(Fundamental) Physics of Elementary Particles

Quantum electrodynamics of leptons and hadrons
Experimental proof of the parton model

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Fundamental Physics of Elementary Particles

Program

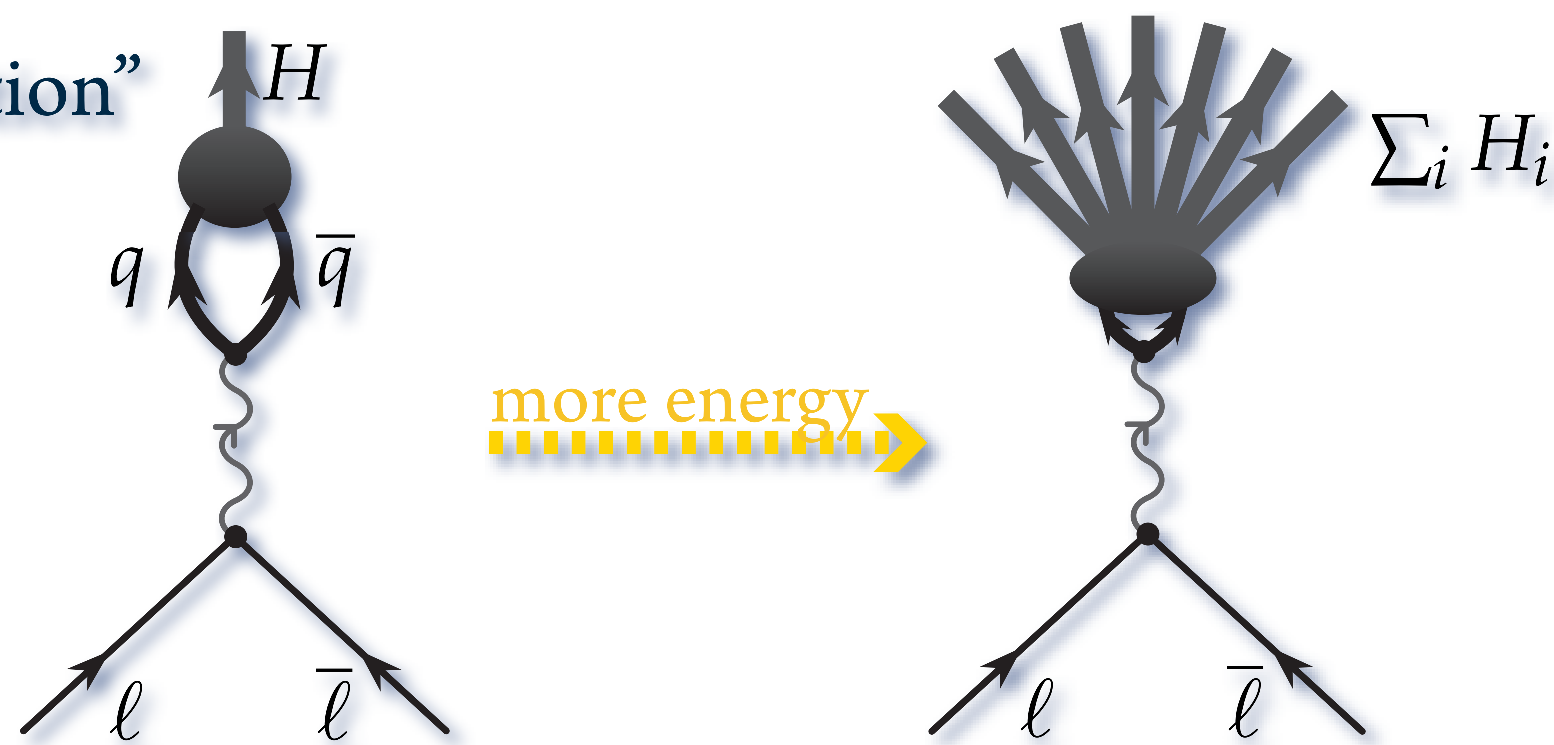
- Hadron Production in Electron-Positron Annihilation
  - Quark-antiquark production threshold
  - Hadron-to-lepton production ratio
  - Fractional charge of quarks
- The Electrodynamics of Lepton-Hadron Scattering
  - Elastic Lepton-Hadron Scattering
  - Deep Inelastic (Light) Lepton-Hadron Scattering
  - Experimental Verification of the Parton Model
- Now, 'bout them Student presentations…
  - Pick a section/topic. Any section, from Ch.–1, 0, … 3 or 4.
  - Start with the material in the text, research it some more
    - other books, journals, Wikipedia, Google …
  - Explain it to your classmates. & critique each other. ***Constructively!***

Hadron Production in e⁻-e⁺ Annihilation

Quark-antiquark production threshold

- The electromagnetic interaction of quarks is described by the analogous Lagrangian as with leptons …
  - … except quarks are not found isolated,
  - … but only within bound states,
  - … where the other constituents complicate matters.

