(Fundamental) Physics of Elementary Particles

Bound states: non-relativistic H-atom and its various corrections as a paradigm, *vs*. positronium

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Fundamental Physics of Elementary Particles

Foreword

- Hadrons = bound states of quarks

Name	q		Mass* (MeV/c^2)	Q	I_3	B	S	C	B'	T	Y
Up	u	:	1.5–3.3	$+\frac{2}{3}$	$+\frac{1}{2}$	$\frac{1}{3}$	0	0	0	0	$+\frac{1}{3}$
Down	d	:	3.5–6.0	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{3}$	0	0	0	0	$+\frac{1}{3}$
Strange	s	:	$105\{^{+25}_{-35}$	$-\frac{1}{3}$	0	$\frac{1}{3}$	-1	0	0	0	$-\frac{2}{3}$
Charm	c	:	$1\,270\{^{+70}_{-110}$	$+\frac{2}{3}$	0	$\frac{1}{3}$	0	$+1$	0	0	$+\frac{4}{3}$
Bottom	b	:	$4\,200\{^{+170}_{-70}$	$-\frac{1}{3}$	0	$\frac{1}{3}$	0	0	-1	0	$-\frac{2}{3}$
Top	t	:	$171\,300\{^{+1\,100}_{-1\,200}$	$+\frac{2}{3}$	0	$\frac{1}{3}$	0	0	0	$+1$	$+\frac{4}{3}$

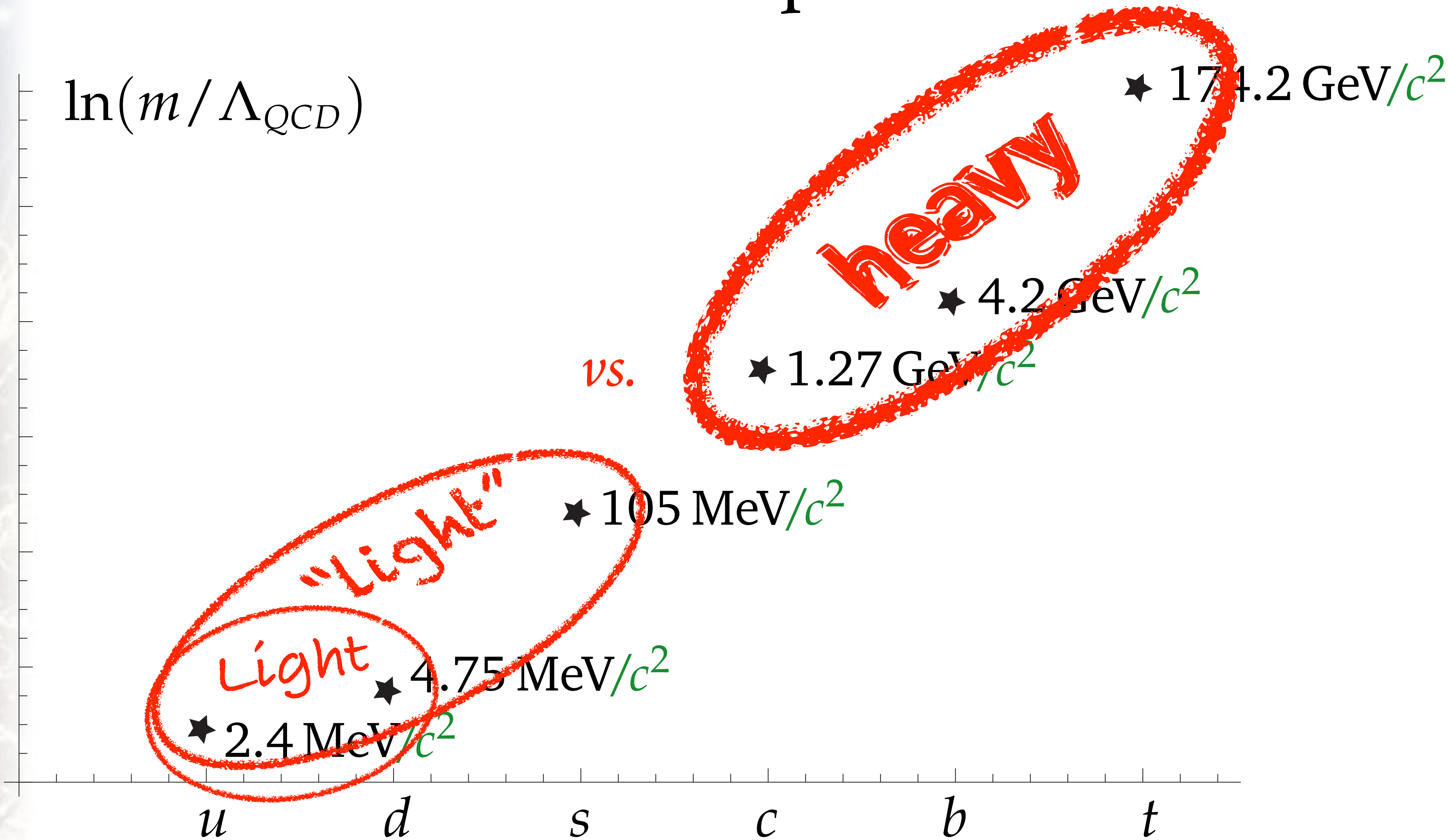
* Inertial mass without the binding energy, which depends on the hadron

$$Q = I_3 + \frac{1}{2}\underbrace{(Baryon + Strange + Charm + B'eauty + Truth)}_{=Y,\ \text{so-called (strong) hypercharge [☞ section 5.2.1]}}$$

