar^2 c^2}\big(f^a{}_{bc} A^b_\mu A^c_\nu\big)\big(f_a{}^{eh} A^\mu_e A^\nu_h\big)
\end{aligned}
$$

non-abelian structure of the $SU(3)_c$ group

nonlinearities

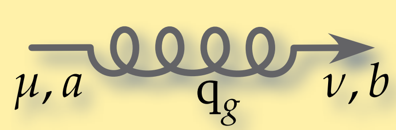
Renormalization in QCD as compared to QED

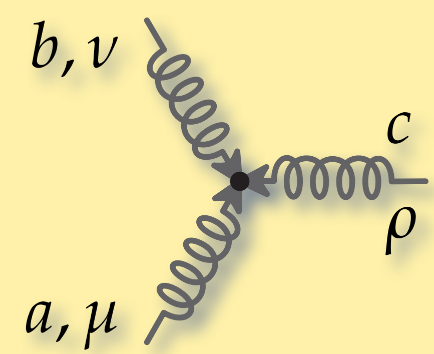
Gluon loops and non-commutativity

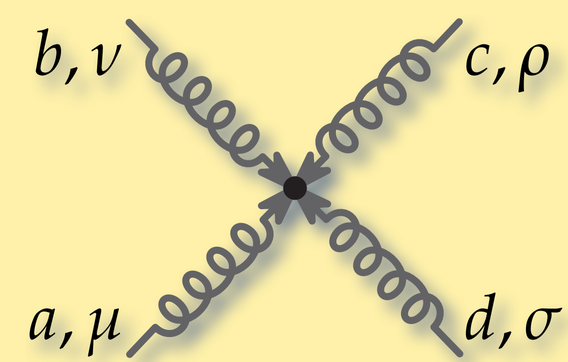
- Non-abelian (non-commutative) QCD: a similar Lagrangian

$$\mathrm{Tr}\left[\mathbb{F}_{\mu\nu}\mathbb{F}^{\mu\nu}\right] = \mathrm{Tr}\Big[\Big(\partial_\mu \mathbb{A}_\nu - \partial_\nu \mathbb{A}_\mu + \tfrac{ig_c}{\hbar c}[\mathbb{A}_\mu, \mathbb{A}_\nu]\Big)\Big(\partial^\mu \mathbb{A}^\nu - \partial^\nu \mathbb{A}^\mu + \tfrac{ig_c}{\hbar c}[\mathbb{A}^\mu, \mathbb{A}^\nu]\Big)\Big]$$

$$= \partial_{[\mu}\mathbb{A}_{\nu]}\partial^{[\mu}\mathbb{A}^{\nu]} + \frac{2ig_c}{\hbar c}\,\partial_{[\mu}\mathbb{A}_{\nu]}[\mathbb{A}^\mu, \mathbb{A}^\nu] - \frac{g_c^2}{\hbar^2 c^2}[\mathbb{A}_\mu, \mathbb{A}_\nu][\mathbb{A}^\mu, \mathbb{A}^\nu]$$





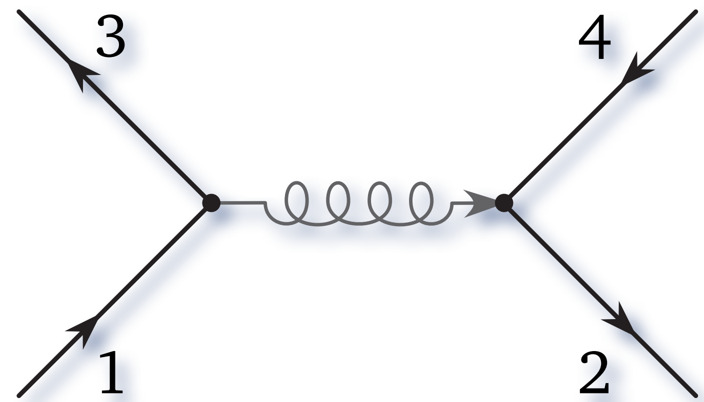


- …where the $SU(3)_c$ trace is implied.