"hadronization"



Hadron Production in e⁻-e⁺ Annihilation

Quark-antiquark production threshold

- Consider a lepton-antilepton annihilation,
- …that produces a quark-antiquark pair:

$$\mathfrak{M}_{\ell\bar{\ell}\to q\bar{q}} = \frac{Q\,g_e^2}{(\mathrm{p}_1+\mathrm{p}_2)^2}[\bar{v}_2\boldsymbol{\gamma}^\mu u_1][\overline{U}_3\boldsymbol{\gamma}_\mu V_4],$$

$$\left\langle|\mathfrak{M}_{\ell\bar{\ell}\to q\bar{q}}|^2\right\rangle = \frac{1}{4}\Big(\frac{Q\,g_e^2}{(\mathrm{p}_1+\mathrm{p}_2)^2}\Big)^2 \mathrm{Tr}\left[\boldsymbol{\gamma}^\mu(\not{p}_1+mc)\boldsymbol{\gamma}^\nu(\not{p}_2-mc)\right]$$
$$\times\,\mathrm{Tr}\left[\boldsymbol{\gamma}^\mu(\not{p}_4-Mc)\boldsymbol{\gamma}^\nu(\not{p}_3+Mc)\right],$$

$$= 8\Big(\frac{Q\,g_e^2}{(\mathrm{p}_1+\mathrm{p}_2)^2}\Big)^2\big[(\mathrm{p}_1\cdot\mathrm{p}_3)(\mathrm{p}_2\cdot\mathrm{p}_4)+(\mathrm{p}_1\cdot\mathrm{p}_4)(\mathrm{p}_2\cdot\mathrm{p}_3)+2(mc)^2(Mc)^2$$
$$+(mc)^2(\mathrm{p}_3\cdot\mathrm{p}_4)+(Mc)^2(\mathrm{p}_1\cdot\mathrm{p}_2)\big],$$

$$= Q^2 g_e^4\Big\{1+\Big(\frac{mc^2}{E}\Big)^2+\Big(\frac{Mc^2}{E}\Big)^2+\Big[1-\Big(\frac{mc^2}{E}\Big)^2\Big]\Big[1-\Big(\frac{Mc^2}{E}\Big)^2\Big]\cos^2\theta\Big\}$$