Fundamental Physics of Elementary Particles

Foreword... cont'd

- Hadrons = bound states of quarks



Fundamental Physics of Elementary Particles

Playbill

- Bound States of Quarks
  - $(Q\text{-}\bar{Q})$ & $(Q\text{-}\bar{q})$ bound states: non-relativistic
  - $(q\text{-}\bar{q})$ bound states: relativistic
  - (qqq), … (QQQ): very complicated
- Non-relativistic H-atom as a paradigm
  - The Coulomb interaction & Bohr's result
  - 1st order (relativistic and magnetic dipole) corrections
  - 2nd order corrections & hierarchy
  - Lamb shift
- Positronium *estimates* by correcting the H-atom
  - Field drag
  - Annihilation: virtual & real
- OZI rule: phenomenological … later, derived from QCD.

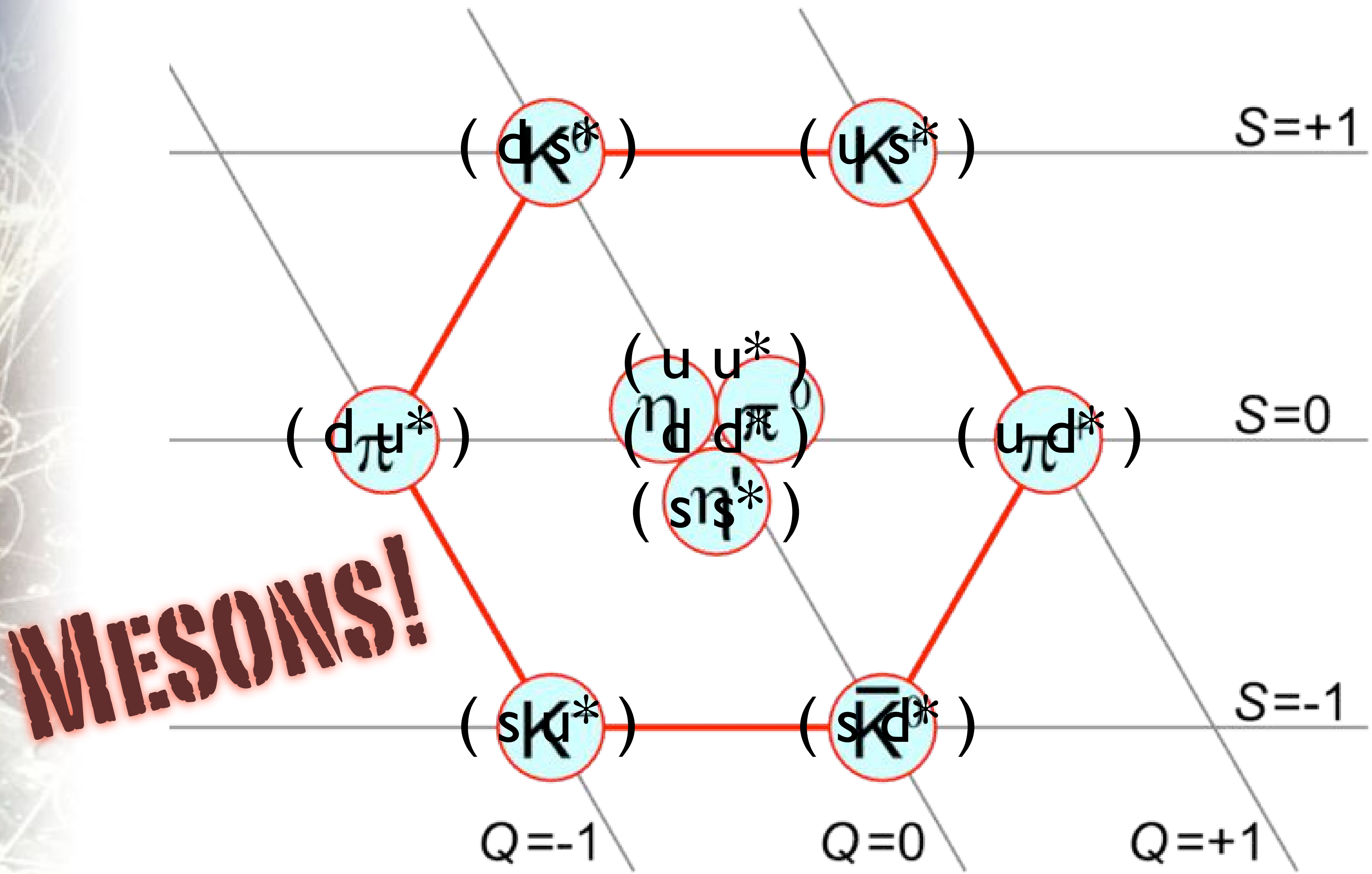
Binding strength $= \dfrac{\text{binding energy}}{\text{rest energy}}$