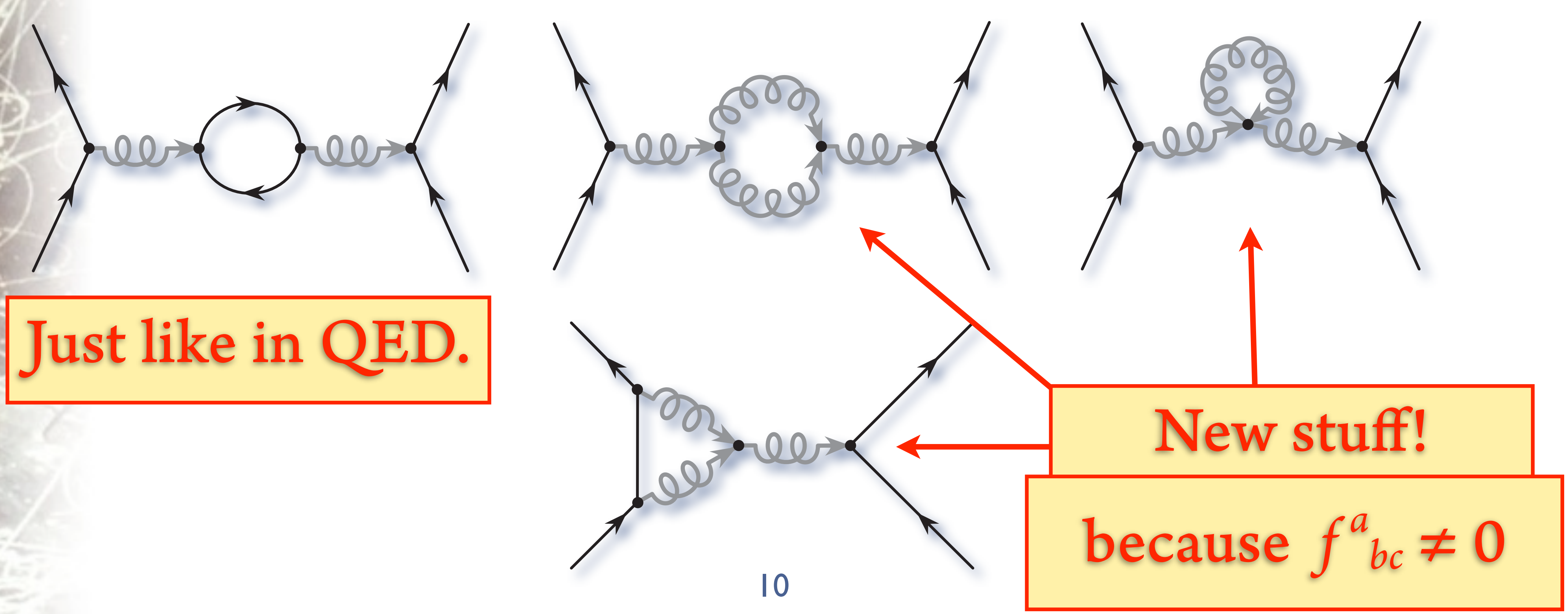
Renormalization in QCD as compared to QED

Gluon loops and non-commutativity

- Owing to these 3-gluon and 4-gluon vertices,



- is being corrected by:



Renormalization in QCD as compared to QED

Non-decoupling in QCD

- Although it is again possible to use the gauge transformation
- $A^a{}_\mu \simeq A^a{}_\mu - c(\partial_\mu \lambda^a)$ with $\lambda^a = -\int \mathrm{d}t\, \Phi^a$,
- … this does not eliminate Φ^a:

$$\mathrm{Tr}\left[\mathbb{F}_{\mu\nu}\mathbb{F}^{\mu\nu}\right] \supset \frac{2g_c}{\hbar c}\left(\dot{\vec{A}}^a - \vec{\nabla}\Phi^a\right) f_{abc}\, \Phi^b (c\vec{A}^c)$$

No decoupling in QCD!

- … nor does $A^a{}_3$ decouple from $A^a{}_1$ and $A^a{}_2$.
- Therefore,
  - owing to the non-abelian nature of SU(3)c,
  - manifested by gluon-gluon couplings & “new” Feynman graphs
- … QCD amplitudes inextricably include non-physical gauge potential components.
- This necessarily violates unitarity.

…but can be “cured” by introducing Fadeev-Popov ghosts & BRST symmetry.

Renormalization in QCD as compared to QED

The SU(n) running coupling parameter

- Suffice it here to merely cite the leading-log result:

$$\alpha_{s,R}(|\mathrm{q}^2|) \approx \frac{\alpha_{s,R}(\mu^2 c^2)}{1 + \frac{\alpha_{s,R}(\mu^2 c^2)}{3\pi}\frac{11n - 2n_f}{4}\ln\left(\frac{|\mathrm{q}^2|}{\mu^2 c^2}\right)}, \qquad |\mathrm{q}^2| \gg \mu^2 c^2$$