Hadron Production in e⁻-e⁺ Annihilation

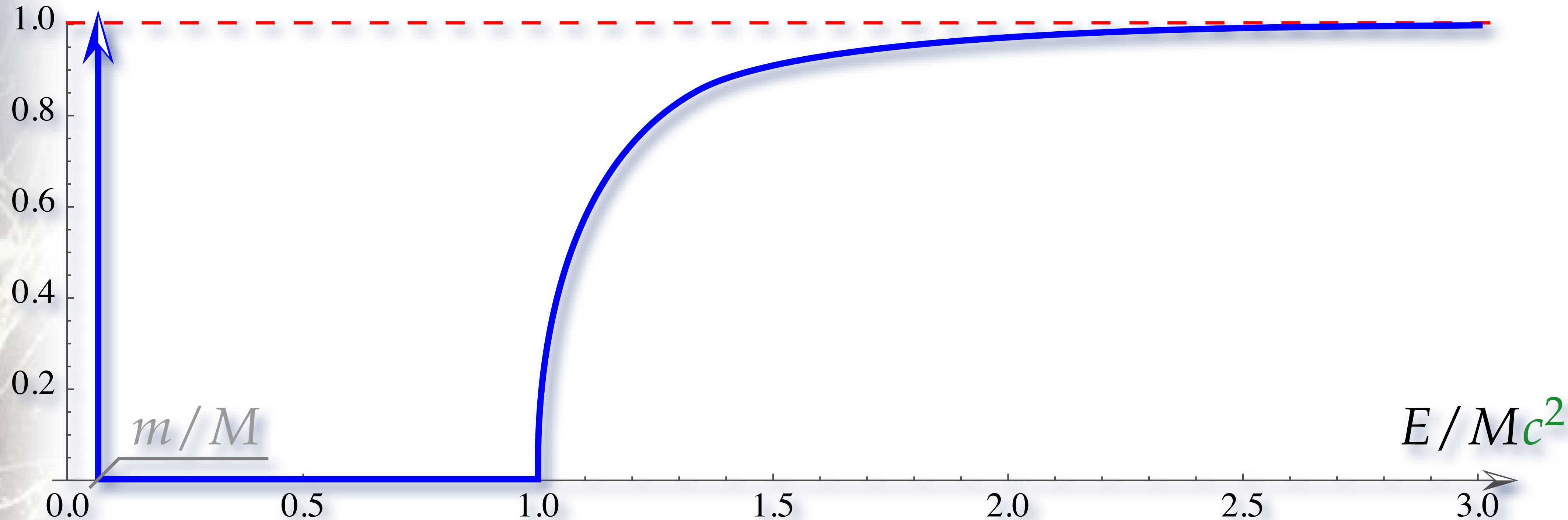
Quark-antiquark production threshold

- Using
$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{S\,|\mathfrak{M}|^2}{(E_A + E_B)^2} \frac{|\vec{p}_f|}{|\vec{p}_i|}$$
- …and integrating over the angles yields
$$\sigma = \frac{\pi}{3}\left(\frac{Q\hbar c\alpha}{E}\right)^2 \sqrt{\frac{1-(Mc^2/E)^2}{1-(mc^2/E)^2}} \left[1+\frac{1}{2}\left(\frac{mc^2}{E}\right)^2\right] \left[1+\frac{1}{2}\left(\frac{Mc^2}{E}\right)^2\right]$$
- where m is the (anti)lepton mass, M the (anti)quark mass.
- Typically $m \ll M$;
- for $E < mc^2$, the above result for σ is real, but unphysical;
- for $mc^2 < E < Mc^2$, σ is purely imaginary;
- for $Mc^2 < E$, σ is real and asymptotically approaches 1;
- higher order corrections sharpen the step-function.

Hadron Production in e⁻-e⁺ Annihilation

Quark-antiquark production threshold

- A sketch of $\sigma(E)$:



- As energies increase, heavier real quarks may be produced.
  - These are "hidden" within the resulting hadrons,
  - … and also decay (within ~10^{-23} s) into lighter quarks …
  - However, with increasingly more available quarks, the probability of producing hadrons (*vs.* leptons) increases.

Hadron Production in e⁻-e⁺ Annihilation

Hadron/lepton production ratio

- The ratio of total number of hadron vs. lepton production
- …grows every time the energy "hits" a new quark threshold.

$$\mathfrak{M}_{\ell\bar{\ell}\to q\bar{q}} = \frac{Q\,g_e^2}{(\mathrm{p}_1+\mathrm{p}_2)^2}[\overline{v}_2\boldsymbol{\gamma}^\mu u_1][\overline{U}_3\boldsymbol{\gamma}_\mu V_4],$$

- …so $\sigma(E) \propto Q^2$, and so

$$R(E) := \frac{\sigma(e^- + e^+ \to \text{hadrons})}{\sigma(e^- + e^+ \to \mu^- + \mu^+)} \approx \Big[3\sum_i Q_i{}^2\Big]_{M_i<E/c^2}.$$

- The factor 3 stems from each quark coming in three "colors."
- Everything else cancels out, since the electromagnetic interaction with muons is the same as with quarks;
- …they are merely heavier,
- …and trapped in the hadron bound states.

Hadron Production in e⁻-e⁺ Annihilation

Fractional charge of quarks

- Are quarks really fractionally charged?
- 1965, M.Y. Han and Y. Nambu:
  - integrally charged quarks

$$Q(u^r) = +1,\ Q(u^y) = +1,\ Q(u^b) = 0,$$
$$Q(d^r) = 0,\quad Q(d^y) = 0,\quad Q(d^b) = -1,$$

  - with the fractional averages +⅔ and –⅓.
  - Processes where the observable depends on the average quark charges cannot differentiate between integral & fractional charges.
    - Such as lepton-hadron (lepton-quark) scattering.
  - Processes that depend nonlinearly on individual quark charges do differentiate between integral & fractional charges.
    - Such as lepton-antilepton → quark-antiquark,
      - hadron production (real quarks) & renormalization (virtual quarks)