Binding strength < 1 ⇒ perturbative

Fundamental Physics of Elementary Particles

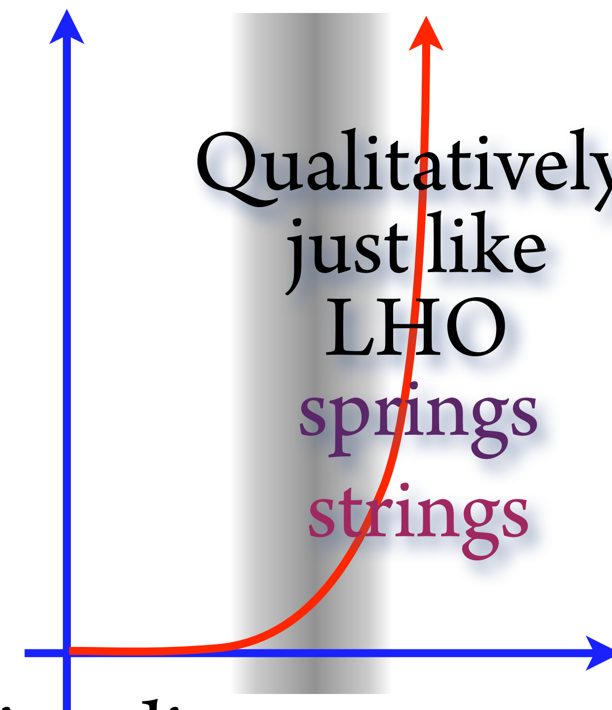
Quark bound states

- Bound $(q\bar{q})$ states with u, d & s quarks, of spin-0:

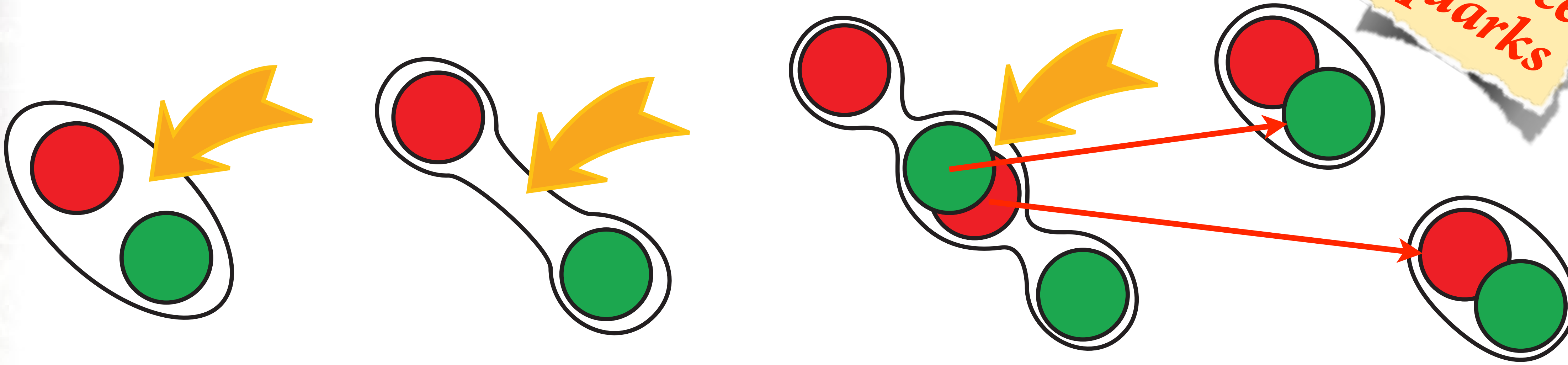


Fundamental Physics of Elementary Particles

Quark bound states



- Mesons: 2-particle bound states
  - Interaction may be modeled:
    - At distances < 10^{-15} m, vanishing force
    - Near ≈ 10^{-15} m, force grows w/distance
    - Just outside 10^{-15} m, force diverges
    - So that trying to extract a quark to a growing distance…
    - …requires a growing amount of energy…
    - …until that energy suffices to produce a $q\bar{q}$-pair:



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Quark bound states

- Mesons
  - Initial approximation: central potential
  - When one of the quarks is much heavier than the other,
    - $\big(\Psi(r,\theta,\phi) = \Sigma_{n,\ell}\, R_n(r)\; Y_\ell{}^m(\theta,\phi)\big)(\text{spin},\ldots)$
    - use $R_n(r)$ from the H-atom in this initial approximation
  - When both quarks have similar (large) masses,
    - use the same-mass (positronium) modification of H-atom as an initial approximation
- Baryons
  - 3-body = ((2-body)+1 body) bound states
    - Considerably more complicated…
    - …leave for later.

Fundamental Physics of Elementary Particles

H-atom, reminder

- The problem and its solution (Bohr/Schrödinger):

$$i\hbar\,\frac{\partial}{\partial t}\Psi(\vec{r},t) = H\Psi(\vec{r},t), \qquad H = \Big[-\frac{\hbar^2}{2m_e}\vec{\nabla}^2 + V(r)\Big],$$

$$\Psi_{n,\ell,m}(\vec{r},t) = e^{-i\omega t}\,\frac{u_{n,\ell}(r)}{r}\,Y_\ell{}^m(\theta,\phi), \qquad \omega = E_{n,\ell,m}/\hbar$$

$$-\frac{\hbar^2}{2m_e}\frac{\mathrm{d}^2u_{n,\ell}}{\mathrm{d}r^2} + \Big[V(r) + \frac{\hbar^2}{2m_e}\frac{\ell(\ell+1)}{r^2}\Big]u_{n,\ell} = E_{n,\ell}\,u_{n,\ell},$$

