(Fundamental) Physics of Elementary Particles

Perturbation !eory in Picutres: Feynman Diagrams

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Program **Fundamental Physics of Elementary Particles**

- Perturbation Theory in Pictures
	- Stationary perturbation theory in Quantum Mechanics
	- · Feynman diagrams in perturbative Quantum Field Theory
		- The Heisenberg Zone and What's Going On
		- A toy-model work-out
- A Few Stray Comments
	- An overview of the subject topic
	- A review of all conservation laws

1.3.2 Perturbation Picturebook

As QUANTUM MECHANICS: WARM-UP CALISTHENICS

Consider a quantum-mechanical system where the **Example 11** Hamiltonian differs from a well-known one by H' . The We want to find solutions to: the relations (–1.11)–(–1.13) are very often listed and derived in all textbooks. Most all textbooks. $\text{t}_\text{t} = \text{t}_\text{t} =$ the relations (–1.11)–(–1.13) are very often listed and derived in all textbooks. Most all textbooks. $\text{t}_\text{t} = \text{t}_\text{t} =$ 104 *Chapter 104* Physics in Spacetime 1. Physics in S **is the We want to find solutions to:** \bullet

 $H|n\rangle = E_n|n\rangle$, $H := H_0 + \lambda H'$ is the "true" Hamiltonian, given as a sum of a "known" Hamiltonian *H*⁰ and a "perturbation" $H|n\rangle = E_n|n\rangle$, and $H:=H_0+\Lambda H$, and where *l* serves to consistently count the order of perturbation. Suppose that for the $H|n\rangle = E_n|n\rangle$, $H := H_0 + \lambda H$

 \mathcal{A} **P** ... where λ is a bookkeeping parameter. \mathbb{Z} is known:

Given that we *do* know:

$$
H_0|n;0\rangle = E_n^{(0)}|n;0\rangle, \qquad \begin{cases} \langle n;0|n';0\rangle &= \delta_{n,n'},\\ \sum_n |n;0\rangle\langle n;0| &= 1, \end{cases}
$$

…we expand as a power-series in *λ*. and the solutions of \mathbf{I} are solutions of \mathbf{I} are source for \mathbf{I} α and α are solutions of α are solutions of α

$$
E_n = \sum_{k=0}^{\infty} \lambda^k E_n^{(k)}, \qquad |n\rangle = \sum_{k=0}^{\infty} \lambda^k |n; k\rangle
$$

Perturbation Picturebook ⇢ ^h*n*; 0*|n*⁰ ; 0i = *dn*,*n*⁰ ,

 Q UANTUM MECHANICS: WARM-UP CALISTHENICS *^l^k [|]n*; *^k*i, (1.85) *^H*0*|n*; 0ⁱ ⁼ *^E*(0) *ⁿ |n*; 0i, Â*ⁿ |n*; 0ih*n*; 0*|* = **1**, (1.84)

We demand that order-by-order, the bases are orthonormalized (always doable, by Gram-Schmidt!) $\overline{1}$ α ^{th community directly</sub>}

$$
\langle m; k | n; k \rangle = \delta_{mn}, \forall m, n,
$$

but also that the "same" kets from different orders of perturbation are orthogonal: *k*=0 *k*=0

$$
\langle n; k | n; \ell \rangle = \delta_{k\ell}, \quad \forall k, \ell.
$$

a

 λ | \prime

|*n*;*k*〉*+λ*|*n*;*k+*1〉

|*n*;*k*〉

n | *n*;*k*+1/

|*n*;*k+*1〉

Why? For consistency of the normalization! $\mathbf n$ $\overline{cy'}$ of the *|m*; 0ih*m*; 0*|* (*E*(0) *ⁿ E*(0) *^m*) $ation!$ \mathbb{Z} why: For consistency of the normalization:

4 so that the superscript in Parties of the b *ⁿ* really behaves as an exponent. With this notation, the standard recursive formulation to the state and energy are corrected to the state and energy are stated to the state and energy are continuous are continuous correction to the state and energy are continuous correction to the state *|n*; *k*i = P \overline{a} *ⁿ H*⁰ *|n*; *k*1i $\begin{array}{c} \n\bullet \text{ } N \\
\downarrow \text{ } N\n\end{array}$ *m*6=*n* $\langle n;k|n;k\rangle + i$ $+ \mathbb{R}^2 \langle n; k - \rangle$ -2 $\mathcal{R}e(\lambda \langle n; k \, | \, n;$ $\langle n; k | n; k \rangle + 2 \Re(\lambda \frac{\langle n; k | n; k+1 \rangle}{\langle n; k | n; k+1 \rangle})$ ^b *^a*+*^b n* , (1.87) *+*|*λ*|2〈*n*;*k+*1|*n*;*k+*1〉

 λ

ⁿ := Â

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Quantum Mechanics: Warm-Up Calisthenics The treatment of the treatment of the general situation with Γ and Γ and Γ and Γ trum is only technically more complicated*14*, and so will not be discussed here. Introduce The treatment of the treatment of the general situation with Γ and Γ and Γ and Γ trum is only technically more complicated*14*, and so will not be discussed here. Introduce

• Now introduce specially weighted projectors:

 \prod \overline{a} $\frac{\alpha}{n}$:= \sum $m{\neq}n$ $\ket{m;0}\bra{m;0}$ $(E_n^{(0)} - E_m^{(0)})$ $\frac{1}{\alpha}$, so Γ $\widehat{\Pi}_n^{\alpha} := \sum_{P \in \mathcal{P}} \frac{|m;0\rangle \langle m;0|}{\langle E^{(0)} \rangle^{R}}$, so $\widehat{\Pi}_n^{\alpha} \widehat{\Pi}_n^{\beta} = \widehat{\Pi}_n^{\alpha+\beta}$ \mathbf{I}_1 $\frac{a}{a}$:= \sum $n \neq n$ $\langle m; 0 \rangle \langle m; 0 |$ $(E^{(0)}_{n} - E^{(0)}_{m})^{\rho}$ \overline{a} , so Π \hat{a} _n $\widehat{\Pi}_n^{\beta} = \widehat{\Pi}_n^{\alpha+\beta}$

They project back to the well-known, initial basis, *away from* $|n;0\rangle$, and curiously normalized. Note the no-degeneracy assumption! With this notation (≈Cohen-Tannoudji, Diu, Laloë), \overline{a} **ready from** $\ket{n;0}$, and curiously normalized. *k*1 *k*1 b **really away from** $\vert n;0\rangle$, and curiously normalized.

$$
H_0 + \lambda H' \left[\left(\sum_{k=0}^{\infty} \lambda^k |n; k \right) \right] = \left(\sum_{k'=0}^{\infty} \lambda^{k'} E_n^{(k')} \right) \left(\sum_{k''=0}^{\infty} \lambda^{k''} |n; k'' \right)
$$

…from where one obtains "order-by-order equations": T first several iterations of the first several iterations of these recursive formulae are: T and T are: T and T are: T and T are: T

ⁿ = h*n*; 0*|H*⁰

h

ⁿ = h*n*; 0*|H*⁰

E(1)

E(1)

Perturbation Picturebook P b *r* haf *m*6=*n |m*; 0ih*m*; 0*|* (*E*(0) *ⁿ E*(0) *^m*) *Diefunce n* and *k* b *ⁿ* := Â *m*6=*n* (*E*(0) *ⁿ E*(0) *^m*) *p*^{*h*}*a***+***b***^{***n***}</sub>^{***h***}^{***a***</sub>**} *ⁿ* , (1.87) (*E*(0) *ⁿ E*(0) *^m*) so that the superscript in P *a ⁿ* really behaves as an exponent. With this notation, the standard ¨ubsch, thubsch@howard.edu, with any comments / suggestions / corrections; thank you! — DRAFT *m*6=*n* (*E*(0) *ⁿ E*(0) *^m*) so that the superscript in P *a ⁿ* really behaves as an exponent. With this notation, the standard ¨ubsch, thubsch@howard.edu, with any comments / suggestions / corrections; thank you! — DRAFT *m*6=*n ⁿ E*(0) s turn supersion b *n* really below this notation, the standard stan ¨ubsch, thubsch@howard.edu, with any comments / suggestions / corrections; thank you! — DRAFT $=$ \blacksquare