Hadron Production in e⁻-e⁺ Annihilation

Fractional charge of quarks

- Comparing the hadron-to-muon production ratio:

$$R(E) \approx 3[(\tfrac{2}{3})^2 + (-\tfrac{1}{3})^2] = \tfrac{5}{3}, \qquad E \leqslant M_{u,d}c^2,$$
$$\widetilde{R}(E) \approx [2(+1)^2 + (-1)^2] = 3, \qquad E \leqslant M_{u,d}c^2,$$

$$R(E) \approx 3[(\tfrac{2}{3})^2 + (-\tfrac{1}{3})^2 + (-\tfrac{1}{3})^2] = 2, \qquad E \leqslant M_s c^2,$$
$$\widetilde{R}(E) \approx [2(+1)^2 + (-1)^2 + (-1)^2] = 4, \qquad E \leqslant M_s c^2,$$

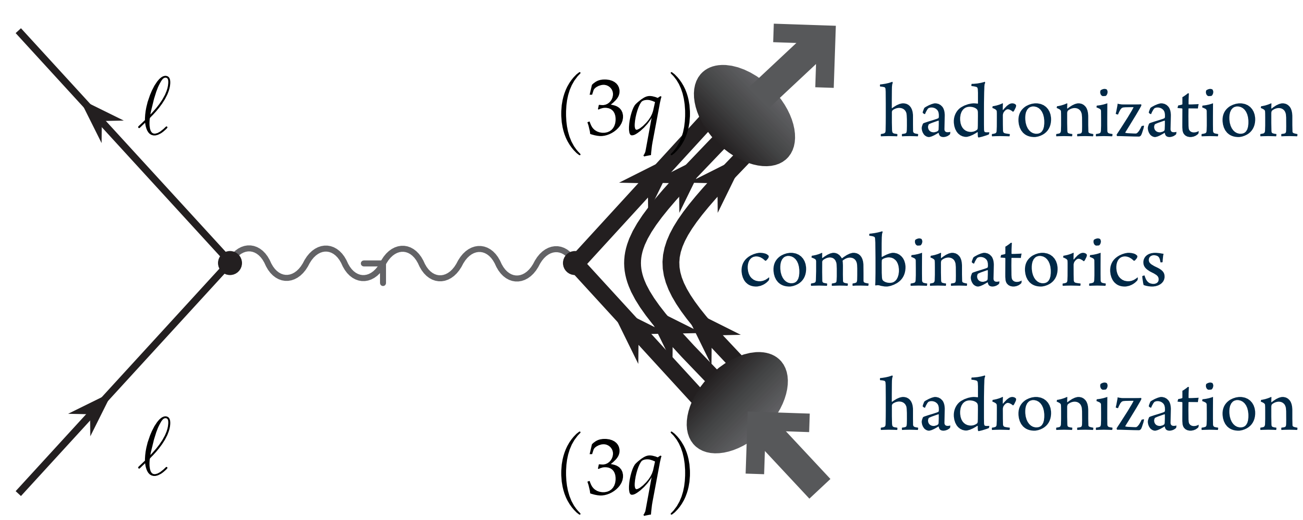
$$R(E) \approx 3[(\tfrac{2}{3})^2 + (-\tfrac{1}{3})^2 + (-\tfrac{1}{3})^2 + (\tfrac{2}{3})^2] = \tfrac{10}{3}, \qquad E \leqslant M_c c^2,$$
$$\widetilde{R}(E) \approx [2(+1)^2 + (-1)^2 + (-1)^2 + 2(+1)^2] = 6, \qquad E \leqslant M_c c^2,$$

- ... and so on.
- The relative incremental increases are clearly different,
- ... and experiments support the Gell-Mann–Zweig model.

The Electrodynamics of Lepton-Hadron Scattering

Elastic Lepton-Hadron Scattering

- Interactions between a lepton "probe" and a hadron "target"
  - …reduces to interactions with individual quarks in the hadron,
  - …modified by their distribution (*form-factors*) in the hadron.
- As the energy grows, however, the outgoing state becomes
  - one or several "jets" of hadrons
  - …none of which need include the original "target,"
  - …of which the original constituents may separate.
- Pictorially:



The Electrodynamics of Lepton-Hadron Scattering

Elastic Lepton-Hadron Scattering

- Modeled after the $e^-+\mu^-\longrightarrow e^-+\mu^-$ scattering,

$$\langle|\mathfrak{M}_{(a)}|^2\rangle = \frac{g_e^4}{(\mathrm{p}_1-\mathrm{p}_3)^4}\, X^{\mu\nu}(1,3;e^-)\, X_{\mu\nu}(2,4;\mu^-),$$

