(Fundamental) Physics of Elementary Particles

QCD: quantum chromodynamics & Feynman rules; Quark-(anti)quark interaction & color antisymmetrization;

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Fundamental Physics of Elementary Particles

Program

- Concrete QCD computations
  - Feynman rules
  - Gluon loops & interactions
    - nonlinearity
    - gauge conditions
  - Quark-quark interaction
    - Color factor computation
    - $(qq)_{\mathbf{3^*}}$ *vs.* $(qq)_{\mathbf{6}}$
    - $f_c(\mathbf{3^*}|\mathbf{3^*}), f_c(\mathbf{3^*}'|\mathbf{3^*}), f_c(\mathbf{6}|\mathbf{3^*}), f_c(\mathbf{3^*}|\mathbf{6}), f_c(\mathbf{6}|\mathbf{6}), f_c(\mathbf{6}'|\mathbf{6})$
  - Quark-antiquark interaction
    - Color factor computation
    - $(qq^*)_{\mathbf{1}}$ *vs.* $(qq^*)_{\mathbf{8}}$
    - $f_c(\mathbf{1}|\mathbf{1}), f_c(\mathbf{8}|\mathbf{1}), f_c(\mathbf{8}|\mathbf{8}), f_c(\mathbf{8}'|\mathbf{8})$
- Conclusion: $SU(3)_c$ formalism

But, First and Foremost

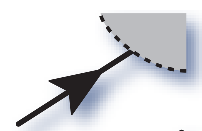
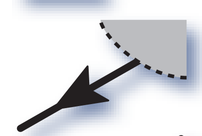
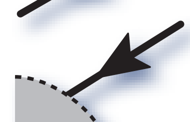
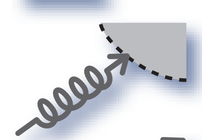
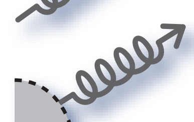
Communication

- When reporting errors in a 344-page document…
- Help locating the error:
  - specify page, paragraph & line, table, figure or equation
  - For example: p. 123, P. 3, l. 2 (page 123, paragraph 3, line 2)
  - Eq. (4.1), Figure 4.1, Table 4.1, …
- State the error/typo:
  - Compare, for example, "Te to the paragraph after 4.50" with
  - "Change 'Since te…' to 'Since the…' four lines after Eq. (4.50)."
  - The latter version allows for an effective electronic search.
- Itemize the suggested corrections.
  - A concatenation of several sentences (even if written clearly)
  - …runs together. (Remember: I need to be able to tell them apart.)

Concrete QCD Computations

Feynman rules

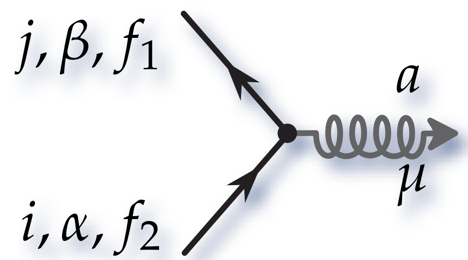
- 1. Notation:
  - 4-momenta: external = $p_1, p_2, \ldots$, internal = $q_1, q_2, \ldots$
  - Orientations:
    - For a spin-½ particle, with 4-momentum
    - For a spin-½ antiparticle, against 4-momentum
    - Gluon lines: external (real) with time, internal (virtual) = arbitrary
  - Polarizations:

Spin-½ quark	incoming		$u^s_f \chi^\alpha$	s = spin projection $= \uparrow, \downarrow$
	outgoing		$\overline{u}_{f,s}\,\chi^\dagger_\alpha$	α = quark color $= r, y, b$ f = quark flavor: $u, d, s, \ldots$
Spin-½ antiquark	incoming		$\overline{v}_{f,s}\,\chi^\dagger_\alpha$	($\cong$ spin-½ quark, travels backwards in time)
	outgoing		$v^s_f \chi^\alpha$	
Gluon	incoming		$\epsilon^\mu \chi^a$	$\epsilon^\mu p_\mu = 0$ and $\epsilon^0 = 0$
	outgoing		$\epsilon^{\mu *} \chi^{a *}$	

Concrete QCD Computations

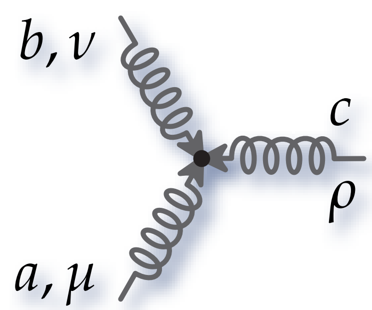
Feynman rules

- 2. Vertices
  - Quark-gluon:



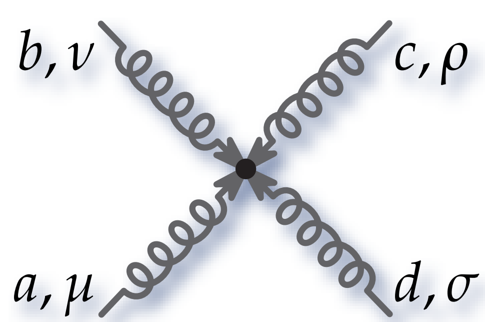
$$\rightarrow \quad -ig_c \boldsymbol{\gamma}^\mu \, \delta^{f_1}_{f_2} \, (\tfrac{1}{2}\lambda_a)^\beta{}_\alpha$$

  - 3-gluons:



$$\rightarrow \quad -g_c f^{abc}[\eta_{\mu\nu}(k_1-k_2)_\rho + \eta_{\nu\rho}(k_2-k_3)_\mu + \eta_{\rho\mu}(k_3-k_1)_\nu]$$

  - 4-gluons:

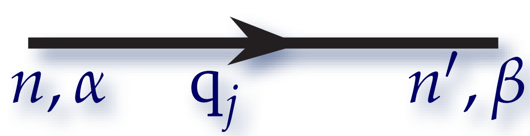


$$\rightarrow \quad -ig_c^2[f^{abe} f^{cd}{}_e(\eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\rho}\eta_{\nu\sigma}) + f^{ace} f^{db}{}_e(\eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}) + f^{ade} f^{bc}{}_e(\eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\rho}\eta_{\nu\sigma})]$$

Concrete QCD Computations

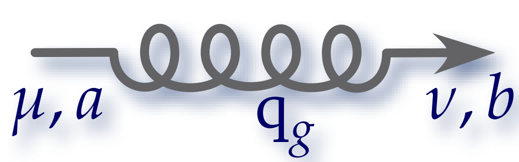
Feynman Rules

- 3. Propagators = internal lines
  - Quarks:



$$\rightarrow \quad \frac{i\delta^{n,n'}\delta^{\beta}_{\alpha}}{\not{q}_j - m_j c} = i\delta^{n,n'}\delta^{\beta}_{\alpha}\frac{\not{q}_j + m_j c \mathbb{1}}{q_j^2 - m_j^2 c^2},$$

  - Gluons:



$$\rightarrow \quad -i\frac{\eta_{\mu\nu}}{q_g^2}\delta^{ab}$$

  - Recall: internal lines represent virtual particles that are off-shell.
- 4. 4-momentum conservation
  - Assign to each vertex $(2\pi)^4\,\delta^4(\Sigma_j\,k_j)$
- 5. Integrate over all internal momenta: $(2\pi)^{-4}\int d^4 q_j$
- 6. Read off: $-i\,\mathfrak{M}\,(2\pi)^4\,\delta^4(\Sigma_j\,p_j)$

