(Fundamental) Physics of Elementary Particles

Non-abelian gauge symmetry; QCD Lagrangian, color-conservation law and equations of motion

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Fundamental Physics of Elementary Particles

Program

- A gauge (local) symmetry principle recap
  - The partial derivative vs. the gauge-covariant derivative
  - General transformation "rules"
- The $SU(3)_c$ transformations
  - Color as a 3-dimensional charge
  - Matrix-valued phases and local symmetry
  - Matrix representations of $SU(3)$
- The $SU(3)_c$-invariant Lagrangian
  - The curvature tensor and the Bianchi identity
  - Equations of motion
  - Color conservation and equation of continuity

Gauge (Local) Symmetry Principle

Partial vs. gauge-covariant derivatives

- First and foremost:
- The mathematical object, $\Psi(\boldsymbol{r}, t)$, used to represent a particle
- depends on (is a function of)
  - external — the position in space and time, → base
  - internal —
    - the phase (as a complex function),
    - additional degrees of freedom
      - spin
      - isospin
      - color
      - …
    → fiber

… all of which are "coordinates," in a broad sense of the word.

- Gauge (local symmetry) principle:
  - internal coordinates are free to depend on external ones.

base, fiber → **fiber bundles/sheaves/ …**

Gauge (Local) Symmetry Principle

Partial vs. gauge-covariant derivatives

- Derivatives compute the rate of change
- So, if $\Psi(\boldsymbol{r}, t)$ also depends on a $(\boldsymbol{r}, t)$-dependent phase,
- then the rate of change stems from
  - varying $\Psi(\boldsymbol{r}, t)$ explicitly, and
  - varying $\Psi(\boldsymbol{r}, t)$ implicitly, *via* varying its phase.
- If "gauge" refers to "fixing" that phase,
- ...then a "gauge-covariant" derivative
- ...must contain two terms: $D_\mu = \partial_\mu + A_\mu(\boldsymbol{r}, t)$,
- where $A_\mu(\boldsymbol{r}, t)$ is the gauge potential, a.k.a. connexion.
- Mathematicians: $\mathrm{d}x^\mu D_\mu = \mathrm{d}x^\mu \partial_\mu + \mathrm{d}x^\mu A_\mu(\boldsymbol{r}, t)$ connexion 1-form
  - is now the (external) coordinate-independent definition.

Gauge (Local) Symmetry Principle

General transformation "rules"

- The unitary gauge (local symmetry) transformation is of the form $U_{\boldsymbol{\varphi}} = \exp\{\mathrm{i}\,\boldsymbol{\varphi}\cdot\mathsf{Q}\}$, $(U_{\boldsymbol{\varphi}}{}^{\dagger} = U_{\boldsymbol{\varphi}}{}^{-1})$
  - where $\boldsymbol{\varphi}$ is the (array of) gauge parameter(s),
  - where Q is the (array of) gauge transformation generator(s).
- Then,
  - $\Psi(\boldsymbol{r}, t) \to U_{\boldsymbol{\varphi}}\, \Psi(\boldsymbol{r}, t)$;
  - $\Psi^{\dagger}(\boldsymbol{r}, t) \to \Psi^{\dagger}(\boldsymbol{r}, t)\, U_{\boldsymbol{\varphi}}{}^{-1}$;
  - $\mathcal{O}(\boldsymbol{r}, t) \to U_{\boldsymbol{\varphi}}\, \mathcal{O}(\boldsymbol{r}, t)\, U_{\boldsymbol{\varphi}}{}^{-1}$;
  - ... and therefore also: $D_{\mu} \to U_{\boldsymbol{\varphi}}\, D_{\mu}\, U_{\boldsymbol{\varphi}}{}^{-1}$.
- Thus,
  - $\partial_{\mu} + A_{\mu}(\boldsymbol{r}, t) \to U_{\boldsymbol{\varphi}}\, (\partial_{\mu} + A_{\mu}(\boldsymbol{r}, t))\, U_{\boldsymbol{\varphi}}{}^{-1}$ implies that
  - $A_{\mu}(\boldsymbol{r}, t) \to U_{\boldsymbol{\varphi}}\, (-i\,(\partial_{\mu}\boldsymbol{\varphi}) + A_{\mu}(\boldsymbol{r}, t))\, U_{\boldsymbol{\varphi}}{}^{-1}$.