- …we now write

$$\left\langle|\mathfrak{M}_{\ell p\to\ell p}|^2\right\rangle = \frac{g_e^4}{(\mathrm{p}_1-\mathrm{p}_3)^4} X^{\mu\nu}(1,3;\ell)\, K_{\mu\nu}(2,4;p^+),$$

$$\begin{aligned} X^{\mu\nu}(1,3;\ell) &= \mathrm{Tr}\left[\boldsymbol{\gamma}^\mu\,(\not{p}_1+m_\ell c\mathbb{1})\,\overline{\boldsymbol{\gamma}}^\nu\,(\not{p}_3+m_\ell c\mathbb{1})\,\right], \\ &= 2\left[p_1^\mu p_3^\nu + p_1^\nu p_3^\mu + \eta^{\mu\nu}\left[m_\ell^2 c^2 - (\mathrm{p}_1\cdot\mathrm{p}_3)\right]\right]. \end{aligned}$$

- The function $X^{\mu\nu}(1, 3; \ell)$ is computed as before, assuming that the lepton is an elementary (point-like) spin-½ Dirac spinor.
- But, protons are not.
- Their structure is inundated with strong interaction effects.

The Electrodynamics of Lepton-Hadron Scattering

Elastic Lepton-Hadron Scattering

- The proton structure modifies $X_{\mu\nu}(2,4;\ell) \to K_{\mu\nu}(2,4;p^{+})$,
- …and we need to determine the form of this function.
- It's a rank-2 tensor that may only depend on p_2, p_4, $q = p_4 - p_2$.
- Write then $p = p_2$, and $p_4 = q + p$. Then $K_{\mu\nu}(2,4;p^{+})$ equals:

$$-K_1\,\eta_{\mu\nu} + \frac{K_2}{M^2c^2}\,p_\mu p_\nu + \frac{K_4}{M^2c^2}\,q_\mu q_\nu + \frac{K_5}{M^2c^2}\,(p_\mu q_\nu + q_\mu p_\nu),$$

- The K_i coefficients are scalars, and so functions of the only scalar variable, $q^2 = (p_4 - p_2)^2$.
  - $p_2{}^2 = p_4{}^2 = M^2\,c^2$, *(on-shell)*
  - $q \cdot p_2 = -\tfrac{1}{2}\,q^2$. *(Prove!)*
- The antisymmetric part of $K_{\mu\nu}(2,4;p^{+})$ is irrelevant, since it is contracted with $X^{\mu\nu}(1,3;\ell)$, which is symmetric.

The Electrodynamics of Lepton-Hadron Scattering

Elastic Lepton-Hadron Scattering

- It turns out that $q^\mu K_{\mu\nu} = 0$. Using this, it follows that

$$K_4 = \frac{M^2 c^2}{\mathrm{q}^2} K_1 + \tfrac{1}{4} K_2, \qquad \text{and} \qquad K_5 = \tfrac{1}{2} K_2.$$

- Thus, $K_{\mu\nu}(2, 4; p^+)$ is well parametrized as:

$$-K_1(\mathrm{q}^2)\Big(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{\mathrm{q}^2}\Big) + \frac{K_2(\mathrm{q}^2)}{M^2 c^2}\Big(p^\mu + \tfrac{1}{2} q^\mu\Big)\Big(p^\nu + \tfrac{1}{2} q^\nu\Big).$$

- This produces

$$\left\langle |\mathfrak{M}_{\ell p \to \ell p}|^2 \right\rangle = \Big(\frac{2{g_e}^2}{\mathrm{q}^2}\Big)^2 \Big[K_1[(\mathrm{p}_1 \cdot \mathrm{p}_3) - 2 m_\ell^2 c^2] + K_2\Big(\frac{(\mathrm{p}_1 \cdot \mathrm{p})(\mathrm{p}_3 \cdot \mathrm{p})}{M^2 c^2} + \frac{\mathrm{q}^2}{4}\Big)\Big],$$

$$\approx \frac{{g_e}^4 c^2}{4EE' \sin^4(\theta/2)} \left(2K_1 \sin^2(\theta/2) + K_2 \cos^2(\theta/2)\right)$$

- where $\mathrm{p} = (Mc, 0, 0, 0)$, and E, E' and θ describe the probe.