$$V(r) = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r} = -\frac{\alpha_e\hbar c}{r}, \quad \alpha_e := \frac{e^2}{4\pi\epsilon_0\hbar c},$$

$$E_n = -\frac{1}{2}\alpha_e^2\,mc^2\,\frac{1}{n^2}, \qquad n = 1,2,3\ldots$$

Fundamental Physics of Elementary Particles

H-atom, reminder

- Degeneracy!
  - $n = 1, 2, 3, 4, \ldots$ ("$n\to\infty$" = continuum/scattering limit),
  - $\ell = 0, 1, 2, \ldots (n-1)$,
  - $|m| \leq \ell$ and $\Delta m \in \mathbb{Z}$ (and since $\ell \in \mathbb{Z}$, then also $m \in \mathbb{Z}$),
  - $m_s = \pm 1/2$.

E_n does not depend on m, ℓ:

$$\sum_{\ell=0}^{n-1}\sum_{m=-\ell}^{\ell} 2 = 2\sum_{\ell=0}^{n-1}(2\ell+1) = 2\,n^2,$$

rotations

for $V(r) = -\frac{\varkappa}{r}$,

$$L_i := \frac{1}{\hbar}(\vec{r}\times\vec{p})_i = -i\,\varepsilon_{ij}{}^{k}\,x^j\frac{\partial}{\partial x^k}$$

Laplace-Runge-Lenz

$$\vec{A} := \vec{p}\times\vec{L} - m\,\varkappa\,\frac{\vec{r}}{r}$$

$$A_j = \frac{1}{\sqrt{2mH}}\Big[\frac{\hbar}{2i}\varepsilon_j{}^{kl}\Big(\frac{\partial}{\partial x^k}L_l + L_l\frac{\partial}{\partial x^k}\Big) - \frac{m\,\varkappa}{\hbar}\,\hat{\mathrm{e}}_j\Big]$$

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H-atom, reminder

- Because of the division by $\sqrt{H}$, the operators A_j are applicable only on eigenstates of H…
- …but those form a complete set, so the operators A_j in fact, may be applied on *any* state.

$$[L_j, L_k] = i\varepsilon_{jk}{}^l L_l, \quad [L_j, A_k] = -i\varepsilon_{jk}{}^l A_l, \quad [A_j, A_k] = \pm i\varepsilon_{jk}{}^l L_l, \quad \text{for } \begin{cases} E<0, \\ E>0; \end{cases}$$

- For $E < 0$ (bound states), this is the $\mathfrak{so}(4)$ algebra,
- For $E > 0$ (scattering states), this is the $\mathfrak{so}(1,3)$ algebra.
- Symmetries count degenerate states. *(In general, not always!)* *(But, all of them, here!)*
- Lifting the degeneracy = breaking a symmetry.

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H-atom, reminder

- Given the non-relativistic initial approximation …
- … check for corrections.
- Relativistic corrections:

These may well break some of the $\mathfrak{so}(4)$ symmetry & lift some of the degeneracy!

$$T_{\rm rel} = m_e c^2\Big[\sqrt{1+(\vec{p}/m_e c)^2}-1\Big] = m_e c^2 \sum_{k=1}^{\infty}\binom{1/2}{k}\Big(\frac{\vec{p}^{\,2}}{m_e^2 c^2}\Big)^k,$$

$$\approx \frac{\vec{p}^{\,2}}{2m_e} - \frac{(\vec{p}^{\,2})^2}{8m_e^3 c^2} + \frac{(\vec{p}^{\,2})^3}{16 m_e^5 c^4} - \dots$$

$$H'_{\rm rel} := -\frac{\hbar^4}{8m_e^3 c^2}(\vec{\nabla}^2)^2, \qquad H''_{\rm rel} := +\frac{\hbar^6}{16 m_e^5 c^4}(\vec{\nabla}^2)^3,$$

- Since

$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e\, e^2} = \frac{\hbar}{\alpha\, m_e c}, \quad \frac{\langle H''\rangle}{\langle H'\rangle} \sim \frac{\hbar^2}{2m_e^2 c^2}\langle\vec{\nabla}^2\rangle \sim \frac{\hbar^2}{2m_e^2 c^2}\frac{1}{a_0^2} = \tfrac{1}{2}\,\alpha^2.$$

Fundamental Physics of Elementary Particles

H-atom, reminder

$$[H,V] = [\tfrac{\hbar^2}{2m_e}\vec{\nabla}^2 + V, V] = \tfrac{\hbar^2}{2m_e}[\vec{\nabla}^2, -\tfrac{\varkappa}{r}],$$
$$= -\tfrac{\hbar^2\varkappa}{2m_e}[\vec{\nabla}^2, \tfrac{1}{r}] = \tfrac{2\pi\hbar^2\varkappa}{m_e}\delta(r)$$

This is nonzero only for $\ell = 0$.

Why?

- In computations, use

$$\frac{\hbar^2}{2m_e}\vec{\nabla}^2 = V(r) - H,$$

- so that:

$$\langle H'_{\rm rel}\rangle = -\frac{1}{2m_e c^2}\langle [V^2 - HV - VH + H^2]\rangle,$$

$$\langle H''_{\rm rel}\rangle = -\frac{1}{2m_e^2 c^4}\langle [V^3 - VHV - V^2H + VH^2$$
$$- HV^2 + H^2V + HVH - H^3]\rangle$$

- Use this also for arbitrary matrix elements, but will have to re-diagonalize the $|n,\ell,m,m_s\rangle$ basis …
- … for the 2nd and higher order perturbations.