The $SU(3)_c$ Transformations

Color as a 3-dimensional charge

- Recall:
  - $\Delta^{++} = (uuu)$,
  - $\Delta^{-} = (ddd)$,
  - $\Omega^{-} = (sss)$.

  } Spin-3/2 baryons; *S*-states; no orbital angular momentum; Spatially symmetric wave-functions
- It follows that:
- either quarks are not fermions (O.W. Greenberg, 1964),

$$[b_i\,,\, b_j^{\dagger}] = \delta_{ij}, \qquad [b_i\,,\, b_j] = 0 = [b_i^{\dagger}, b_j^{\dagger}], \qquad \text{bosons,}$$

$$\{f_i\,,\, f_j^{\dagger}\} = \delta_{ij}, \qquad \{f_i\,,\, f_j\} = 0 = \{f_i^{\dagger}, f_j^{\dagger}\}, \qquad \text{fermions,}$$

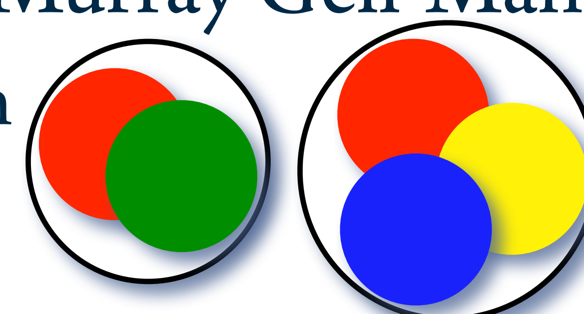
$$\left.\begin{aligned} \{\tilde{f}_{i,\alpha}\,,\, \tilde{f}^{\dagger}_{j,\alpha}\} &= \delta_{ij}, & \{\tilde{f}_{i,\alpha}\,,\, \tilde{f}_{j,\alpha}\} &= 0 = \{\tilde{f}^{\dagger}_{i,\alpha}\,,\, \tilde{f}^{\dagger}_{j,\alpha}\}, \\ [\tilde{f}_{i,\alpha}\,,\, \tilde{f}^{\dagger}_{j,\beta}] &= \delta_{ij}, & [\tilde{f}_{i,\alpha}\,,\, \tilde{f}_{j,\beta}] &= 0 = [\tilde{f}^{\dagger}_{i,\alpha}\,,\, \tilde{f}^{\dagger}_{j,\alpha}], \quad \alpha \neq \beta, \end{aligned}\right\} \text{para-fermions}$$

- …or…

The $SU(3)_c$ Transformations

Color as a 3-dimensional charge

- Quarks are fermions,
- …but have an additional degree of freedom.
  - January 1965: Boris V. Struminsky, Dubna (Moscow, Russia)
  - …then with N. Bogolyubov + Albert Tavchelidze
  - May 1965, A. Tavchelidze: ICTP, Trieste (Italy)
  - December 1965, Moo-Young Han + Yoichiro Nambu
    - integrally charged, colored quarks + 8 (color-anticolor) gluons
  - Final version (w/fractionally charged quarks): 1974, William Bardeen, Harald Fritzsch & Murray Gell-Mann
  - Quark: $\Psi_n{}^{aA}(\boldsymbol{r}, t)$, where:
    - n = $u, d, s, c,$ … indicates the flavor
    - a = *red*, *blue*, *yellow* indicates the "color"
    - A = 1, 2, 3, 4 indicates the component of the Dirac spinor