- Here,
  - n = number of colors [$n = 3$ for $SU(3)_c$]
  - n_f = number of n-color fermions [$n_f = 6$ flavors of 3-color quarks for q > 171.3 GeV (top quark mass; for lower energies $n_f < 6$]
- This is ***unlike*** the electromagnetic one:

$$\alpha_{e,R}(|\mathrm{q}^2|) \approx \frac{\alpha_{e,R}(0)}{1 - \frac{\alpha_{e,R}(0)}{3\pi}\ln\left(\frac{|\mathrm{q}^2|}{m_e^2 c^2}\right)}, \qquad |\mathrm{q}^2| \gg m_e^2 c^2$$

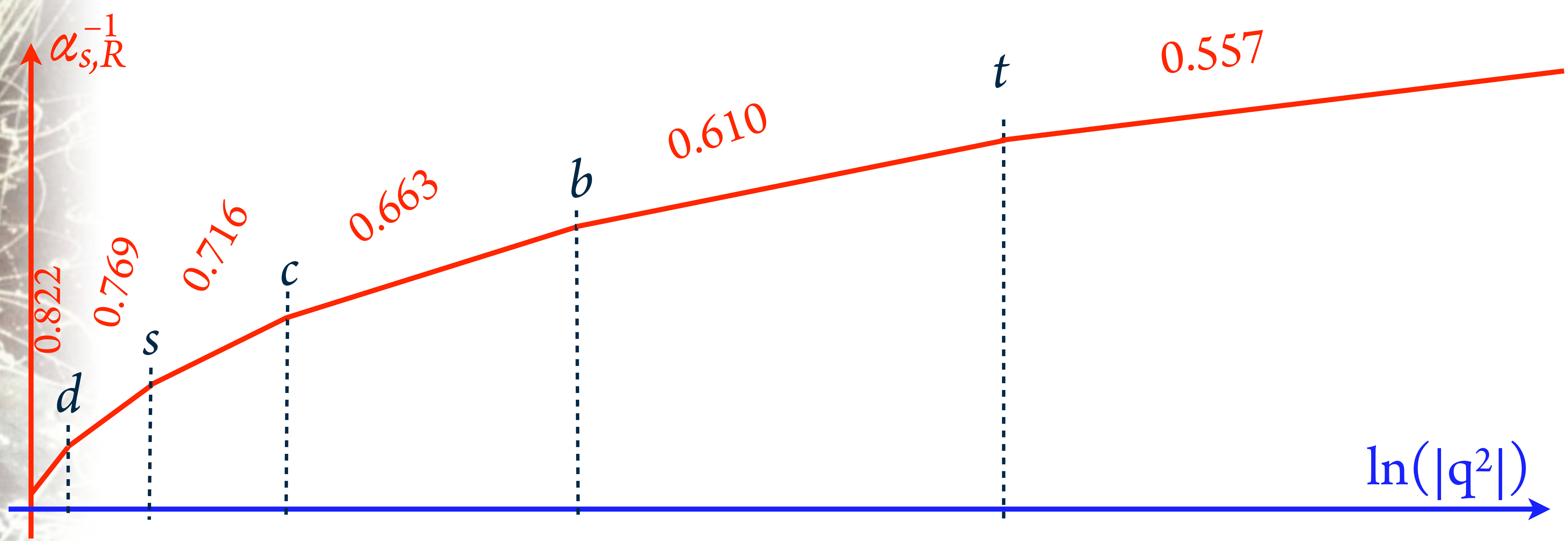
Renormalization in QCD as compared to QED

The SU(n) running coupling parameter

- Suffice it here to merely cite the leading-log result:

$$\alpha_{s,R}(|q^2|) \approx \frac{\alpha_{s,R}(\mu^2 c^2)}{1 + \frac{\alpha_{s,R}(\mu^2 c^2)}{3\pi} \frac{11n - 2n_f}{4} \ln\left(\frac{|q^2|}{\mu^2 c^2}\right)}, \qquad |q^2| \gg \mu^2 c^2$$