Concrete QCD Computations

Feyman rules

- 7. Each fermion loop = one (–1) factor
- 8. Amplitudes for partial processes that are related by an exchange of an odd pair of fermions have a relative – sign.
- As before, we can order the Feynman diagrams by:
  - counting the orders of g_c,
  - and counting loops.
- These amplitudes cannot be used as in electromagnetism,
- … because quarks do not appear as free particles.
- Nevertheless, they *can* indicate *relative* probabilities,
- …somewhat akin to applying the Wigner-Eckardt theorem.

$$\frac{\sigma_{\text{proces 1}}}{\sigma_{\text{proces 2}}} = \frac{|\mathfrak{M}_1|^2}{|\mathfrak{M}_2|^2} = \frac{|(\text{spin})_1 \cdot (\text{isospin})_1 \cdot (\text{color})_1 \cdot (\text{other})_1|^2}{|(\text{spin})_2 \cdot (\text{isospin})_2 \cdot (\text{color})_2 \cdot (\text{other})_2|^2}$$

Concrete QCD Computations

Gluon loops & interactions

- The gluon Lagrangian involves

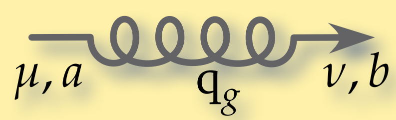
$$\mathbb{F}_{\mu\nu} := \frac{\hbar c}{i g_c}[\partial_\mu + \frac{i g_c}{\hbar c}\mathbb{A}_\mu, \partial_\nu + \frac{i g_c}{\hbar c}\mathbb{A}_\nu] = \partial_\mu \mathbb{A}_\nu - \partial_\nu \mathbb{A}_\mu + \frac{i g_c}{\hbar c}[\mathbb{A}_\mu, \mathbb{A}_\nu],$$

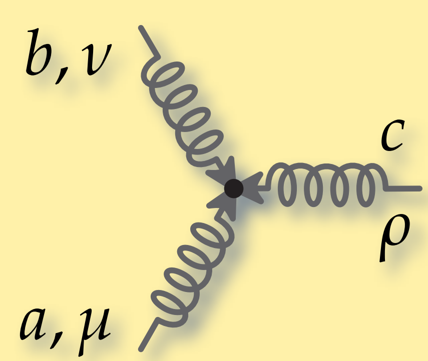
$$= \partial_{[\mu}\mathbb{A}_{\nu]} + \frac{i g_c}{\hbar c}[\mathbb{A}_\mu, \mathbb{A}_\nu]$$

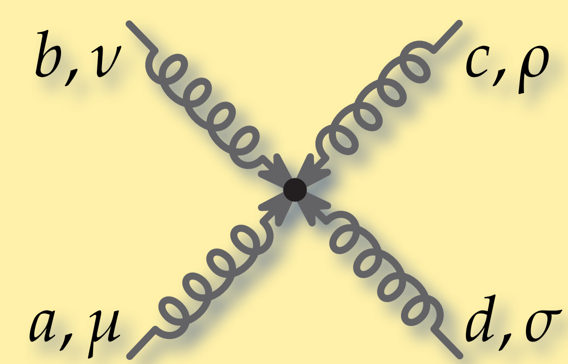
- which when squared produces:

$$\mathbb{F}_{\mu\nu}\mathbb{F}^{\mu\nu} = \left(\partial_{[\mu}\mathbb{A}_{\nu]} + \frac{i g_c}{\hbar c}[\mathbb{A}_\mu, \mathbb{A}_\nu]\right)\left(\partial^{[\mu}\mathbb{A}^{\nu]} + \frac{i g_c}{\hbar c}[\mathbb{A}^\mu, \mathbb{A}^\nu]\right)$$

$$= \partial_{[\mu}\mathbb{A}_{\nu]}\partial^{[\mu}\mathbb{A}^{\nu]} + \frac{2 i g_c}{\hbar c}\partial_{[\mu}\mathbb{A}_{\nu]}[\mathbb{A}^\mu, \mathbb{A}^\nu] - \frac{g_c^2}{\hbar^2 c^2}[\mathbb{A}_\mu, \mathbb{A}_\nu][\mathbb{A}^\mu, \mathbb{A}^\nu]$$





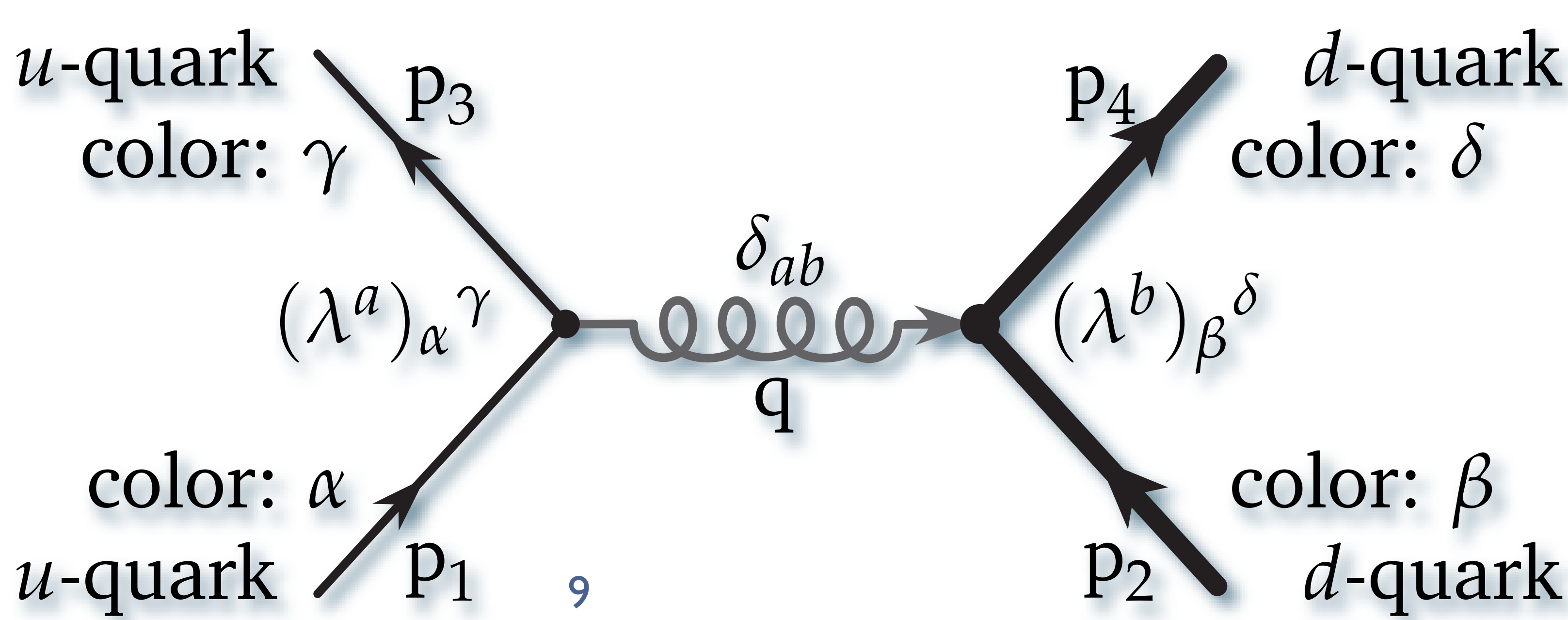


keep in mind, for later…

Concrete QCD Computations

Quark-quark interaction

- Consider a concrete process, such as $p^{+}+n^{0} \longrightarrow p^{+}+n^{0}$.
- Analyze as $(uud)+(udd) \longrightarrow (uud)+(udd)$,
  - where the strong interaction is dominating
  - so consider quark-quark interactions
  - $(u+u)\longrightarrow(u+u) \approx (u+d)\longrightarrow(u+d) \approx (d+d)\longrightarrow(d+d)$
  - …up to corrections $O(|m_u - m_d|/(m_u+m_d)) \approx 33\%$…
  - Then also: $(p+p)\longrightarrow(p+p) \approx (p+n)\longrightarrow(p+n) \approx (n+n)\longrightarrow(n+n)$,
  - …as Heisenberg initially observed, introducing isospin.
- Consider:



Monday, November 7, 11

Concrete QCD Computations

Quark-quark interaction

- The amplitude computation differs from that in electromagnetism only by color factors:

$$\mathfrak{M}_{u+d\to u+d} = -\frac{g_s^2}{2}\frac{1}{\mathrm{q}^2}\left[\overline{u}_3\,\boldsymbol{\gamma}^{\mu}\,u_1\right]\left[\overline{u}_4\,\boldsymbol{\gamma}_{\mu}\,u_1\right]\left(\chi_3^{\dagger}\,\boldsymbol{\lambda}^{a}\,\chi_1\right)\left(\chi_4^{\dagger}\,\boldsymbol{\lambda}_{a}\,\chi_2\right),$$

old stuff | new stuff

- Re-use the electromagnetism computation, with the $g_e \longrightarrow g_c$ replacement,
- Compute the color factor, $f_c(3,4|1,2) = \frac{1}{4}(\chi_3^{\dagger}\boldsymbol{\lambda}^{a}\chi_1)(\chi_4^{\dagger}\boldsymbol{\lambda}_{a}\chi_2)$
- …for all the different possible cases.
- Since the EM amplitude would have given $\frac{1}{4\pi\epsilon_0}\frac{e^2}{r} = \frac{\alpha_e\hbar c}{r}$
- …the QCD amplitude will yield

$$V_{qq}(r) = f_c\,\frac{\alpha_s\hbar c}{r}$$

remaining to be determined

Concrete QCD Computations

Quark-quark interaction

- So, consider computing

$$f_c(3,4|1,2) = \tfrac{1}{4}(\chi_3^\dagger \boldsymbol{\lambda}^a \chi_1)(\chi_4^\dagger \boldsymbol{\lambda}_a \chi_2) \;=\; \tfrac{1}{4} \underbrace{\chi^\dagger_{3\gamma}\,\chi^\dagger_{4\delta}}_{\text{out}} (\lambda^a)_\alpha{}^\gamma\,(\lambda_a)_\beta{}^\delta \underbrace{\chi_1^\alpha\,\chi_2^\beta}_{\text{in}}.$$

- … for the different possible two-quark in- and out-states.
- Use the color—tensor—matrix notation translations:

$$\chi^r \leftrightarrow \delta^\alpha_1 \leftrightarrow \begin{bmatrix}1\\0\\0\end{bmatrix}, \quad \chi^y \leftrightarrow \delta^\alpha_2 \leftrightarrow \begin{bmatrix}0\\1\\0\end{bmatrix}, \quad \chi^b \leftrightarrow \delta^\alpha_3 \leftrightarrow \begin{bmatrix}0\\0\\1\end{bmatrix}.$$

- Use also that

$$(\mathbf{3}\otimes\mathbf{3})_A = \mathbf{3}^* \quad \chi_1^{[\alpha}\chi_2^{\beta]} := \tfrac{1}{\sqrt{2}}(\delta^\alpha_\gamma\delta^\beta_\delta - \delta^\beta_\gamma\delta^\alpha_\delta)\,\chi_1^\gamma\chi_2^\delta \qquad \alpha\neq\beta,\ \alpha,\beta = 1,2,3$$

$$(\mathbf{3}\otimes\mathbf{3})_S = \mathbf{6} \quad \chi_1^{(\alpha}\chi_2^{\beta)} := \left\{\begin{matrix}\tfrac{1}{\sqrt{2}}(\delta^\alpha_\gamma\delta^\beta_\delta + \delta^\beta_\gamma\delta^\alpha_\delta)\\ \delta^\alpha_\gamma\delta^\beta_\delta\end{matrix}\right\}\chi_1^\gamma\chi_2^\delta \quad \left\{\begin{matrix}\alpha\neq\beta,\\ \alpha=\beta,\end{matrix}\right. \quad \alpha,\beta = 1,2,3$$

Concrete QCD Computations

Some $SU(3)_c$ representations

- The fundamental representation
  - denoted **3**, for a complex 3-dimensional vector space,
  - … spanned by (t^1, t^2, t^3): $c_1 t^1 + c_2 t^2 + c_3 t^3$, *i.e.*, $\mathbb{C}^3 = \{c_1, c_2, c_3\}$
  - … which are transformed one into another by $SU(3)_c$.
- The antisymmetric product = antisymmetric rank-2 tensor
  - may be identified with **3***: $t_\alpha = \varepsilon_{\alpha\beta\gamma}\, t^{[\beta\gamma]}$
  - represented by linear combinations of $t^{[12]}$, $t^{[13]}$ and $t^{[23]}$,
  - … which are transformed one into another by $SU(3)_c$.
- The symmetric product = symmetric rank-2 tensor
  - may be identified with **6**:
  - represented by linear combinations of
  - $t^{(11)}$, $t^{(22)}$, $t^{(33)}$, $t^{(12)}$, $t^{(13)}$ and $t^{(23)}$,
  - … which are transformed one into another by $SU(3)_c$.

Concrete QCD Computations

Quark-quark interaction

- Cases of $f_c(3{,}4|1{,}2)$ to be examined
- where $(1{,}2)$ and $(3{,}4)$ range over:
  - two copies of the same element of $\mathbf{3^*}$: $f_c(\mathbf{3^*}|\mathbf{3^*})$, e.g., [13]|[13];
  - two different elements of $\mathbf{3^*}$: $f_c(\mathbf{3^{*\prime}}|\mathbf{3^*})$, e.g., [12]|[13];
  - one element of $\mathbf{3^*}$ & one of $\mathbf{6}$: $f_c(\mathbf{6}|\mathbf{3^*})$, e.g.,
    (11)|[12], (33)|[12], (13)|[13] & (12)|[13];
  - two copies of the same element of $\mathbf{6}$: $f_c(\mathbf{6}|\mathbf{6})$, e.g., (11)|(11);
  - two different elements of $\mathbf{6}$: $fv(\mathbf{6'}|\mathbf{6})$. e.g., (11)|(33).
- There are plenty of other choices, but they may all be transformed into one of the ***eight*** above, by $SU(3)_c$.
- It then suffices to work with the above eight representatives.