meson baryon

- P.S.: Greenberg subsequently proved equivalence…

The $SU(3)_c$ Transformations

Matrix-valued phases and local symmetry

- Without spelling out the Dirac components,

$$\boldsymbol{\Psi}_n(\mathrm{x}) = \hat{\mathrm{e}}_\alpha \Psi_n^\alpha(\mathrm{x}) = \hat{\mathrm{e}}_{\mathrm{r}} \Psi_n^{\mathrm{r}}(\mathrm{x}) + \hat{\mathrm{e}}_{\mathrm{y}} \Psi_n^{\mathrm{y}}(\mathrm{x}) + \hat{\mathrm{e}}_{\mathrm{b}} \Psi_n^{\mathrm{b}}(\mathrm{x}) = \begin{bmatrix} \Psi_n^{\mathrm{r}}(\mathrm{x}) \\ \Psi_n^{\mathrm{y}}(\mathrm{x}) \\ \Psi_n^{\mathrm{b}}(\mathrm{x}) \end{bmatrix},$$

- … where $n = u, d, s, c, b, t$ indicates the "flavor."
- Arranging the colors in a matrix format,
  - the quark wave-function phase becomes 3×3 matrix-valued,
  - as does the unitary phase-transformation operator $U_{\boldsymbol{\varphi}}$.

$$\boldsymbol{\Psi}_n(\mathrm{x}) \to e^{i g_c \boldsymbol{\varphi}(\mathrm{x})/\hbar}\, \boldsymbol{\Psi}_n(\mathrm{x}), \qquad \boldsymbol{\varphi}(\mathrm{x}) := \varphi^a(\mathrm{x})\, Q_a,$$

  - where Q_a are 3×3 matrices
    - Hermitian, so $U_{\boldsymbol{\varphi}}$ would be unitary,
    - traceless, so $U_{\boldsymbol{\varphi}}$ would be unimodular.

Diagonal phase-transformation pertains to electromagnetism …

The $SU(3)_c$ Transformations

Matrix-valued phases and local symmetry

- Gauge (local symmetry) transformations

$$[i\hbar \not{D} - mc]\boldsymbol{\Psi}_n(\mathrm{x}) = 0 \;\to\; [i\hbar \not{D}' - mc]\boldsymbol{\Psi}'_n(\mathrm{x}) = 0,$$

$$D_\mu \;\to\; D'_\mu \;:=\; U_{\boldsymbol{\varphi}}\, D_\mu\, U_{\boldsymbol{\varphi}}^{-1},$$

$$U_{\boldsymbol{\varphi}} := e^{ig_c \boldsymbol{\varphi}/\hbar},$$

- Notice the multi-component-ness:

$$\not{D}\boldsymbol{\Psi}_n \equiv \boldsymbol{\gamma}^\mu D_\mu \boldsymbol{\Psi}_n, \quad (\not{D}\boldsymbol{\Psi}_n)^\alpha \equiv \boldsymbol{\gamma}^\mu D_{\mu}{}^{\alpha}{}_{\beta}\, \Psi_n^\beta, \quad (\not{D}\boldsymbol{\Psi}_n)^{\alpha A} \equiv (\gamma^\mu)^A{}_B\, D_{\mu}{}^{\alpha}{}_{\beta}\, \Psi_n^{\beta B},$$

- ... which is usually suppressed in notation.
- In general:

$$D_\mu := \mathbb{1}\,\partial_\mu + \frac{ig_c}{\hbar c} A^a_\mu\, Q_a,$$

- where however the *form* of Q_a depends on what it acts upon.