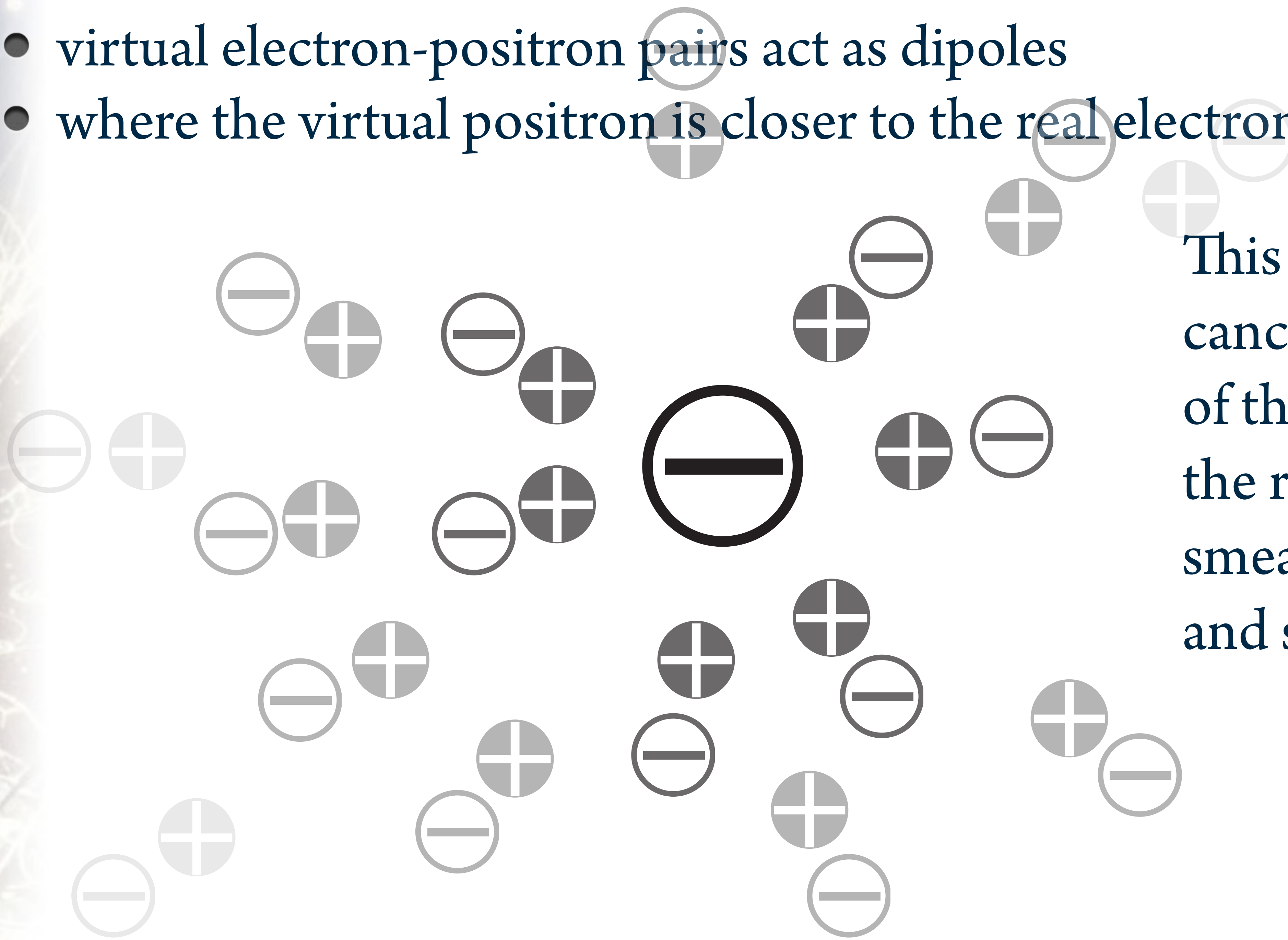
- On a logarithmic scale, $\alpha_{s,R}$ then looks like:



Screening of charge

Fermion-antifermion vacuum polarization

- In QED, around a real electron,
  - virtual electron-positron pairs act as dipoles
  - where the virtual positron is closer to the real electron



This effectively cancels out some of the charge of the real electron, smearing it out and screening it.