Quantum Mechanics: Warm-Up Calisthenics **WOANTUM MECHANICS: WARM-UP GALISTHENICS SO THANTUM MECH** \mathbf{E} **ANICS: WARM-UP CALISTHENICS** b **ANTUM MECHANICS: WARM-UP CALISTHENICS** b **ANTUM MECHANICS: WARM-UP CALISTHENICS ANTUM MECHANICS: WARM-UP CALISTHENICS**

• The "order-by-order" equations: recursive formulae for the *k*th correction to the state and energy are: \int_0^L equations:

$$
|n;k\rangle = \widehat{\Pi}_n^1 H'|n;k-1\rangle - \sum_{i=1}^{k-1} E_n^{(i)} \widehat{\Pi}_n^1 |n;k-i\rangle, \qquad k > 0,
$$

$$
E_n^{(k)} = \langle n;0|H'|n;k-1\rangle.
$$

The first few of these are easy: \blacksquare The first few of these are easy: T_{net} formula formulate formulae areas recovered as T_{net}

$$
E_n^{(1)} = \langle n; 0 | H' | n; 0 \rangle,
$$

\n
$$
|n; 1 \rangle = \hat{\Pi}_n^1 H' |n; 0 \rangle,
$$

\n
$$
E_n^{(2)} = \langle n; 0 | H' \hat{\Pi}_n^1 H' | n; 0 \rangle
$$

\n
$$
|n; 2 \rangle = \hat{\Pi}_n^1 (H' - E_n^{(1)}) |n; 1 \rangle,
$$

\n
$$
= \hat{\Pi}_n^1 H' \hat{\Pi}_n^1 H' |n; 0 \rangle - \hat{\Pi}_n^1 \hat{\Pi}_n^1 H' |n; 0 \rangle \langle n; 0 | H' |n; 0 \rangle,
$$

\n
$$
= [\hat{\Pi}_n^1 H' \hat{\Pi}_n^1 - \hat{\Pi}_n^2 H' |n; 0 \rangle \langle n; 0 |] H' |n; 0 \rangle,
$$

 \mathbb{X}

 ϕ

 \mathbb{Z}

 \hat{z}

|n; 1i = P

|n; 1i = P

|n; 2i = P

|n; 2i = P

Perturbation Picturebook *[|]n*; 1ⁱ ⁼ ^P^b ¹ *ⁿ H*⁰ *|n*; 0i, (1.89b) **r** du pau di Figuuraboon \mathcal{L}

Quantum Mechanics: Warm-Up Calisthenics *ⁿ*(*H*⁰ *^E*(1) *ⁿ*)*|n*; 1i, \Box UANTUM

Soon enough, the complications grow: ⁼ ^P^b ¹ *ⁿ ^H*⁰ ^P^b ¹ *ⁿ H*⁰ *[|]n*; 0i ^P^b ¹ *ⁿ* ^P^b ¹ *ⁿ H*⁰ *|n*; 0ih*n*; 0 *H*⁰ nough, the complications grow:

…so the energy correction also requires subtractions… *^E*(0) *^m E*(0) *n* $E_n^{(3)} = \langle n; 0 | H^n | n; 2 \rangle$ $= \langle n; 0|H'[\hat{\Pi}^1_n H' \hat{\Pi}^1_n - \hat{\Pi}^2_n H'|n; 0\rangle \langle n; 0|]H'|n; 0\rangle$ $\mathcal{I} = \langle h, \mathbf{0} | \mathbf{1} \mathbf{$ *m***. so the energy correction also requires subtractions...** $= \langle n;0|H'\,\widehat{\Pi}^1_n\,H'\,\widehat{\Pi}^1_n\,H'|n;0\rangle - \langle n;0|H'\,\widehat{\Pi}^2_n\,H'|n;0\rangle\langle n;0|H'|n;0\rangle,$ le energy correction also requires subtractions…