The $SU(3)_c$ Transformations

Matrix representations of SU(3)

- As obtained in the "general" formalism:

$$A'^a_\mu Q_a = A^a_\mu \, U_{\boldsymbol{\varphi}} Q_a U_{\boldsymbol{\varphi}}^{-1} + \frac{\hbar c}{i g_c} U_{\boldsymbol{\varphi}} (\partial_\mu U_{\boldsymbol{\varphi}}^{-1}) = A^a_\mu \, U_{\boldsymbol{\varphi}} Q_a U_{\boldsymbol{\varphi}}^{-1} - c(\partial_\mu \varphi^a) Q_a,$$

- … where

$$\mathbb{A}'_\mu = U_{\boldsymbol{\varphi}} \mathbb{A}_\mu U_{\boldsymbol{\varphi}}^{-1} - c(\partial_\mu \boldsymbol{\varphi}), \qquad \mathbb{A}_\mu := A^a_\mu Q_a,$$

- … and where

$$[Q_a, Q_b] = i f_{ab}{}^c \, Q_c.$$

- are the 3×3 matrices that act upon the (quark) color 3-vector.
- But, what about the Q_a's acting on the 8 $A_\mu{}^a$'s or 8 φ^a's?

$$\delta A^a_\mu = -(D_\mu \boldsymbol{\varphi})^a := -c(\partial_\mu \varphi^a) + \frac{i g_c}{\hbar c} A^b_\mu \, (\widetilde{Q}_b)_c{}^a \varphi^c \quad a,b,c = 1,\dots,8$$

$$(\widetilde{Q}_b)_c{}^a = i f_{bc}{}^a, \qquad = -(\partial_\mu \varphi^a) - \frac{g_c}{\hbar c} A^b_\mu f_{bc}{}^a \varphi^c,$$

The $SU(3)_c$-invariant Lagrangian

The curvature tensor and the Bianchi identity

- Notice the differences: $D'_\mu = U_\varphi D_\mu U_\varphi^{-1}$ implies

$$\Rightarrow \qquad A'_\mu = A_\mu - (\partial_\mu \varphi) \quad \text{for electromagnetism,}$$

$$\Rightarrow \quad (A')^a_\mu = A^a_\mu - (D_\mu \varphi^a) = A^a_\mu - (\partial_\mu \varphi^a) + \frac{g_c}{\hbar c} A^b_\mu f_{bc}{}^a \varphi^c.$$

nonlinear

- Also,

$$F_{\mu\nu}(A') = F_{\mu\nu}(A),$$

- But, for the non-abelian (matrix-valued) case:

$$(\partial_\mu (A')^a_\nu - \partial_\nu (A')^a_\mu) \neq (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu),$$

$$(\partial_\mu (A')^a_\nu - \partial_\nu (A')^a_\mu) \neq U_\varphi (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) U_\varphi^{-1}.$$

- Recall however that

$$D'_\mu = U_\varphi D_\mu U_\varphi^{-1},$$

The $SU(3)_c$-invariant Lagrangian

The curvature tensor and the Bianchi identity

- In electrodynamics:

$$\big[\,D_\mu\,,\,D_\nu\,\big] = \big[\,\partial_\mu + \tfrac{iq}{\hbar c}A_\mu\,,\,\partial_\nu + \tfrac{iq}{\hbar c}A_\nu\,\big] = +\tfrac{iq}{\hbar c}(\partial_\mu A_\nu - \partial_\nu A_\mu) = \tfrac{iq}{\hbar c}F_{\mu\nu}.$$

- It must be a commutator, so that the result would not be a differential operator, but an "ordinary" function.
- A commutator also computes the mismatch in … well,
- … commuting.
- In general: $[D, D]$ = (torsion)·D + (curvature).
- So:

$$\mathbb{F}_{\mu\nu} := \tfrac{\hbar c}{ig_c}\big[\,D_\mu\,,\,D_\nu\,\big] = \tfrac{\hbar c}{ig_c}\big[\,\partial_\mu + \tfrac{ig_c}{\hbar c}A^b_\mu\,Q_b\,,\,\partial_\nu + \tfrac{ig_c}{\hbar c}A^c_\nu\,Q_c\,\big],$$