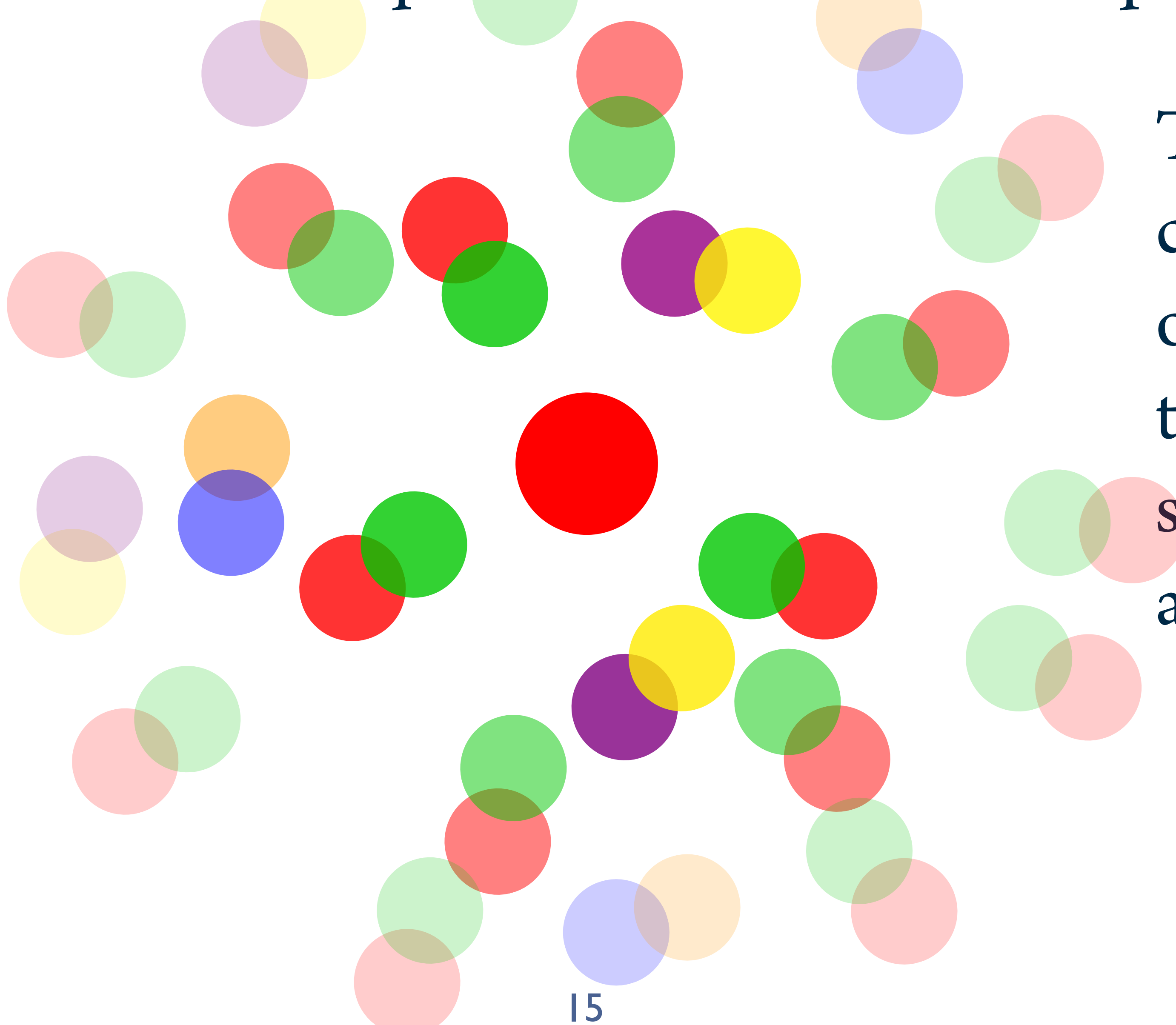
Screening of charge

Fermion-antifermion vacuum polarization

- Just as in QED, around a real quark,
  - virtual quark-antiquark pairs act as "dipoles"
  - where the virtual antiquark is closer to the real quark

The red-antired pairs are polarized

Other pairs are not polarized



This effectively cancels out some of the color of the real quark, smearing it out and screening it.

Non-abelian anti-screening of charge

Non-abelian Gauss's law

- But, in QCD the gluons contribute too!
- Recall:

$$\left(D_\mu F^{a\,\mu\nu} = \partial_\mu F^{a\,\mu\nu} - \tfrac{g_c}{\hbar c} f_{bc}{}^{a} A^b_\mu F^{c\,\mu\nu} \right) = j^{a\,\nu}_{(q)}$$

- where the $\nu = 0$ component is the quark color density.
- Define:

$$\vec{E}^a := \hat{e}_i F^{a\,i0}, \qquad \rho^a_{(q)} := j^{a\,0}_{(q)}, \qquad \vec{A}^a := -\hat{e}^i A^a_i$$

- …and obtain:

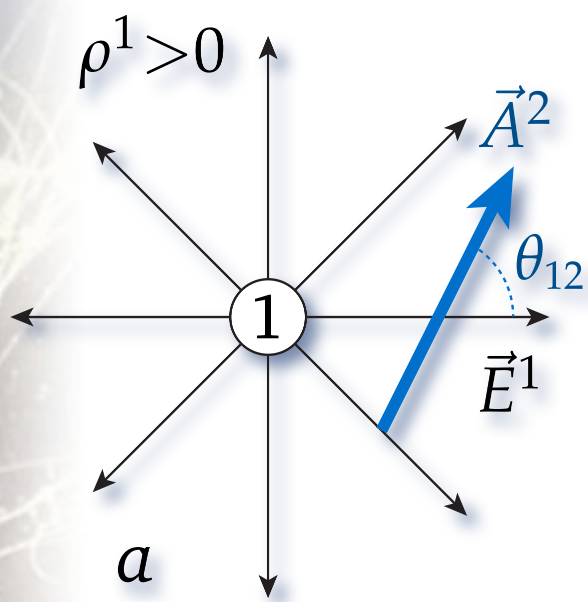
$$\vec{\nabla}\cdot\vec{E}^a = \rho^a_{(q)} - \tfrac{g_c}{\hbar c} f^a{}_{bc} \vec{A}^b \cdot \vec{E}^c$$