Concrete QCD Computations

Quark-quark interaction

- Consider a representative of $f(\mathbf{3}^*|\mathbf{3}^*)$:

$$\left\{\tfrac{1}{4}\,(\chi^\dagger_{3\gamma}\,\chi^\dagger_{4\delta})_{\mathbf{3}}\,(\lambda^a)_\alpha{}^\gamma\,(\lambda_a)_\beta{}^\delta\,(\chi^\alpha_1\,\chi^\beta_2)_{\mathbf{3}^*}\right\}$$

$$\supset \frac{1}{4}\,\frac{1}{\sqrt{2}}\left(\delta^1_\gamma\delta^3_\delta - \delta^1_\delta\delta^3_\gamma\right)(\lambda^a)_\alpha{}^\gamma\,(\lambda_a)_\beta{}^\delta\,\frac{1}{\sqrt{2}}\left(\delta^\alpha_1\delta^\beta_3 - \delta^\beta_1\delta^\alpha_3\right),$$

$$= \frac{1}{8}\left[\lambda^a{}_1{}^1\,\lambda_{a3}{}^3 - \lambda^a{}_3{}^1\,\lambda_{a1}{}^3 - \lambda^a{}_1{}^3\,\lambda_{a3}{}^1 + \lambda^a{}_3{}^3\,\lambda_{a1}{}^1\right]$$

$$= \frac{1}{4}\left[\lambda^a{}_1{}^1\,\lambda_{a3}{}^3 - \lambda^a{}_3{}^1\,\lambda_{a1}{}^3\right].$$

$$\boldsymbol{\lambda}_1 = \begin{bmatrix}0&1&0\\1&0&0\\0&0&0\end{bmatrix},\quad \boldsymbol{\lambda}_2 = \begin{bmatrix}0&-i&0\\i&0&0\\0&0&0\end{bmatrix},\quad \boldsymbol{\lambda}_3 = \begin{bmatrix}1&0&0\\0&-1&0\\0&0&0\end{bmatrix},\quad \boldsymbol{\lambda}_4 = \begin{bmatrix}0&0&1\\0&0&0\\1&0&0\end{bmatrix},$$

$$\boldsymbol{\lambda}_5 = \begin{bmatrix}0&0&-i\\0&0&0\\i&0&0\end{bmatrix},\quad \boldsymbol{\lambda}_6 = \begin{bmatrix}0&0&0\\0&0&1\\0&1&0\end{bmatrix},\quad \boldsymbol{\lambda}_7 = \begin{bmatrix}0&0&0\\0&0&-i\\0&i&0\end{bmatrix},\quad \boldsymbol{\lambda}_8 = \frac{1}{\sqrt{3}}\begin{bmatrix}1&0&0\\0&1&0\\0&0&-2\end{bmatrix}.$$

$$= \frac{1}{4}\left[\lambda^8{}_1{}^1\,\lambda_{83}{}^3 - \lambda^4{}_3{}^1\,\lambda_{41}{}^3 - \lambda^5{}_3{}^1\,\lambda_{51}{}^3\right]$$

$$= \frac{1}{4}\left[\frac{1}{\sqrt{3}}\cdot\frac{-2}{\sqrt{3}} - 1\cdot 1 - i\cdot(-i)\right] = -\frac{2}{3}.$$

Concrete QCD Computations

Quark-quark interaction

- So: $f_c(\mathbf{3}^*|\mathbf{3}^*)$, represented by $f_c([13]|[13])$, $= -⅔$ (attractive!)
  - Similarly, $f_c(\mathbf{3}^{*\prime}|\mathbf{3}^*)$ represented by $f_c([12]|[13])$,

$$\tfrac{1}{4}\,\tfrac{1}{\sqrt{2}}\left(\delta^1_\gamma\delta^2_\delta - \delta^1_\delta\delta^2_\gamma\right)\,(\lambda^a)_\alpha{}^\gamma\,(\lambda_a)_\beta{}^\delta\,\tfrac{1}{\sqrt{2}}\left(\delta^\alpha_1\delta^\beta_3 - \delta^\beta_1\delta^\alpha_3\right)$$

$$= \tfrac{1}{8}\left[\lambda^a{}_1{}^1\,\lambda_{a3}{}^2 - \lambda^a{}_3{}^1\,\lambda_{a1}{}^2 - \lambda^a{}_1{}^2\,\lambda_{a3}{}^1 + \lambda^a{}_3{}^2\,\lambda_{a1}{}^1\right]$$

$$= \tfrac{1}{4}\left[\lambda^a{}_1{}^1\,\lambda_{a3}{}^2 - \lambda^a{}_3{}^1\,\lambda_{a1}{}^2\right] = 0$$ — not happening!

- $f_c(\mathbf{6}|\mathbf{3}^*)$, represented by $f_c((11)|[12])$, $= 0$
- $f_c(\mathbf{6}|\mathbf{3}^*)$, represented by $f_c((33)|[12])$, $= 0$
- $f_c(\mathbf{6}|\mathbf{3}^*)$, represented by $f_c((13)|[13])$, $= 0$
- $f_c(\mathbf{6}|\mathbf{3}^*)$, represented by $f_c((13)|[13])$, $= 0$

} — not happening!

- $f_c(\mathbf{6}|\mathbf{6})$, represented by $f_c((11)|(11))$, $= +⅓$ (repulsive!)
- $f_c(\mathbf{6}'|\mathbf{6})$, represented by $f_c((11)|(33))$, $= 0$ — not happening!

Concrete QCD Computations

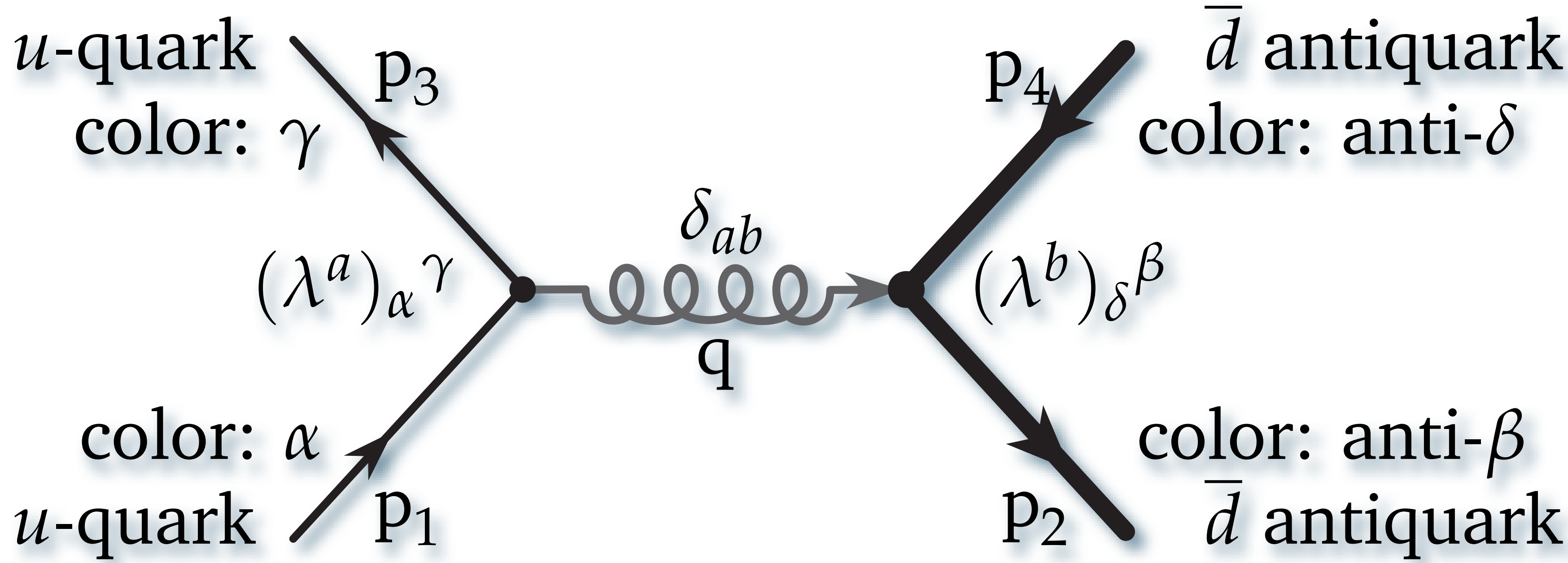
Quark-quark interaction

- To summarize:
- The quark-quark 1-gluon-exchange interaction is
  - ***attractive*** when the quarks' colors are antisymmetrized
  - — and stay in the same particular state,
  - ***repulsive*** when the quarks' colors are symmetrized
  - — and stay in the same particular state,
  - ***forbidden*** (*verboten*) in all other cases.
- More-gluons' exchange interaction ***does*** follow this pattern.
- In a baryon, there are three quarks.
  - For the color of each pair to be antisymmetrized,
  - …the triple color factor has to be fully antisymmetrized.
  - $(\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3})_A = \mathbf{1}$, *i.e.*, $(t^\alpha\, t^\beta\, t^\gamma)_A \propto \varepsilon^{\alpha\beta\gamma}$, which is an $SU(3)$-invariant.
  - Ψbaryon $= [\Psi(\text{space})\cdot\chi(\text{spin})\cdot\chi(\text{flavor})]_S \cdot \chi_A(\text{color})$