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H-atom, reminder

- The 1st relativistic correction, to 1st order produces:

$$E_n^{(1,r_1)} = -\frac{1}{2m_e c^2}\Big[\langle V^2\rangle - 2E_n^{(0)}\langle V\rangle + (E_n^{(0)})^2\Big],$$

$$= -\alpha_e^4\, m_e c^2\,\frac{1}{4n^4}\Big[\frac{2n}{(\ell+\frac{1}{2})} - \frac{3}{2}\Big].$$

- Compared with $E_n = -\frac{1}{2}\alpha_e^2\, m c^2\,\frac{1}{n^2}$,
- …the correction is indeed of $O(\alpha^4)$.
- It depends on ℓ, so it breaks the Laplace-Runge-Lenz symmetry and lifts some of the degeneracy.
- Higher corrections will refine this result.

Fundamental Physics of Elementary Particles

H-atom, reminder

- Consider now the magnetic corrections.
- Both e^- and p^+ have intrinsic magnetic dipole moments.
- They are charged, so their relative motion produces an (orbital) electric current and a magnetic field.
- These three magnetic fields interact pair-wise, and contribute to the energy:

$$\vec{\mu} = \frac{q}{2m}\vec{L} \quad\Rightarrow\quad \vec{\mu}_e =: \frac{(-e)}{2m_e}\vec{S}_e, \quad etc.$$

$$H_{S_eO} = -\vec{\mu}_e \cdot (\tfrac{1}{2}\vec{B}),$$
$$H_{S_pO} = -\vec{\mu}_p \cdot \vec{B},$$
$$H_{S_eS_p} = -\frac{\mu_0}{4\pi}\Big[\Big(3(\vec{\mu}_e\cdot\hat{r})(\vec{\mu}_p\cdot\hat{r}) - \vec{\mu}_e\cdot\vec{\mu}_p\Big)\frac{1}{r^3} + \frac{8\pi}{3}\vec{\mu}_e\cdot\vec{\mu}_p\,\delta^3(\vec{r}\,)\Big].$$

- The dipole moments were attributed to spinning…

…but, there is no teensy spinning sphere.

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H-atom, reminder

- For what it is worth, the observable magnetic dipole moment and the half-integral "spin" of an electron are related:

$$\vec{\mu}_e = 2\Big(1 + \frac{\alpha_e}{2\pi} + \dots\Big)\frac{(-e)}{2m_e}\,\vec{S}$$

$$\vec{\mu}_e = -g_e \mu_B \vec{S}_e, \qquad \mu_B := \frac{e}{2m_e}, \qquad g_e = 2.002\,319\,304\,361\,7(15) \approx 2;$$

$$\vec{\mu}_p = +g_p \mu_N \vec{S}_p, \qquad \mu_N := \frac{e}{2m_p}, \qquad g_p = 2.7928,$$

$$H_{S_eO} = -\Big(\frac{g_e(-e)}{2m_e}\hbar\vec{S}_e\Big)\cdot\Big(\tfrac{1}{2}\frac{e}{4\pi\epsilon_0 m_e r^3}\hbar\vec{L}\Big) \approx \frac{e^2}{4\pi\epsilon_0}\frac{\hbar^2}{2m_e^2c^2}\frac{1}{r^3}\,\vec{L}\cdot\vec{S}_e,$$

$$H_{S_pO} = \frac{g_p\, e^2}{4\pi\epsilon_0}\frac{\hbar^2}{m_e m_p c^2}\frac{1}{r^3}\,\vec{L}\cdot\vec{S}_p,$$

$\sim 1/2{,}000$

$$H_{S_eS_p} \approx \frac{g_p\, e^2}{4\pi\epsilon_0}\frac{\hbar^2}{m_e m_p c^2}\Big[\Big(3(\vec{S}_e\cdot\hat{r})(\vec{S}_p\cdot\hat{r}) - \vec{S}_e\cdot\vec{S}_p\Big)\frac{1}{r^3} + \frac{8\pi}{3}\vec{S}_e\cdot\vec{S}_p\,\delta^3(\vec{r}\,)\Big]$$

Fundamental Physics of Elementary Particles

H-atom, reminder

- The S_eO correction, to 1st order produces:

$$\langle H_{S_eO}\rangle = \frac{e^2}{4\pi\epsilon_0}\frac{\hbar^2}{2m_e^2c^2}\left\langle \frac{1}{r^3}\,\vec{L}\cdot\vec{S}_e\right\rangle \sim \frac{\alpha_e\hbar^3}{2m_e^2c}\cdot\frac{1}{a_0{}^3} = \frac{\alpha_e^4 m_e c^2}{2}$$

$$E_n^{(1,SO)} = \alpha_e^4 m_e c^2\,\frac{j(j+1)-\ell(\ell+1)-\frac{3}{4}}{4n^3\ell(\ell+\frac{1}{2})(\ell+1)}$$

$$\vec{J} := \vec{L}+\vec{S} \qquad \Rightarrow \qquad \vec{L}\cdot\vec{S} = \tfrac{1}{2}\left[J^2 - L^2 - S^2\right]$$

- …which is commensurate with the 1st relativistic correction, to the first order, and they add up:

$$E_n^{(1,r_1)} + E_n^{(1,SO)} = -\alpha_e^4\, m_e c^2\,\frac{1}{4n^4}\left[\frac{2n}{(j+\frac{1}{2})} - \frac{3}{2}\right], \qquad \begin{cases} j &= \ell + {}^1\!/_2, \\ j &= \ell - {}^1\!/_2; \end{cases}$$

- …resulting in the $O(\alpha^4)$ fine structure (splitting).