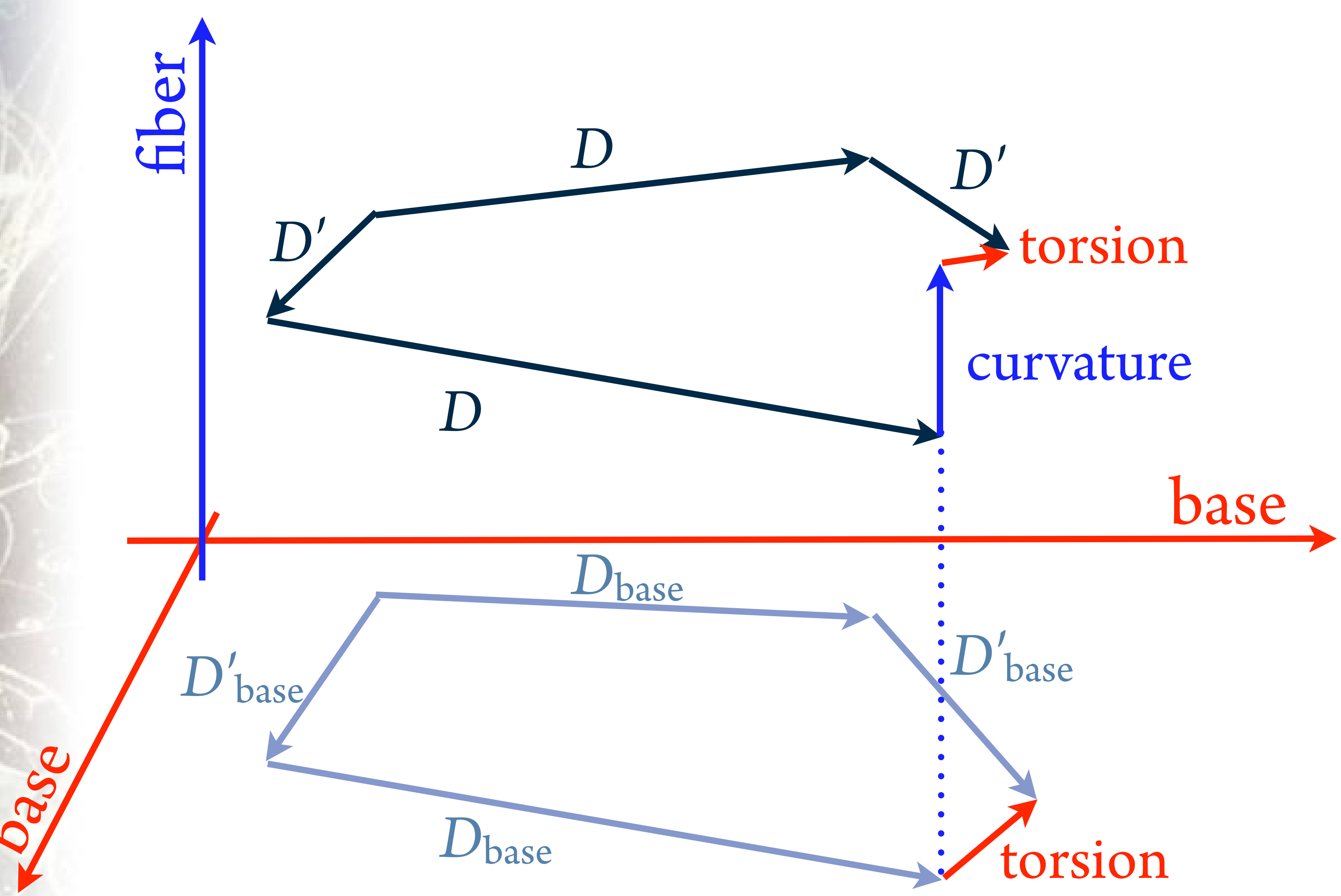
$$= (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)Q_a + \tfrac{\hbar c}{ig_c}\big(\tfrac{ig_c}{\hbar c}\big)^2 A^b_\mu\,A^c_\nu\,[Q_b, Q_c] \;=\; F^a_{\mu\nu}Q_a,$$

$$F^a_{\mu\nu} := (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) - \tfrac{g_c}{\hbar c}\,f^a{}_{bc}\,A^b_\mu\,A^c_\nu,$$