 $\hat{\Pi}_{n}^{1}((H'-E_{n}^{(1)})|n;2\rangle - E_{n}^{(2)}|n;1\rangle)$ *n*, 0i Pb 1 $=\widehat{\Pi}_{n}^{1}H^{\prime}|n;2\rangle - \widehat{\Pi}_{n}^{1}|n;2\rangle\langle n;0|H^{\prime}|n;0\rangle - \widehat{\Pi}_{n}^{1}|n;1\rangle\langle n;0|H^{\prime}\widehat{\Pi}|$ \hat{H} ¹ \hat{H} ¹ \hat{H} ¹ \hat{H} ¹ \hat{H} ¹ \hat{H} ² \hat{H} ² *ⁿ H*⁰ \overline{a} *n*; 0i $-\hat{\Pi}_{n}^{2}H'\hat{\Pi}_{n}^{1}H'_{n}$: 0) $\langle n:0|H'|n:0\rangle - \hat{\Pi}_{n}^{2}H'|n:0\rangle\langle n:0|H'\hat{\Pi}_{n}^{1}H'|n:0\rangle$ $-\widehat{\Pi}_{n}^{3} H'|n;0\rangle\langle n;0|H'|n;0\rangle^{2},$ $|n;3\rangle = \widehat{\Pi}_n^1$ $|n;3\rangle = \widehat{\Pi}_{n}^{1}((H'-E_{n}^{(1)})|n;2\rangle - E_{n}^{(2)}|n;1\rangle),$ $=\widehat{\Pi}_{n}^{1} H^{\prime}|n;2\rangle - \widehat{\Pi}_{n}^{1}|n;2\rangle\langle n;0|H^{\prime}|n;0\rangle - \widehat{\Pi}_{n}^{1}|n;1\rangle\langle n;0|H^{\prime}\widehat{\Pi}_{n}^{1} H^{\prime}|n;0\rangle,$ $=\widehat{\Pi}_{n}^{1} H^{\prime} \widehat{\Pi}_{n}^{1} H^{\prime} \widehat{\Pi}_{n}^{1} H^{\prime} |n;0\rangle - \widehat{\Pi}_{n}^{1} H^{\prime} \widehat{\Pi}_{n}^{2} H^{\prime} |n;0\rangle \langle n;0|H^{\prime} |n;0\rangle$ $-\widehat{\Pi}_{n}^{2} H^{\prime}\widehat{\Pi}_{n}^{1} H^{\prime}|n;0\rangle\langle n;0|H^{\prime}|n;0\rangle - \widehat{\Pi}_{n}^{2} H^{\prime}|n;0\rangle\langle n;0|H^{\prime}\widehat{\Pi}_{n}^{1} H^{\prime}|n;0\rangle$ $\overline{}$ $((H'-E_n^{(1)})|n;2\rangle - E_n^{(2)}|n;1\rangle),$ $\overline{1}$, (1.89f)

…and the subtractions multiply combinatorially. The *combinatorial* growth gives a clue: f_{max} and the original subtractions, in the form of f_{max} *l* growth gives a clue: **b** 1 *ⁿ* [*H*⁰ $\bullet\,$ … and the subtractions multiply combinatorially. **• The combinatorial growth gives a clue: diagrams!**