Fundamental Physics of Elementary Particles

H-atom, reminder

- The next order corrections are:

$$E_n^{(1,r_2)} = \langle H''_{\rm rel}\rangle \sim \frac{1}{m_e^2 c^4}\left\langle\left(\frac{e^2}{4\pi\epsilon_0\, r}\right)^3\right\rangle \sim \frac{\alpha_e^6\, m_e c^2}{n^3};$$

$$\left(\frac{m_e}{m_p}\right) > \alpha_e{}^2$$

$$E_n^{(2,r_1)} = \sum_{n'\cdots\neq n\cdots}\frac{|\langle n',\cdots|H'_{\rm rel}|n,\cdots\rangle|^2}{E_n^{(0)} - E_m^{(0)}} \sim \frac{\alpha_e^6 m_e c^2}{n^4};$$

$$E_n^{(1,S_pO)} = \langle H_{S_pO}\rangle = \frac{g_p\, e^2}{4\pi\epsilon_0}\frac{\hbar^2}{m_e m_p c^2}\left\langle\frac{1}{r^3}\vec{L}\cdot\vec{S}_p\right\rangle \sim g_p\left(\frac{m_e}{m_p}\right)\frac{\alpha_e^4 m_e c^2}{n^3};$$

$$E_n^{(1,S_eS_p)} = \langle H_{S_eS_p}\rangle \sim \frac{g_p\, e^2}{4\pi\epsilon_0}\frac{\hbar^2}{m_e m_p c^2}\left\langle\vec{S}_e\cdot\vec{S}_p\frac{1}{r^3}\right\rangle \sim g_p\left(\frac{m_e}{m_p}\right)\frac{\alpha_e^4 m_e c^2}{n^3}.$$

$$E_n^{(1,r_2)} : E_n^{(2,r_1)} : E_n^{(1,S_pO)} : E_n^{(1,S_eS_p)} \approx n\alpha_e^2 : \alpha_e^2 : g_p\left(\frac{m_e}{m_p}\right) : g_p\left(\frac{m_e}{m_p}\right)$$

Fundamental Physics of Elementary Particles

H-atom, reminder

- So the hyper-fine structure is given by:

$$E_n^{(1,S_eS_p)} + E_n^{(1,S_pO)} = \Big(\frac{m_e}{m_p}\Big)\alpha_e^4\, m_e c^2\, \frac{g_p}{2n^3}\, \frac{\pm 1}{(f+\frac{1}{2})(\ell+\frac{1}{2})}, \qquad \begin{cases} f &= j + {}^1\!/\!_2, \\ f &= j - {}^1\!/\!_2; \end{cases}$$

$$\vec{F} := \vec{J} + \vec{S}_p = \vec{L} + \vec{S}_e + \vec{S}_p$$

- Note:

$$\vec{Z} := \vec{S}_e + \vec{S}_p$$

$$\frac{\pm 1}{(f+\frac{1}{2})(\ell+\frac{1}{2})} = \begin{cases} +\frac{4}{3}, \\ -4; \end{cases} \qquad \begin{cases} z &= 1 \ \ \text{(triplet)}, \\ z &= 0 \ \ \text{(singlet)}. \end{cases}$$

- After this there is the $O(\alpha^6)$ hyper-hyper-fine structure, and so on.
- But, that's not all.

Fundamental Physics of Elementary Particles

H-atom, reminder

- Lamb shift: recall that

$$\vec{\mu}_e = 2\big(1+\frac{\alpha_e}{2\pi}+\dots\big)\frac{(-e)}{2m_e}\vec{S}$$

- … so that the $O(\alpha)$ correction in the magnetic dipole moment produces an $O(\alpha^5)$ correction to the S_eO term:

$$E_n^{\text{Lamb}} = \begin{cases} \alpha^5 m_e c^2 \frac{1}{4n^3}\, k(n,0) & \ell = 0; \\ \alpha^5 m_e c^2 \frac{1}{4n^3}\Big[k(n,\ell) \pm \frac{1}{\pi(j+\frac{1}{2})(\ell+\frac{1}{2})}\Big], \quad j=\ell\pm\frac{1}{2}, & \ell \neq 0. \end{cases}$$

slowly varying functions

- Since $\alpha > (m_e/m_p) > \alpha^2$, the Lamb shift is ~5 times ***bigger*** than the hyperfine shift.

$$E_n^{(1,r_2)} : E_n^{(2,r_1)} : E_n^{(1,S_pO)} : E_n^{(1,S_eS_p)} : E_n^{(QED)} \approx n\alpha_e^2 : \alpha_e^2 : g_p\Big(\frac{m_e}{m_p}\Big) : g_p\Big(\frac{m_e}{m_p}\Big) : \alpha_e,$$

$$\approx (5.33\times10^{-5}\cdot n) : (5.33\times10^{-5}) : (1.52\times10^{-3}) : (1.52\times10^{-3}) : (7.30\times10^{-3}).$$