- where the nonlinear coupling (owing to the non-abelian structure) serves as an additional (*gluonic*) source of the chromo-electric field.

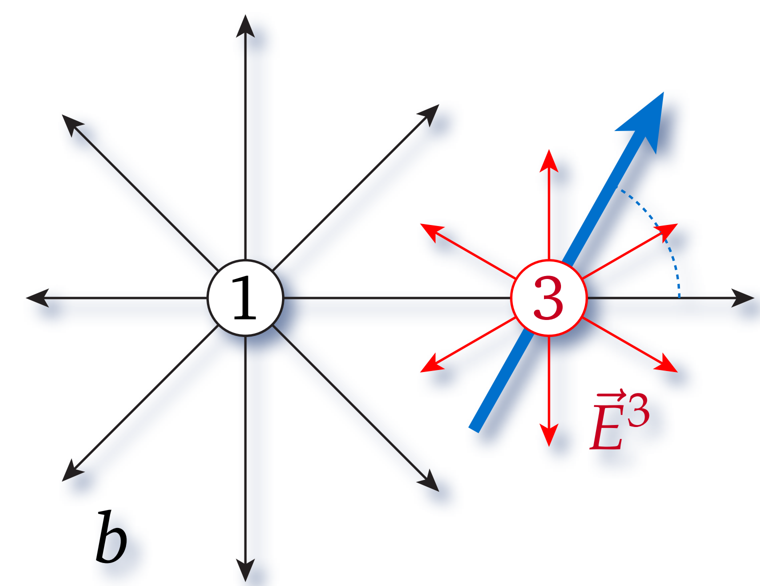
Non-abelian anti-screening of charge

Non-abelian Gauss's law

- Following Peskin & Schroeder, consider a color-1 quark, and a color-2 virtual gluon 3-vector potential:



The color-1 chromo-electric field from the quark, together with the virtual color-2 potential act as a source for the color-3 chromo-electric field.



$$\vec{\nabla}\cdot\vec{E}^3 = -\frac{g_c}{\hbar c} f^3{}_{21}\vec{A}^2\cdot\vec{E}^1 = -\frac{g_c}{\hbar c}(-1)|\vec{A}^2||\vec{E}^1|(\cos\theta_{12} = +\tfrac{1}{2})$$

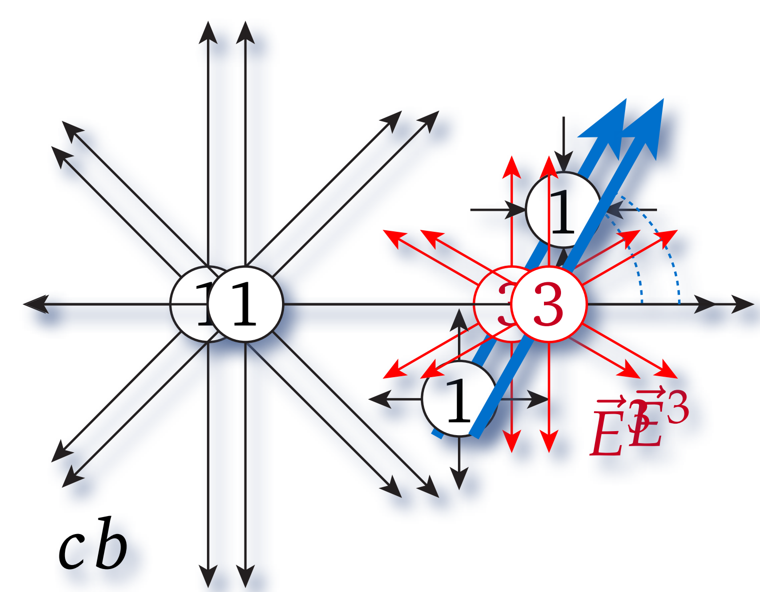
$$= +\frac{g_c}{2\hbar c}|\vec{A}^2||\vec{E}^1|$$