The Electrodynamics of Lepton-Hadron Scattering

Elastic Lepton-Hadron Scattering

- Kinematics fixes

$$E' = \frac{E}{1 + (2E/Mc^2)\sin^2(\theta/2)}.$$

- Thus:

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha\hbar}{4ME\sin^2(\theta/2)}\right)^2 \frac{E'}{E}\left(2K_1\sin^2(\theta/2) + K_2\cos^2(\theta/2)\right),$$

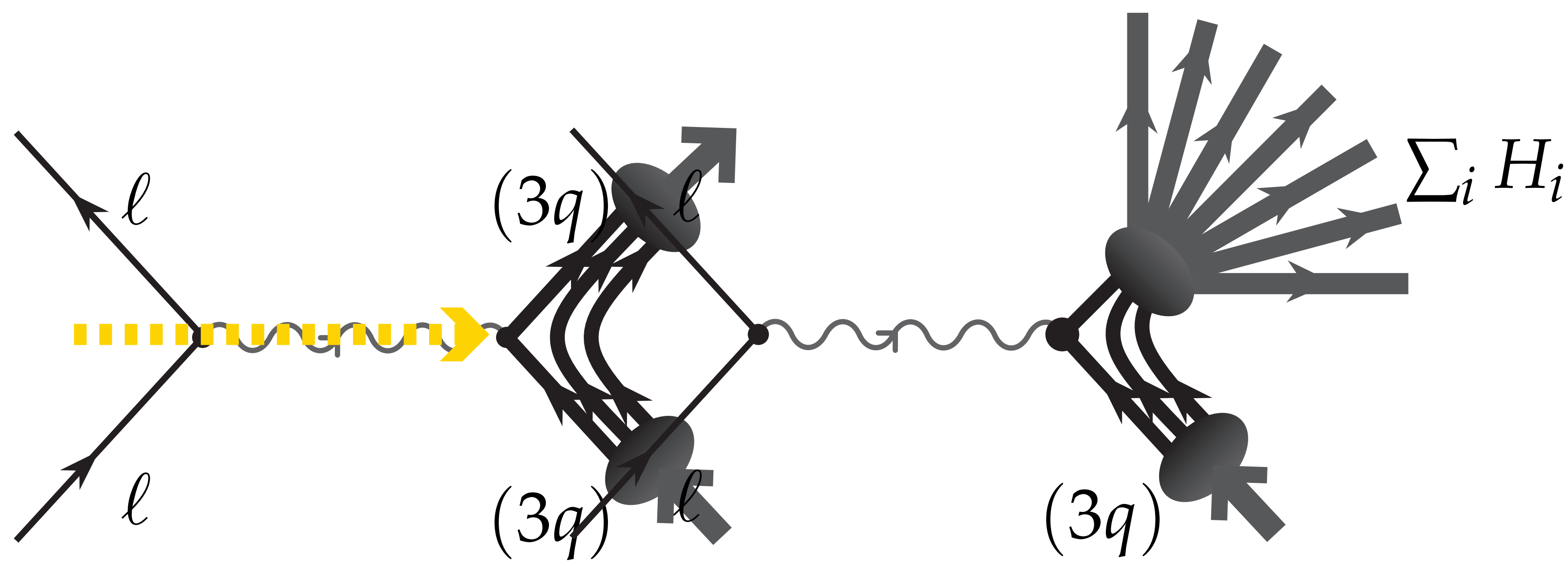
$$= \left(\frac{\alpha\hbar}{4ME\sin^2(\theta/2)}\right)^2 \frac{2K_1\sin^2(\theta/2) + K_2\cos^2(\theta/2)}{1 + (2E/Mc^2)\sin^2(\theta/2)},$$

- as computed in 1950 by M.N. Rosenbluth.
- The two functions, K_1 and K_2, are determined experimentally,
- …and describe the distribution of the quarks in the hadron.
- They are called "form factors," and are also the aim of the strong interaction theory—to be considered shortly.