Concrete QCD Computations

Quark-antiquark interaction

- A 1-gluon exchange:



- produces the amplitude

$$\mathfrak{M}_{u+\bar{d}\to u+\bar{d}} = -\frac{g_c^2}{4q^2}[\bar{u}_3\boldsymbol{\gamma}^\mu u_1][\bar{v}_2\boldsymbol{\gamma}_\mu v_4](\chi_3^\dagger\,\boldsymbol{\lambda}^a\,\chi_1)(\chi_2^\dagger\,\boldsymbol{\lambda}_a\,\chi_4),$$

- with the color factor

$$f_c(3,\bar{4}|1,\bar{2}) = \tfrac{1}{4}(\chi_3^\dagger\boldsymbol{\lambda}^a\chi_1)(\chi_2^\dagger\boldsymbol{\lambda}_a\chi_4)$$

Concrete QCD Computations

Quark-antiquark interaction

- The incoming and outgoing quarks may now have the colors in the
  - color-singlet ($SU(3)_c$-invariant) state $(\chi_1\chi_2^\dagger)^\alpha{}_\beta = \delta^\alpha_\beta\,\mathring{\boldsymbol{\chi}}$
  - or the (traceless hermitian matrix) color-octet state:

$$\{\chi_{12}{}^\alpha{}_\beta = \sqrt{1+\tfrac{1}{2}\delta^\alpha_\beta}\,(\chi^\alpha_1\chi^\dagger_{2\beta} - \tfrac{1}{\sqrt{3}}\delta^\alpha_\beta\,\mathring{\boldsymbol{\chi}}),\ \ \alpha,\beta = red, yellow, blue = 1,2,3\},$$

$$= \Big\{\sqrt{\tfrac{3}{2}}(\delta^\alpha_1\delta^1_\beta - \mathring{\boldsymbol{\chi}}),\ \sqrt{\tfrac{3}{2}}(\delta^\alpha_2\delta^2_\beta - \mathring{\boldsymbol{\chi}}),\ \sqrt{\tfrac{3}{2}}(\delta^\alpha_3\delta^3_\beta - \mathring{\boldsymbol{\chi}}),$$

$$(\delta^\alpha_1\delta^2_\beta),\ (\delta^\alpha_1\delta^3_\beta),\ (\delta^\alpha_2\delta^1_\beta),\ (\delta^\alpha_2\delta^3_\beta),\ (\delta^\alpha_3\delta^1_\beta),\ (\delta^\alpha_3\delta^2_\beta)\Big\},$$

- Symbolically:
  - $\mathbf{3}\otimes\mathbf{3}^* = \mathbf{1}\oplus\mathbf{8}$
  - $t^\alpha\otimes s_\beta = [\tfrac{1}{3}\,\delta^\alpha{}_\beta\,(t^\gamma s_\gamma)] + [t^\alpha s_\beta - \tfrac{1}{3}\,\delta^\alpha{}_\beta\,(t^\gamma s_\gamma)]$

Concrete QCD Computations

Quark-antiquark interaction

- Since the color charge of an antiquark is ***opposite*** of the color of the corresponding quark,
- …the 1-gluon exchange gives rise to the potential

$$V_{q\bar{q}}(r) = -f_c \frac{\alpha_c \hbar c}{r},$$

- Need to compute $f_c(3,\bar{4}|1,\bar{2})$ for:
  - $f_c(\mathbf{8}|\mathbf{8})$, represented by $f_c({}^{1}{}_{3}|{}^{1}{}_{3})$,
  - $f_c(\mathbf{8}'|\mathbf{8})$, represented by $f_c({}^{3}{}_{1}|{}^{1}{}_{3})$,
  - $f_c(\mathbf{8}|\mathbf{1})$, represented by $f_c({}^{1}{}_{3}|\mathbf{1})$,
  - $f_c(\mathbf{1}|\mathbf{1})$, represented by $f_c(\mathbf{1}|\mathbf{1})$.
- Proceed as before: $\frac{1}{4}\,(\delta^{1}_{\gamma}\delta^{\delta}_{3})\,(\lambda^{a})_{\alpha}{}^{\gamma}\,(\lambda_{a})_{\delta}{}^{\beta}\,(\delta^{\alpha}_{1}\delta^{3}_{\beta}),$

$$= \frac{1}{4}\,\lambda^{a}{}_{1}{}^{1}\,\lambda_{a3}{}^{3} = \frac{1}{4}\,\lambda^{8}{}_{1}{}^{1}\,\lambda_{83}{}^{3} = \frac{1}{4}\,\frac{1}{\sqrt{3}}\,\frac{-2}{\sqrt{3}} = -\frac{1}{6},$$

Concrete QCD Computations

Quark-antiquark interaction

- Obtain:
  - $f_c(\mathbf{8}|\mathbf{8})$, represented by $f_c({}^1{}_3|{}^1{}_3)$, = –⅙ — repulsive!
  - $f_c(\mathbf{8}'|\mathbf{8})$, represented by $f_c({}^3{}_1|{}^1{}_3)$, = 0
  - $f_c(\mathbf{8}|\mathbf{1})$, represented by $f_c({}^1{}_3|\mathbf{1})$, = 0
  - $f_c(\mathbf{1}|\mathbf{1})$, represented by $f_c(\mathbf{1}|\mathbf{1})$:

$$V_{q\bar{q}}(r) = -f_c \frac{\alpha_c \hbar c}{r},$$

$$\tfrac{1}{4}\,(\chi^{\dagger}_{3\gamma}\,\chi^{\delta}_{4})_{\mathbf{1}}\,(\lambda^{a})_{\alpha}{}^{\gamma}\,(\lambda_{a})_{\delta}{}^{\beta}\,(\chi^{\alpha}_{1}\,\chi^{\dagger}_{2\beta})_{\mathbf{1}}$$

$$= \frac{1}{4}\,\frac{1}{\sqrt{3}}(\delta^{1}_{\gamma}\delta^{\delta}_{1} + \delta^{2}_{\gamma}\delta^{\delta}_{2} + \delta^{3}_{\gamma}\delta^{\delta}_{3})\,(\lambda^{a})_{\alpha}{}^{\gamma}\,(\lambda_{a})_{\delta}{}^{\beta}\,\frac{1}{\sqrt{3}}(\delta^{\alpha}_{1}\delta^{1}_{\beta} + \delta^{\alpha}_{2}\delta^{2}_{\beta} + \delta^{\alpha}_{3}\delta^{3}_{\beta}),$$

$$= \frac{1}{12}\,\lambda^{a}{}_{\alpha}{}^{\gamma}\,\lambda_{a\gamma}{}^{\alpha} = \frac{1}{12}\,\delta_{ab}\,\mathrm{Tr}(\boldsymbol{\lambda}^{a}\,\boldsymbol{\lambda}^{b}) = \frac{1}{12}\,\delta_{ab}\,2\delta^{ab} = \frac{1}{6}\,8 = \frac{4}{3}$$

attractive!