Non-abelian anti-screening of charge

Non-abelian Gauss's law

- Following Peskin & Schroeder, consider a color-1 quark, and a color-2 virtual gluon 3-vector potential:

The coupling of color-3 chromo-electric field and the color-2 gauge potential act as a ***dipole*** of color-1 sources.



$$\vec{\nabla}\cdot\vec{E}^1 = -\frac{g_c}{\hbar c} f^1{}_{23}\vec{A}^2\cdot\vec{E}^3 = -\frac{g_c}{\hbar c}(+1)|\vec{A}^2||\vec{E}^3|\cos\theta_{32},$$

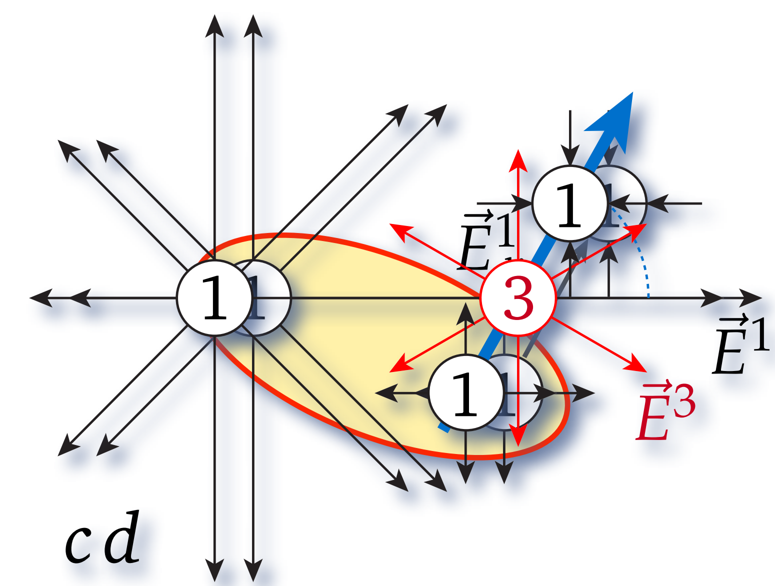
where $\cos\theta_{32}$ is positive SW from the color-3 "source", and negative on the other, NE side of it.

Non-abelian anti-screening of charge

Non-abelian Gauss's law

- Following Peskin & Schroeder, consider a color-1 quark, and a color-2 virtual gluon 3-vector potential:

For clarity, focus on the color-1 sources only:



The virtual color-1 dipole (mimicked by the nonlinear coupling of the virtual color-2 gluon) does not screen the color-1 of the original (quark) source, but anti-screens (reinforces) it.

Non-abelian anti-screening of charge

The Landau pole & dimensional transmutation

- The renormalized coupling parameter

$$\alpha_{s,R}(|\mathrm{q}^2|) \approx \frac{\alpha_{s,R}(\mu^2 c^2)}{1 + \frac{\alpha_{s,R}(\mu^2 c^2)}{3\pi}\frac{11n-2n_f}{4}\ln\left(\frac{|\mathrm{q}^2|}{\mu^2 c^2}\right)}, \qquad |\mathrm{q}^2| \gg \mu^2 c^2,$$

- depends on two parameters:
  - the "reference" mass parameter μ,
  - the value of $\alpha_{s,R}$ at the 4-momentum $\sqrt{|\mathrm{q}^2|} = \mu c$.
- We may instead introduce a mass scale, Λ_{QCD}:

$$\ln(\Lambda^2_{QCD}) := \ln(\mu^2 c^2) - \frac{12\pi}{(11n-2n_f)\alpha_{s,R}(\mu^2 c^2)},$$

$$\alpha_{s,R}(|\mathrm{q}^2|) \approx \frac{12\pi}{(11n-2n_f)\ln\left(\frac{|\mathrm{q}^2|}{\Lambda^2_{QCD}}\right)}$$