The Electrodynamics of Lepton-Hadron Scattering

Deep Inelastic (Light) Lepton-Hadron Scattering

- Now:



$$d\sigma = \frac{\hbar^2 \left\langle |\mathfrak{M}|^2_{\ell p \to \ell X} \right\rangle}{4\sqrt{(\mathrm{p}_1 \cdot \mathrm{p}_2) - (m_1 m_2 c^2)^2}} \prod_{i=3}^{n} \left(\frac{c\,\mathrm{d}^3 \vec{p}_i}{(2\pi)^3 2E_i} \right) (2\pi)^4 \delta^4 (\mathrm{p}_1 + \mathrm{p}_2 - \sum_{j=3}^{n} \mathrm{p}_j),$$

$$\left\langle |\mathfrak{M}|^2_{\ell p \to \ell X} \right\rangle = \frac{g_e^4}{\mathrm{q}^4} X^{\mu\nu}(1,3;\ell \text{ (lepton)}) K_{\mu\nu}(2,4; X \text{ (hadrons)})$$

- If only the (lepton) probe deflection angle (distribution) and energy is measured, all else must be summed over.

The Electrodynamics of Lepton-Hadron Scattering

Deep Inelastic (Light) Lepton-Hadron Scattering

- This produces

$$d\sigma = \frac{4\pi M \hbar^2 g_e{}^4\, X^{\mu\nu}(1,3;\ell)}{4\mathrm{q}^4\sqrt{(\mathrm{p}_1\cdot\mathrm{p}_2)^2-(m_1 m_2 c^2)^2}}\Big(\frac{c\,d^3\vec{p}_3}{(2\pi)^3 2E_3}\Big)\,W_{\mu\nu},$$

$$W_{\mu\nu} := \frac{1}{4\pi M}\sum_X \int\cdots\int \prod_{i=4}^{n}\Big(\frac{c\,d^3\vec{p}_i}{(2\pi)^3 2E_i}\Big) \times (2\pi)^4\delta^4(\mathrm{p}_1+\mathrm{p}_2-\sum_{j=3}^{n}\mathrm{p}_j)\,K_{\mu\nu}(2,4;X)$$

- For the proton initially at rest, $\mathrm{p}_2=(Mc,\mathbf{0})$; $\mathrm{p}_1=(E/c,\boldsymbol{p}_i)$.
- Then, $\mathrm{p}_1\cdot\mathrm{p}_2 = E\,M$, and

$$\sqrt{(\mathrm{p}_1\cdot\mathrm{p}_2)^2-(m_1 m_2 c^2)^2} = \sqrt{M^2(E^2-m_\ell^2 c^4)} \approx ME$$

- since typically $m_\ell\, c^2 \ll E$ (relativistic probes).

The Electrodynamics of Lepton-Hadron Scattering

Deep Inelastic (Light) Lepton-Hadron Scattering

- We may thus approximate $m_\ell \approx 0$, and have

$$\mathrm{p}_1 = E(1, \hat{p}_i)/c \text{ and } \mathrm{p}_3 = E'(1, \hat{p}_f)/c.$$

- With this,

$$\mathrm{d}^3\vec{p}_f = |\vec{p}_f|^2 \mathrm{d}|\vec{p}_f| \mathrm{d}\Omega \approx c^{-3}(E')^2\, \mathrm{d}E' \mathrm{d}\Omega,$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E'\,\mathrm{d}\Omega} = \left(\frac{\alpha\hbar}{c\mathrm{q}^2}\right)^2 \frac{E'}{E}\, X^{\mu\nu}(1,3;\ell)\, W_{\mu\nu}.$$

- There is typically more than one resulting hadron (X), and so

$$\mathrm{p}_{\mathrm{tot}}^2 \neq M^2c^2, \quad \text{and} \quad E' \neq \frac{E}{1 + (2E/Mc^2)\sin^2(\theta/2)}.$$

- Also, we no longer have the relation $\mathrm{p}\cdot\mathrm{q} = -\tfrac{1}{2}\mathrm{q}^2$, so one defines

$$x := -\frac{\mathrm{q}^2}{2\mathrm{q}\cdot\mathrm{p}}.$$

The Electrodynamics of Lepton-Hadron Scattering

Deep Inelastic (Light) Lepton-Hadron Scattering

- The "form-factors" now stem from writing $W_{\mu\nu}(2,4;p^+)$ as:

$$W_1(\mathrm{q}^2,x)\Big(-\eta^{\mu\nu}+\frac{q^\mu q^\nu}{\mathrm{q}^2}\Big)+\frac{W_2(\mathrm{q}^2,x)}{M^2c^2}\Big(p^\mu+\tfrac{1}{2x}q^\mu\Big)\Big(p^\nu+\tfrac{1}{2x}q^\nu\Big),$$

- …which allows a Rosenbluth-like computation:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E'\mathrm{d}\Omega}=\Big(\frac{\alpha\hbar}{2ME\sin^2(\theta/2)}\Big)^2\frac{E'}{E}\big(2W_1\sin^2(\theta/2)+W_2\cos^2(\theta/2)\big).$$