- The quark-antiquark 1-gluon exchange potential is:
  - ***attractive*** for in- and out-state color-singlets,
  - ***repulsive*** for in- and out-state (*same!*) color octets,
  - ***forbidden*** (verboten) otherwise.

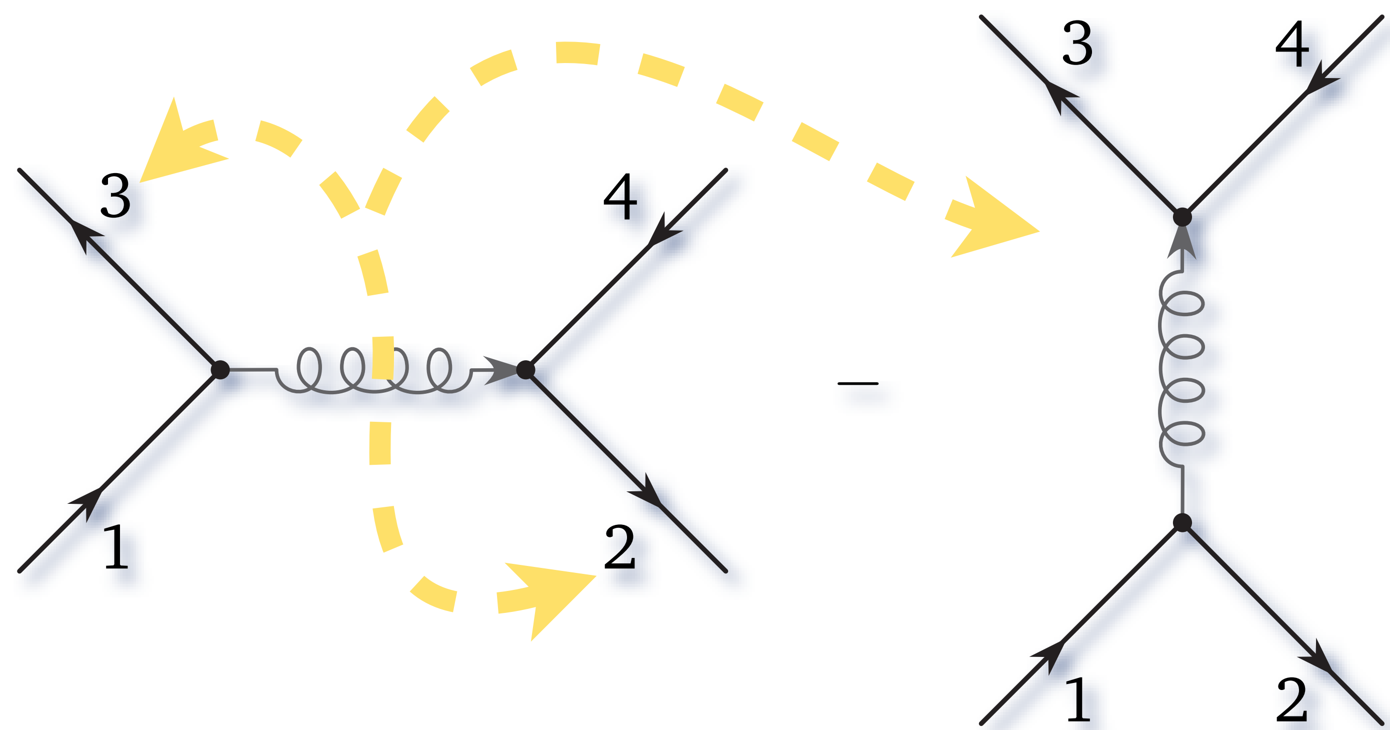
Mesons must be $SU(3)_c$-invariant.

Concrete QCD Computations

Quark-antiquark interaction

- How about the possible (virtual) annihilation + re-creation?

$$\mathfrak{M}_{u+\bar{u}\to u+\bar{u}} =$$



$$\mathfrak{M}_{u+\bar{u}\to u+\bar{u}} = -\frac{g_c^2}{4(\mathrm{p}_1-\mathrm{p}_3)^2}[\bar{u}_3\boldsymbol{\gamma}^\mu u_1][\bar{v}_2\boldsymbol{\gamma}_\mu v_4](\chi_3^\dagger\boldsymbol{\lambda}^a\chi_1)(\chi_2^\dagger\boldsymbol{\lambda}_a\chi_4)$$

$$+\frac{g_c^2}{4(\mathrm{p}_1+\mathrm{p}_2)^2}[\bar{v}_2\boldsymbol{\gamma}^\mu u_1][\bar{u}_3\boldsymbol{\gamma}_\mu v_4](\chi_2^\dagger\boldsymbol{\lambda}^a\chi_1)(\chi_3^\dagger\boldsymbol{\lambda}_a\chi_4),$$

Concrete QCD Computations

Quark-antiquark interaction

- How about the possible (virtual) annihilation + re-creation?
- The color factors are now:
  - $f_c(\mathbf{8}|\mathbf{8})$:

$$\left\{\tfrac{1}{4}\,(\chi^{\dagger}_{3\gamma}\,\chi^{\delta}_{4})_{\mathbf{8}}\,(\lambda^{a})_{\alpha}{}^{\beta}\,(\lambda_{a})_{\delta}{}^{\gamma}\,(\chi^{\alpha}_{1}\,\chi^{\dagger}_{2\beta})_{\mathbf{8}}\right\} \supset \tfrac{1}{4}\,(\delta^{1}_{\gamma}\delta^{\delta}_{3})\,(\lambda^{a})_{\alpha}{}^{\beta}\,(\lambda_{a})_{\delta}{}^{\gamma}\,(\delta^{\alpha}_{1}\delta^{3}_{\beta}),$$

$$= \tfrac{1}{4}\,\lambda^{a}{}_{1}{}^{3}\,\lambda_{a3}{}^{1} = \tfrac{1}{4}\,(\lambda^{4}{}_{1}{}^{3}\,\lambda_{43}{}^{1} + \lambda^{5}{}_{1}{}^{3}\,\lambda_{53}{}^{1}) = \tfrac{1}{4}\,(1\cdot 1 + (-i)\cdot(i)) = \tfrac{1}{2},$$

  - $f_c(\mathbf{8}'|\mathbf{8})$:

$$\left\{\tfrac{1}{4}\,(\chi^{\dagger}_{3\gamma}\,\chi^{\delta}_{4})_{\mathbf{8}'}\,(\lambda^{a})_{\alpha}{}^{\beta}\,(\lambda_{a})_{\delta}{}^{\gamma}\,(\chi^{\alpha}_{1}\,\chi^{\dagger}_{2\beta})_{\mathbf{8}}\right\} \supset \tfrac{1}{4}\,(\delta^{3}_{\gamma}\delta^{\delta}_{1})\,(\lambda^{a})_{\alpha}{}^{\beta}\,(\lambda_{a})_{\delta}{}^{\gamma}\,(\delta^{\alpha}_{1}\delta^{3}_{\beta}),$$

$$= \tfrac{1}{4}\,\lambda^{a}{}_{1}{}^{3}\,\lambda_{a1}{}^{3} = \tfrac{1}{4}(\lambda^{4}{}_{1}{}^{3}\,\lambda_{41}{}^{3} + \lambda^{5}{}_{1}{}^{3}\,\lambda_{51}{}^{3}) = \tfrac{1}{4}\,(1\cdot 1 + (-i)\cdot(-i)) = 0,$$