Fundamental Physics of Elementary Particles

Positronium, a la H-atom

- Mesons, where the quark and the antiquark have similar masses, are well approximated by positronium.
- Now,

$$\frac{m_{e^+} m_{e^-}}{m_{e^+} + m_{e^-}} = \tfrac{1}{2} m_e$$

$$E_n(e^+, e^-) = \tfrac{1}{2} E_n(H) = -\alpha_e^2 m_e c^2 \, \frac{1}{4n^2}$$

- The wave-functions look identical, except that the Bohr radius $a_0 \to 2a_0$.
- The 1st relativistic correction to the Hamiltonian (kinetic energy) doubles, as e^- and e^+ both contribute…
- …but $\langle (\vec{p}^2)^2 \rangle \propto (m_e c)^4$ is reduced by $(½)^4$.
- The relativistic energy correction acquires a ⅛ factor.

Fundamental Physics of Elementary Particles

Positronium, a la H-atom

- The magnetic corrections acquire factors that all stem from the facts that
  - $m_e/m_p \to m_{e-}/m_{e+} = 1,\ g_e/g_p \to g_{e-}/g_{e+} = 1,$ and
  - Thomas precession is symmetric between e^- and e^+.
- This variety of changed ratios makes, e.g., the "hyperfine" splitting as large as the "fine" splitting…
- but leaves the Lamb shift $\alpha \sim 1/_{137}$ times smaller,
- and the $O(\alpha^6)$ corrections smaller still.
- But there also exist two profoundly novel corrections to the energy, with no analogue in the H-atom.

Positronium Novelties

- Field latency:
  - As e^{+} is as light as e^{-}, their orbiting each other is symmetric.
  - As one moves in the Coulomb field created by the other…
  - …that Coulomb field itself moves (with the other), and the changes in it propagate (lag) at the speed of light.

$$H_{\rm lat} = -\frac{e^2}{4\pi\epsilon_0}\frac{1}{2m_e^2c^2}\frac{1}{r}\left(p^2 + (p\cdot\hat{r})^2\right)$$

$$E_n^{(\rm lat)} = \langle H_{\rm lat}\rangle = \alpha_e^4 m_e c^2\,\frac{1}{2n^3}\left[\frac{11}{32n} - \frac{2+\epsilon}{\ell+{}^1\!/_2}\right]$$

$$\text{…where}\quad \epsilon = \begin{cases} 0 & \text{for } j=\ell, \quad s=0 \\ -\frac{3\ell+4}{(\ell+1)(2\ell+3)} & \text{for } j=\ell+1, \\ \frac{1}{\ell(\ell+1)} & \text{for } j=\ell, \\ \frac{3\ell-1}{\ell(2\ell-1)} & \text{for } j=\ell-1, \end{cases} \quad \Big\} \; s=1$$

Fundamental Physics of Elementary Particles

Positronium Novelties II

- Virtual annihilation:
- Unlike in the H-atom, the e^- and e^+ in the positronium can annihilate and be re-created.
- For this, they must be in the same location, so $\propto |\Psi(0)|^2$, i.e., $\ell = 0$ only. (Why?)
- Since spin$(\gamma) = 1$, spin$(e^-,e^+) = 1$ too, i.e., the positronium must be in its *triplet* state(s).

$$E_n^{(ann)} = \alpha_e^4 m_e c^2 \frac{1}{4n^3}, \qquad \ell = 0,\ s = 1.$$

- … which is of the order of fine structure, but exclusive to "spin-triplet S-states."

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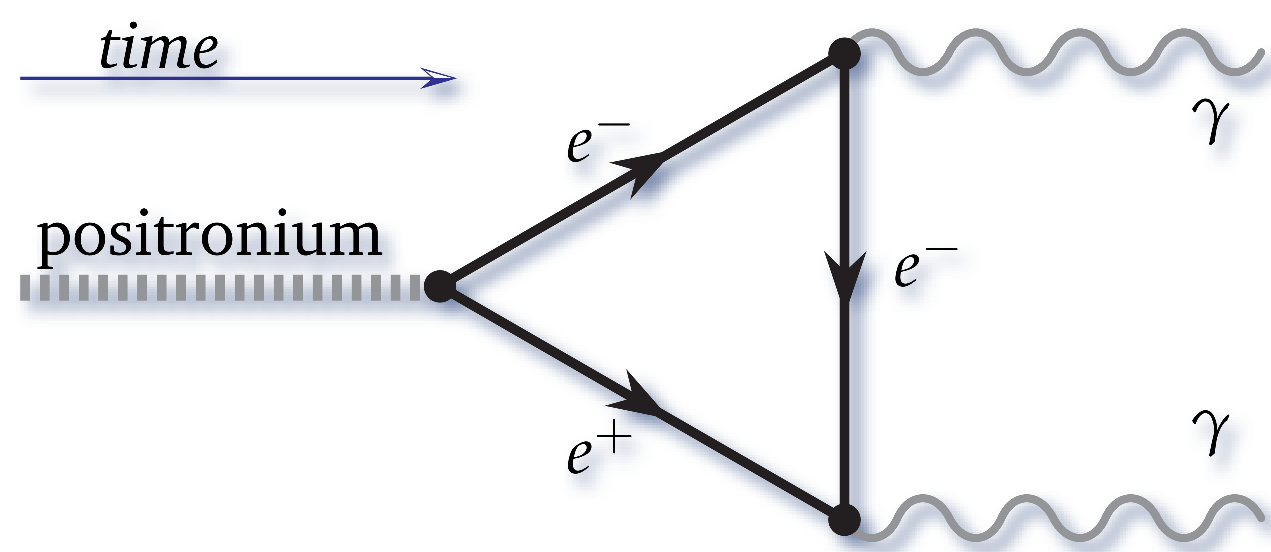
Positronium Novelties II