Digression

Curvature and torsion

- Picture the result of computing $[D, D'] = DD' - D'D$
- …in the total space of a fiber bundle:



Generally, the rate-of-change operators (D) will fail to commute both in the fiber-space direction (= curvature: at the same "place" but of a different "value") and in the base-space direction (= torsion: not even at the same "place").

The $SU(3)_c$-invariant Lagrangian

The curvature tensor and the Bianchi identity

- Of course, we have that

$$\mathbb{F}_{\mu\nu} \;\to\; \mathbb{F}'_{\mu\nu} := \frac{i\hbar c}{g_c}\big[\,D'_\mu\,,\,D'_\nu\,\big] = \frac{i\hbar c}{g_c}\big[\,U_{\boldsymbol{\varphi}}D_\mu U_{\boldsymbol{\varphi}}^{-1}\,,\,U_{\boldsymbol{\varphi}}D_\nu U_{\boldsymbol{\varphi}}^{-1}\,\big]$$

$$= \frac{i\hbar c}{g_c}U_{\boldsymbol{\varphi}}\big[\,D_\mu\,,\,D_\nu\,\big]U_{\boldsymbol{\varphi}}^{-1},$$

$$= U_{\boldsymbol{\varphi}}\,\mathbb{F}_{\mu\nu}\,U_{\boldsymbol{\varphi}}^{-1}.$$

- Independently,

$$D_\mu(\mathbb{F}_{\nu\rho}) = [D_\mu, \mathbb{F}_{\mu\nu}] = \frac{\hbar c}{i g_c}\big[\,D_\mu\,,\big[\,D_\nu\,,\,D_\rho\,\big]\big]$$

$$\big[\,A\,,\big[\,B\,,\,C\,\big]\big] + \big[\,B\,,\big[\,C\,,\,A\,\big]\big] + \big[\,C\,,\big[\,A\,,\,B\,\big]\big] \equiv 0,$$

$$\varepsilon^{\mu\nu\rho\sigma}D_\mu(\mathbb{F}_{\nu\rho}) = \frac{\hbar c}{i g_c}\varepsilon^{\mu\nu\rho\sigma}\big[\,D_\mu\,,\big[\,D_\nu\,,\,D_\rho\,\big]\big] = 0,$$

- And, for all $SU(n)$:

$$\mathrm{Tr}[\mathbb{F}_{\mu\nu}] = F^k_{\mu\nu}\,\mathrm{Tr}[Q_k] = 0.$$

The $SU(3)_c$-invariant Lagrangian

Equations of motion

- Since the matrix-valued ($\mathfrak{su}(3)$ algebra-valued) curvature transforms by similarity transformation,
- …with respect to which the trace function is invariant,

$$\begin{aligned}\mathrm{Tr}[\mathbb{F}_{\mu\nu}\mathbb{F}^{\mu\nu}] \;\to\; \mathrm{Tr}[\mathbb{F}'_{\mu\nu}\mathbb{F}'^{\,\mu\nu}] &= \mathrm{Tr}[U_{\boldsymbol{\varphi}}\mathbb{F}_{\mu\nu}U_{\boldsymbol{\varphi}}^{-1}U_{\boldsymbol{\varphi}}\mathbb{F}^{\mu\nu}U_{\boldsymbol{\varphi}}^{-1}]\\ &= \mathrm{Tr}[\mathbb{F}_{\mu\nu}\mathbb{F}^{\mu\nu}U_{\boldsymbol{\varphi}}^{-1}U_{\boldsymbol{\varphi}}],\\ &= \mathrm{Tr}[\mathbb{F}_{\mu\nu}\mathbb{F}^{\mu\nu}]\end{aligned}$$