  - $f_c(\mathbf{1}|\mathbf{1})$:

$$\tfrac{1}{4}\,(\chi^{\dagger}_{3\gamma}\,\chi^{\delta}_{4})_{\mathbf{1}}\,(\lambda^{a})_{\alpha}{}^{\beta}\,(\lambda_{a})_{\delta}{}^{\gamma}\,(\chi^{\alpha}_{1}\,\chi^{\dagger}_{2\beta})_{\mathbf{1}}$$

$$= \tfrac{1}{4}\,\tfrac{1}{\sqrt{3}}(\delta^{1}_{\gamma}\delta^{\delta}_{1} + \delta^{2}_{\gamma}\delta^{\delta}_{2} + \delta^{3}_{\gamma}\delta^{\delta}_{3})\,(\lambda^{a})_{\alpha}{}^{\beta}\,(\lambda_{a})_{\delta}{}^{\gamma}\,\tfrac{1}{\sqrt{3}}(\delta^{\alpha}_{1}\delta^{1}_{\beta} + \delta^{\alpha}_{2}\delta^{2}_{\beta} + \delta^{\alpha}_{3}\delta^{3}_{\beta}),$$

$$= \tfrac{1}{12}\,\lambda^{a}{}_{\alpha}{}^{\alpha}\,\lambda_{a\gamma}{}^{\gamma} = \tfrac{1}{12}\,\mathrm{Tr}(\boldsymbol{\lambda}^{a})\,\mathrm{Tr}(\boldsymbol{\lambda}_{a}) = 0,$$

Concrete QCD Computations

Quark-antiquark interaction

- How about the possible (virtual) annihilation + re-creation?
- The algebraic sum (actually difference) of the two amplitudes is

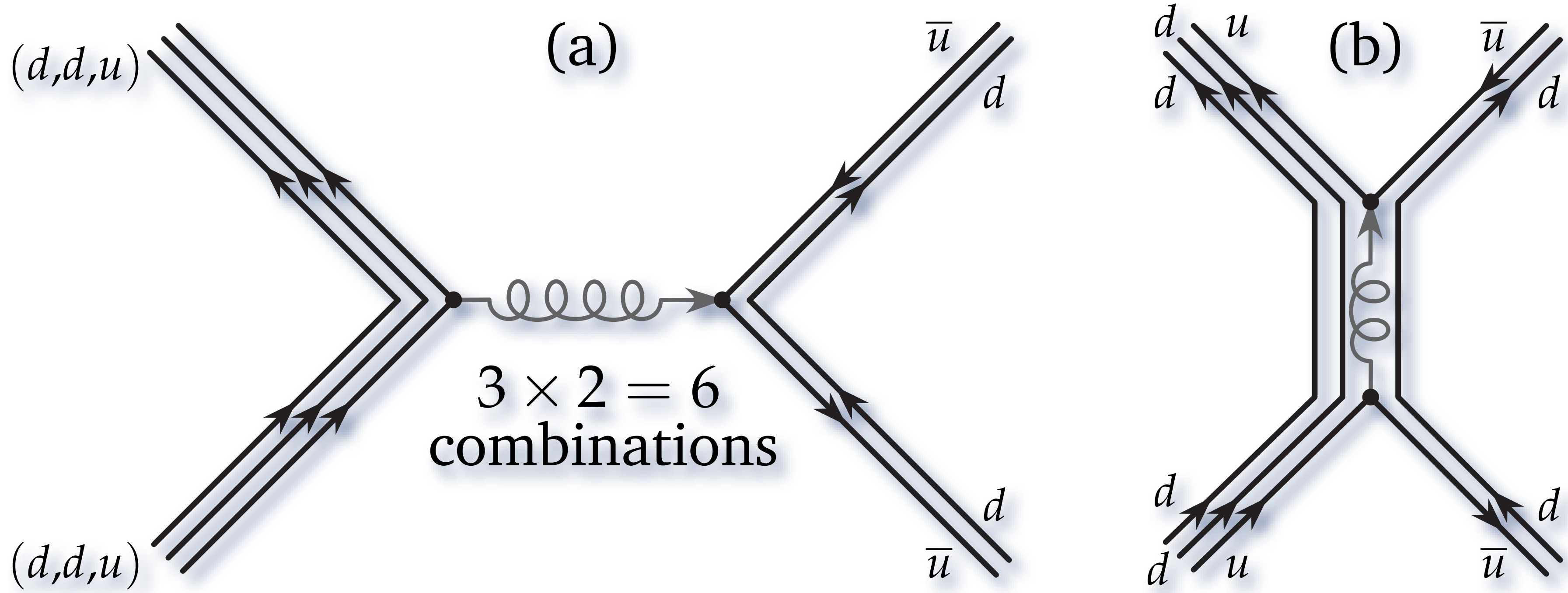
$$\mathfrak{M}_{u+\bar{u}\to u+\bar{u}} = -\frac{g_c^2}{(\mathrm{p}_1-\mathrm{p}_3)^2}\begin{Bmatrix} -\frac{1}{6} \\ +\frac{4}{3} \end{Bmatrix}[\bar{u}_3\boldsymbol{\gamma}^\mu u_1][\bar{v}_2\boldsymbol{\gamma}_\mu v_4] + \frac{g_c^2}{(\mathrm{p}_1+\mathrm{p}_2)^2}\begin{Bmatrix} \frac{1}{2} \\ 0 \end{Bmatrix}[\bar{v}_2\boldsymbol{\gamma}^\mu u_1][\bar{u}_3\boldsymbol{\gamma}_\mu v_4], \quad \text{if} \quad \begin{cases} \boldsymbol{\chi}_{12} \subset \mathbf{8}, \\ \boldsymbol{\chi}_{12} = \mathbf{1}. \end{cases}$$

- An $SU(3)_c$-invariant quark-antiquark pair cannot decay into a single gluon—even virtually—by color-conservation.
- Similarly, ($SU(3)_c$-invariant) hadrons can neither emit nor absorb a single gluon—by color-conservation.
- All hadron-hadron interaction must be mediated by $SU(3)_c$-invariant objects: ($n\geq2$)-gluons and/or quark-antiquark pairs.

Concrete QCD Computations

Quark-antiquark interaction

- So, in a $n^0 + \pi^- \longrightarrow n^0 + \pi^-$ scattering, 1-gluon exchange could happen as follows:

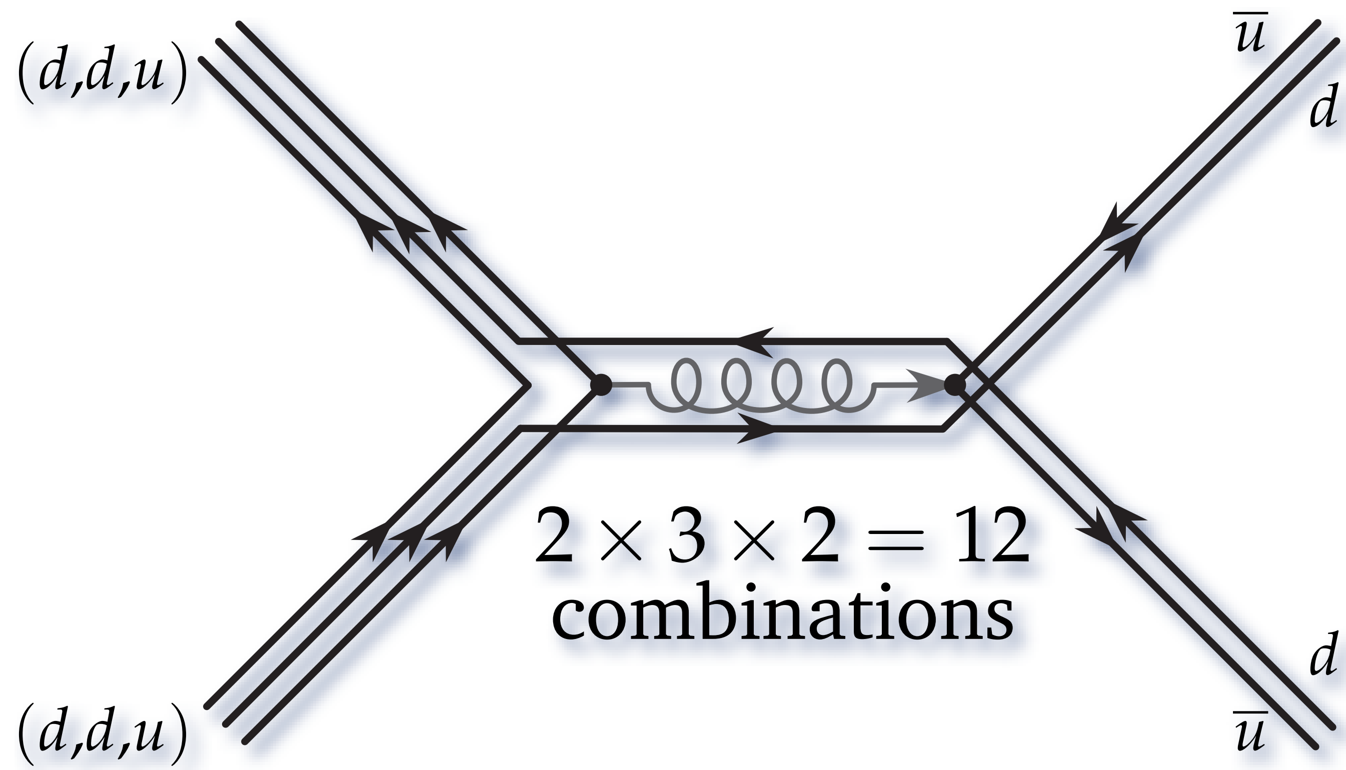


- ...except that two $SU(3)_c$-invariant hadrons cannot exchange an $SU(3)_c$-***variant*** gluon and stay $SU(3)_c$-invariant.
- So, the processes depicted in (a) must additionally involve an exchange of at least one more gluon, or a *d*-quark ...

Concrete QCD Computations

Quark-antiquark interaction

- So, in a $n^0 + \pi^- \longrightarrow n^0 + \pi^-$ scattering, 1-gluon exchange will include

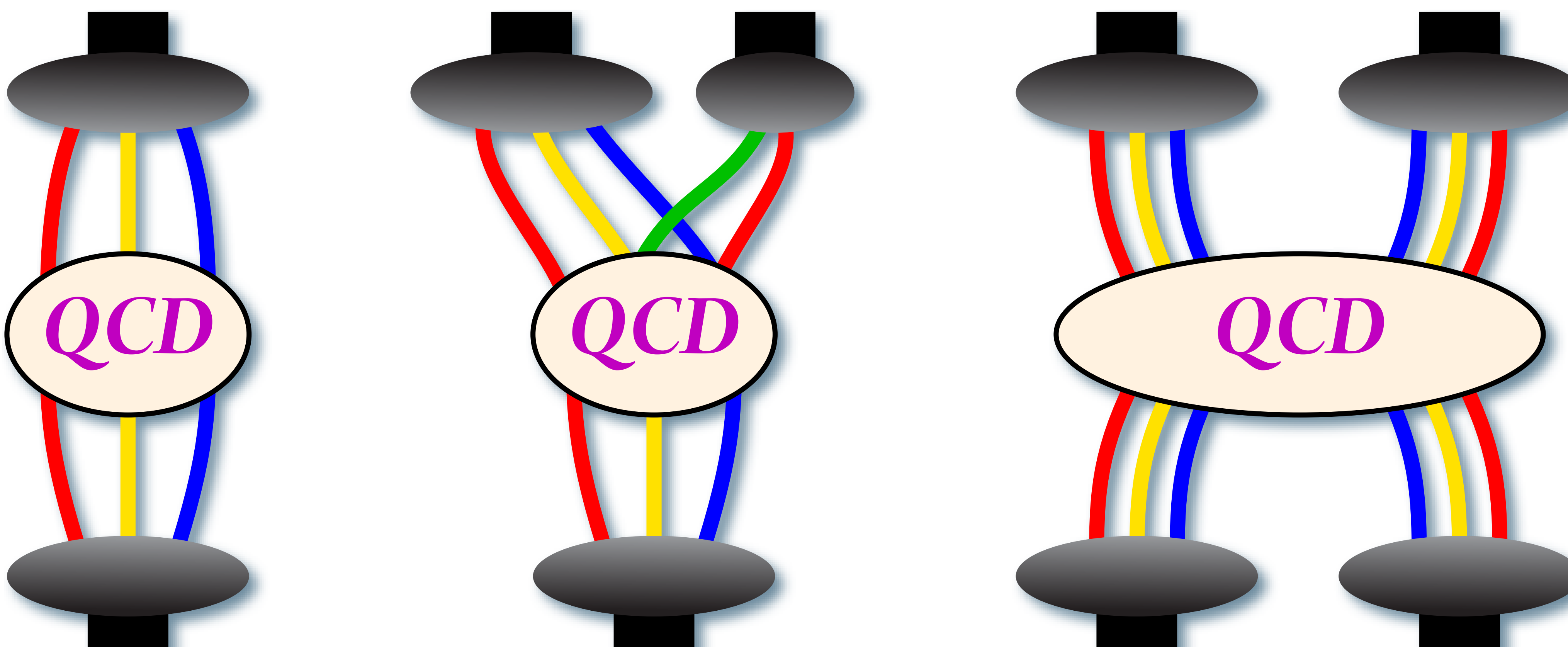


- …which is still $O(g_c{}^2)$, but is significantly complicated by the d-quark exchange. The mediating particle effectively becomes another hadron (π^0, or its P-wave excitation, ρ^0, or…).

Concrete QCD Computations

Conclusions

- Generally speaking,
- the QCD interactions must proceed so as to
- …not change the color-invariance of the hadrons involved
- …nor any other (real) intermediate state.



Concrete QCD Computations

Conclusions

- QCD interactions favor color-antisymmetrization:
- In a baryon, the three quarks attract each other by way of QCD precisely if they form an $SU(3)_c$-invariant state.
  - That is the color factor must be totally antisymmetric.
- In a meson, the quark-antiquark pair attract each other by way of QCD precisely if they form an $SU(3)_c$-invariant state.
- Two $SU(3)_c$-invariant hadrons cannot exchange an $SU(3)_c$-***variant*** gluon and stay $SU(3)_c$-invariant.
- Thus, two hadrons can interact only by exchanging
  - $SU(3)_c$-invariant objects, consisting of 2 or more of
  - …gluons and/or quark-antiquark pairs.
  - The hadron-hadron force is thus a "remnant" (*à la* van der Waals).

Concrete QCD Computations

Conclusions

- The 1-gluon exchange produces a reasonable qualitative statement (antisymmetrization ⇔ attraction).
- But, it is indicative of:
  - neither large-distance ($\geq 10^{-15}$m) ***confinement***
  - nor short-distance ($\ll 10^{-15}$m) ***asymptotic freedom*** (next time)
- Confinement is a large-distance feature
  - akin to the Coulomb (static) field in EM
  - …formed as a condensate of indefinitely many quanta
  - …essentially a non-perturbative phenomenon
- Asymptotic freedom is a perturbative result
  - 1973, David Gross & Frank Wiczek, & David Politzer
  - …a year before the "November (1974) revolution."

Thanks!

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