- Real annihilation:



$$\Gamma = \sigma\, v\, |\Psi(\vec{0}, t)|^2$$

$$\sigma = \int \mathrm{d}^2\Omega \left(\frac{\hbar c}{8\pi}\right)^2 \frac{|\mathfrak{M}|^2}{(E_{e^-} + E_{e^+})^2} \left|\frac{\vec{p}_f}{\vec{p}_i}\right| = 4\pi \frac{\alpha_e^2 \hbar^2}{m_e^2 c\, v}.$$

$$\tau = \frac{1}{\Gamma} = \frac{2\,\hbar\, n^3}{\alpha_e^5 m_e c^2} = (1.24494 \times 10^{-10}\,\mathrm{s}) \times n^3,$$

- …which agrees with experiments.

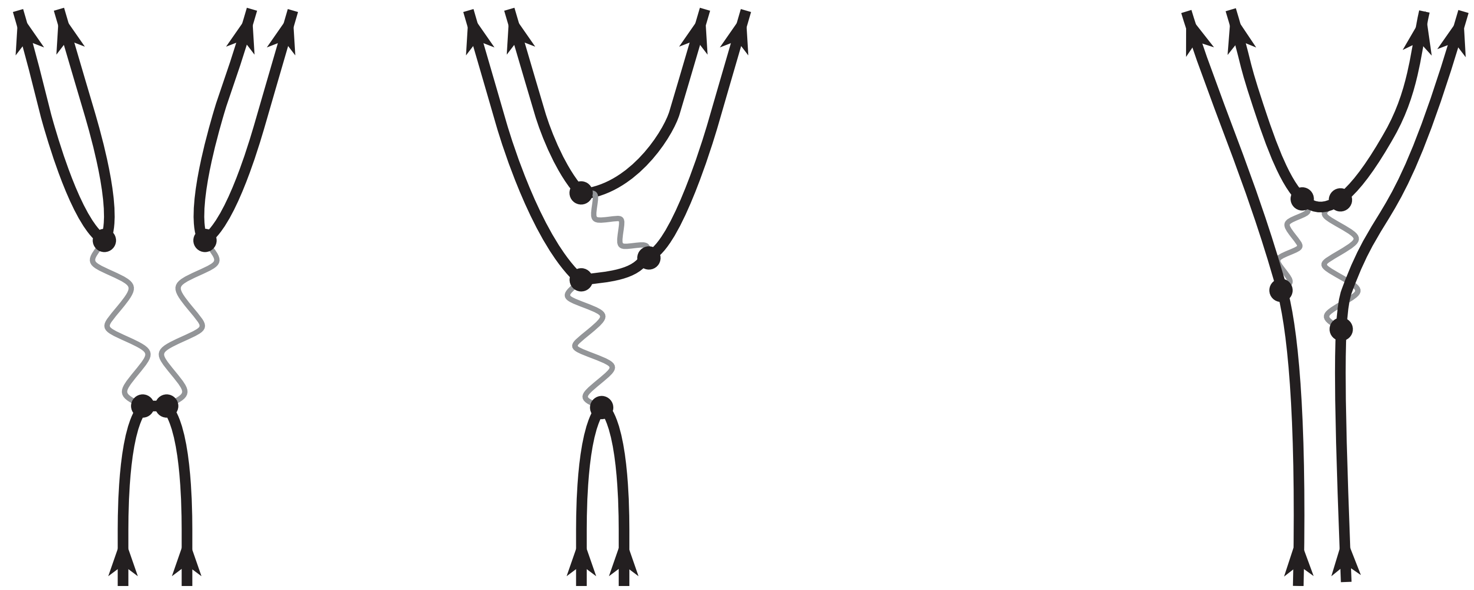
Fundamental Physics of Elementary Particles

Finally, Something Else...

- The "OZI rule" (Okubo-Zweig-Iizuka, 1960's):

> Decays that require the annihilation of all initially present partons are delayed.

- Or, in terms of Feynman graphs,



Decays are delayed ... as compared to this.

Fundamental Physics of Elementary Particles

Quark Bound States

- The general idea is to use the results from the H-atom and positronium, and adapt them by:
  - varying the masses of the constituent quarks
  - changing the fine structure constant, α,

    from the electromagnetic ($\sim 1/_{137}$) to the strong ($\sim 1/_{10}$–1)
  - adapting the spin orientations (triplet for positronium annihilation) as needed (to include *flavor* and *color*)
- …and this works remarkably well!
- …so people talked of "paper-writing algorithm,"
  - recycling (preferably obscure) papers on E&M
  - into papers on strong interactions.

& don't forget to change the Authors' names.

Thanks!

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http://homepage.mac.com/thubsch/