- …so one chooses:

$$\begin{aligned}\mathscr{L}_{QCD} &= \sum_n \mathrm{Tr}\left[\overline{\boldsymbol{\Psi}}_n(\mathrm{x})\,[i\hbar c\,\not{D} - m_n c^2]\,\boldsymbol{\Psi}_n(\mathrm{x})\right] - \tfrac{1}{4}\,\mathrm{Tr}\left[\mathbb{F}_{\mu\nu}\mathbb{F}^{\mu\nu}\right],\\ &= \sum_n \overline{\Psi}_{\alpha\,n}(\mathrm{x})\left[i\boldsymbol{\gamma}^{\mu}\left(\hbar c\,\delta^{\alpha}_{\beta}\partial_{\mu} + i g_c\,A^a_{\mu}(\tfrac{1}{2}\lambda_a)^{\alpha}{}_{\beta}\right) - m_n c^2\delta^{\alpha}_{\beta}\right]\Psi^{\beta}_n(\mathrm{x})\\ &\quad - \tfrac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu}.\end{aligned}$$

The $SU(3)_c$-invariant Lagrangian

Equations of motion

- Variation by $A^a{}_\mu$ yields:

$$D_\mu F^{a\,\mu\nu} = g_c \sum_n \overline{\Psi}_{n\alpha A} (\gamma^\nu)^A{}_B (\tfrac{1}{2}\lambda^a)^\alpha{}_\beta \Psi_n^{\beta B},$$

$$j^{a\,\mu}_{(q)} := g_c \sum_n \overline{\Psi}_{n\alpha A} (\gamma^\mu)^A{}_B (\tfrac{1}{2}\lambda^a)^\alpha{}_\beta \Psi_n^{\beta B}.$$

- But, while

$$(D_\mu F^{\mu\nu} = \partial_\mu F^{\mu\nu}) = g_e \overline{\Psi}_A (\gamma^\nu)^A{}_B \Psi^A,$$

- implies

$$\partial_\nu j^\nu_e = \frac{4\pi\epsilon_0 c}{4\pi} \partial_\nu \partial_\mu F^{\mu\nu} \equiv 0, \qquad \text{since} \qquad F_{\mu\nu} = -F_{\nu\mu}$$

- the same is not true of $D_\mu F^{a\,\mu\nu}$.
- Instead:

$$D_\nu\, j^{a\,\nu}_{(q)} = D_\nu\, D_\mu\, F^{a\,\mu\nu} = -\tfrac{1}{2}[D_\mu, D_\nu] F^{a\,\mu\nu} = -\tfrac{1}{2} f^a{}_{bc} F^b_{\mu\nu} F^{c\,\mu\nu} = 0,$$

The $SU(3)_c$-invariant Lagrangian

Color conservation and equation of continuity

- This does not lead to a conserved color:

$$0 = D_\mu j^{a\,\mu}_{(q)} = \partial_\mu j^{a\,\mu}_{(q)} - \tfrac{g_c}{\hbar c} f^a{}_{bc}\, A^b_\mu\, j^{c\,\mu}_{(q)},$$

$$\Rightarrow \qquad \frac{\mathrm{d}}{\mathrm{d}t}\Big(\int_V \mathrm{d}^3\vec{r}\, j^{a\,0}_{(q)}\Big) = -\oint_{\partial V} \mathrm{d}^2\vec{r}\cdot \vec{j}^{\,a}_{(q)} + \tfrac{g_c}{\hbar c} f^a{}_{bc}\Big(\int_V \mathrm{d}^3\vec{r}\, A^b_\mu\, j^{c\,\mu}_{(q)}\Big),$$