diverges at Λ_{QCD}

…where perturbative computations fail…

The effective QCD potential revisited

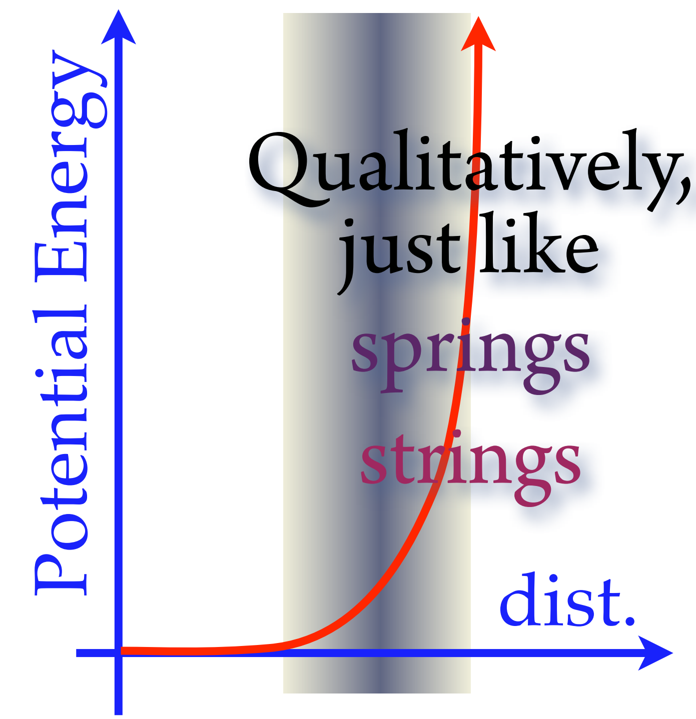
Asymptotic (ultraviolet) freedom

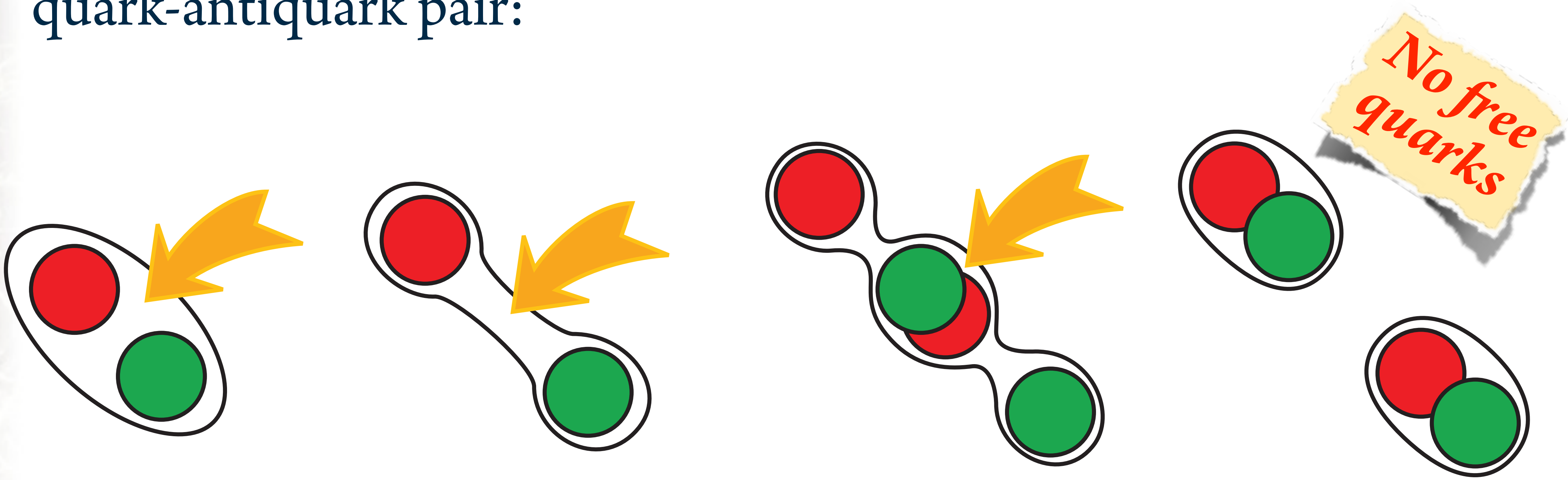
- The magnitude of $\alpha_{s,R}(|q^2|)$ decreases as $|q^2|$ increases,
- *i.e.*, as the distance of interaction shrinks.
- That is, the strength of the strong interaction decreases to zero with the distance between the two interacting quarks.
- This was dubbed "asymptotic freedom"
  - counterpoint to the fact that quarks are confined within hadrons
  - quarks cease to be "held" by the strong interaction…
  - …when they don't try to leave the hadron.
  - … Just like the restoring force of a spring.
- The effective potential describing their interaction is then approximately flat (constant) at small distance.

The effective QCD potential revisited

Infrared confinement

- In turn, when $\sqrt{|q^2|} \ll \mu c$, or $\sqrt{|q^2|} \searrow \Lambda_{QCD}$,
- … all perturbative computations fail.
- Experimentally, separating (anti)quarks
  - to larger and larger distances
  - requires larger and larger $\sqrt{|q^2|}$,
  - … which eventually creates a quark-antiquark pair:





Thanks!

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