- Rosenbluth's result is obtained in the special case when

$$W_i(\mathrm{q}^2,x)=-\frac{K_i(\mathrm{q}^2)}{2M\mathrm{q}^2}\,\delta(x-1),$$

- where the δ-function not only fixes $x=1$, but also implies

$$E'=\frac{E}{1+(2E/Mc^2)\sin^2(\theta/2)}.$$

The Electrodynamics of Lepton-Hadron Scattering

Experimental Verification of the Parton Model

- For an elastic $A+B \longrightarrow A'+B'$ collision, in target (B) frame:

$$\mathrm{p}_1 = (E/c, \vec{p}_i), \quad \mathrm{p}_3 = (E'/c, \vec{p}_f),$$
$$\mathrm{p}_2 = (Mc, \vec{0}), \quad \mathrm{p}_4 = (E''/c, \vec{P}_f).$$

- Write $m_A = m_{A'} = m$, and $m_B = m_{B'} = M$. Then:

$$\mathrm{q} = (\mathrm{p}_1 - \mathrm{p}_3) = \Big((E-E')/c, (\vec{p}_i - \vec{p}_f)\Big) = (\mathrm{p}_4 - \mathrm{p}_2) = \Big(E''/c - Mc, \vec{P}_f\Big),$$
$$\mathrm{q}\cdot\mathrm{p}_2 = M(E-E'),$$
$$\mathrm{q}^2 \approx -4\frac{EE'}{c^2}\sin^2(\theta/2) \qquad \text{when} \qquad mc^2 \ll E, E'.$$

- 1960's, J. Bjorken, within the quark model:

$$F_1(x) := M\,W_1(\mathrm{q}^2, x), \qquad \text{and} \qquad F_2(x) := \frac{-\mathrm{q}^2}{2Mc^2x}W_2(\mathrm{q}^2, x)$$

- … are asymptotically q^2-independent.

The Electrodynamics of Lepton-Hadron Scattering

Experimental Verification of the Parton Model

- This, "Bjorken scaling"

$$W_1(\mathrm{q}^2, x) \sim \tfrac{1}{M} F_1(x), \quad \text{and} \quad W_2(\mathrm{q}^2, x) \sim -\frac{2Mc^2x}{\mathrm{q}^2} F_2(x),$$

- ... was soon confirmed in deep inelastic scattering.
- It indicates that:
  - the 4-momentum transfer is mostly to one of the three quarks,
  - the quarks are much smaller than the proton,
  - ... so that quarks may as well be treated as point-like (elementary, substructure-less) particles.
- In 1969, C. Callan and D. Gross derived $2xF_1(x) = F_2(x)$,
- ... which soon confirmed experimentally.
- It indicates that quarks have spin ½$\hbar$.

The Electrodynamics of Lepton-Hadron Scattering

Experimental Verification of the Parton Model

- And, that's not all.
  - Assuming that each of the three quarks acquires a fraction, z_i, of the transferred momentum,
  - using the Bjorken scaling and Callan-Gross relations between $F_1(x)$ and $F_2(x)$,
  - representing the form-factor functions in terms of quark-in-hadron probability distributions, $f_i(x)$:

$$F_1(x) = \frac{1}{2}\sum_i {Q_i}^2 f_i(x), \qquad F_2(x) = x\sum_i {Q_i}^2 f_i(x),$$

  - …and using experimental data for $F_1(x)$ and $F_2(x)$, one obtains:

$$\int_0^1 \mathrm{d}x\; x f_d(x) \approx 0.18 \quad \text{and} \quad \int_0^1 \mathrm{d}x\; x f_u(x) \approx 0.36.$$

  - This adds up to 54% of the transferred momentum.
  - Hadrons contain something else, chargeless, that picks up the rest.

Gluons!

Now, 'bout Them *Student Presentations*

General Expectations

- Purpose
  - To explain to your class-mates the material you have chosen
  - To explore the topic starting with the text, but going beyond
    - Work out & present the details of a computation
    - Research additional literature and summarize
  - To convince the instructor you've done the the above
- Format
  - 30-minute presentation (blackboard or projection)
  - followed by 15 minutes of questions
    - If the students do not ask, the instructor will.
  - Audience will fill a 1-page questionnaire about each presentation
    - ...handed over to the presenter, for the presenter's benefit

Thanks!

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