- … as the additional right-hand side term doesn't vanish.
- However, use that

$$D_\mu F^{a\,\mu\nu} = \partial_\mu F^{a\,\mu\nu} - \tfrac{g_c}{\hbar c} f_{bc}{}^a A^b_\mu F^{c\,\mu\nu},$$

$$D_\mu F^{a\,\mu\nu} = j^{a\,\nu}_{(q)} \quad \Rightarrow \quad \partial_\mu F^{a\,\mu\nu} = J^{a\,\nu}_{(c)}, \quad \Rightarrow \quad \partial_\nu J^{a\,\nu}_{(c)} = 0,$$

$$J^{a\,\nu}_{(c)} := j^{a\,\mu}_{(q)} + \tfrac{g_c}{\hbar c} f_{bc}{}^a A^b_\mu F^{c\,\mu\nu},$$

- … so both quarks and gluons contribute to the color charge:

$$Q^a_{(c)} := \int \mathrm{d}^3\vec{r}\, J^{a\,0}_{(c)} = g_c \int \mathrm{d}^3\vec{r}\, \Big(\sum_n [\overline{\Psi}_n\, \boldsymbol{\gamma}^\mu\, \tfrac{1}{2}\boldsymbol{\lambda}^a\, \Psi_n] + \tfrac{1}{\hbar c} f_{bc}{}^a A^b_\mu F^{c\,\mu\nu}\Big)$$

The $SU(3)_c$-invariant Lagrangian

Color conservation and equation of continuity

- This changes the analogues of Gauss-Ampère laws.
- Consider the $\nu = 0$ case of the equation $D_\mu F^{a\,\mu\nu} = j_{(q)}{}^{a\nu}$:

$$\partial_\mu F^{a\,\mu 0} - \tfrac{g_c}{\hbar c} f^a{}_{bc} A^b_\mu F^{c\,\mu 0} = j^{a\,0}_{(q)},$$

- … and define:

$$\vec{E}^a := \hat{\mathrm{e}}_i F^{a\,i0}, \qquad \rho^a_{(q)} := j^{a\,0}_{(q)}, \qquad \vec{A}^a := -\hat{\mathrm{e}}^i A^a_i,$$

- Then,

$$\vec{\nabla}\cdot\vec{E}^a = \rho^a_{(q)} - \tfrac{g_c}{\hbar c} f^a{}_{bc} \vec{A}^b \cdot \vec{E}^c,$$

- and
  - it is impossible to write analogues of Maxwell's equations with no reference to the gauge potentials
  - both quarks and gluons serve as "sources" for the color force-field
  - the equations are nonlinear.

The $SU(3)_c$-invariant Lagrangian

Color conservation and equation of continuity

- To sum up:

$$D_\mu \mathbb{F}^{\mu\nu} = \mathbb{J}^\nu_{(q)} \qquad \text{and} \qquad \varepsilon^{\mu\nu\rho\sigma} D_\mu(\mathbb{F}_{\nu\rho}) = 0,$$

$$\mathbb{J}^\nu_{(q)} := g_c \Big(\sum_n \overline{\Psi}_{n\alpha A} (\gamma^\mu)^A{}_B (\tfrac{1}{2}\lambda^a)^\alpha{}_\beta \Psi_n^{\alpha A} \Big) Q_a$$

$$\partial_\mu \mathbb{F}^{\mu\nu} = \mathbb{J}^\nu_{(c)}, \qquad \mathbb{J}^\nu_{(c)} := \mathbb{J}^\nu_{(q)} + \tfrac{ig_c}{\hbar c}[\mathbb{A}_\mu, \mathbb{F}^{\mu\nu}],$$

$$\partial_\nu \mathbb{J}^\nu_{(c)} = 0, \qquad \frac{\mathrm{d}}{\mathrm{d}t}\int_V \mathrm{d}^3\vec{r}\, \mathbb{J}^0_{(c)} = -\oint_{\partial V} \mathrm{d}^2\vec{\sigma}\cdot\vec{\mathbb{J}}_{(c)}.$$

- are the matrix-valued analogues of
  - the Maxwell's equations
  - the conserved color-current & color-charge.

Thanks!

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