Herturbation Picturebook |n n ⁼ ^P^b ¹ *ⁿ ^H*⁰ ^P^b ¹ *ⁿ ^H*⁰ ^P^b ¹ *ⁿ H*⁰ *[|]n*; 0i ^P^b ¹ *ⁿ ^H*⁰ ^P^b ² *ⁿ H*⁰ *|n*; 0ih*n*; 0*|H*⁰ ^P^b ² *ⁿ ^H*⁰ ^P^b ¹ *ⁿ H*⁰ *|n*; 0ih*n*; 0*|H*⁰ *[|]n*; 0i ^P^b ² *ⁿ H*⁰ *|n*; 0ih*n*; 0 ⁼ ^P^b ¹ *[|]n*; 2i ^P^b ¹ *ⁿ [|]n*; 2ih*n*; 0 *H*⁰ *n*; 0i ^P^b ¹ *ⁿ [|]n*; 1ih*n*; 0 ⁼ ^P^b ¹ *ⁿ ^H*⁰ ^P^b ¹ *ⁿ ^H*⁰ ^P^b ¹ *ⁿ H*⁰ *[|]n*; 0i ^P^b ¹ *ⁿ ^H*⁰ ^P^b ² *ⁿ H*⁰ *|n*; 0ih*n*; 0*|H*⁰ ^P^b ² *ⁿ ^H*⁰ ^P^b ¹ *ⁿ H*⁰ *|n*; 0ih*n*; 0*|H*⁰ *[|]n*; 0i ^P^b ² *ⁿ H*⁰ *|n*; 0ih*n*; 0 *1.3. Feynman's Diagrams and Calculus* 105 ⁼ ^P^b ¹ *ⁿ H*⁰ *[|]n*; 2i ^P^b ¹ *ⁿ [|]n*; 2ih*n*; 0 *H*⁰ *n*; 0i ^P^b ¹ *ⁿ [|]n*; 1ih*n*; 0 ⁼ ^P^b ¹ *ⁿ ^H*⁰ ^P^b ¹ *ⁿ ^H*⁰ ^P^b ¹ *ⁿ H*⁰ *[|]n*; 0i ^P^b ¹ *ⁿ ^H*⁰ ^P^b ² *ⁿ H*⁰ *|n*; 0ih*n*; 0*|H*⁰ ⁼ ^P^b ¹ *ⁿ H*⁰ *[|]n*; 2i ^P^b ¹ *ⁿ [|]n*; 2ih*n*; 0 *H*⁰ *n*; 0i ^P^b ¹ *ⁿ [|]n*; 1ih*n*; 0 ⁼ ^P^b ¹ *ⁿ ^H*⁰ ^P^b ¹ *ⁿ ^H*⁰ ^P^b ¹ *ⁿ H*⁰ *[|]n*; 0i ^P^b ¹ *ⁿ ^H*⁰ ^P^b ² *ⁿ H*⁰ *|n*; 0ih*n*; 0*|H*⁰ ^P^b ² *ⁿ ^H*⁰ ^P^b ¹ *[|]n*; 0i ^P^b ² *|n*; 0ih*n*; 0 *ⁿ H*⁰ *|n*; 0ih*n*; 0*|H*⁰ *ⁿ H*⁰ ⁼ ^h*n*; 0*|H*⁰ ^P^b ¹ *ⁿ ^H*⁰ ^P^b ¹ *ⁿ H*⁰ *[|]n*; 0ih*n*; 0*|H*⁰ ^P^b ² *ⁿ H*⁰ *|n*; 0ih*n*; 0*|H*⁰ *[|]n*; 3ⁱ ⁼ ^P^b ¹ (*H*⁰ *^E*(1) *ⁿ*)*|n*; 2i *^E*(2) *ⁿ |n*; 1i , *1.3. Feynman's Diagrams and Calculus* 105 ⁼ ^h*n*; 0*|H*⁰ ^P^b ¹ *ⁿ ^H*⁰ ^P^b ¹ *ⁿ H*⁰ *[|]n*; 0ih*n*; 0*|H*⁰ ^P^b ² *ⁿ H*⁰ *|n*; 0ih*n*; 0*|H*⁰ ⁼ ^h*n*; 0*|H*⁰ ^P^b ¹ *ⁿ ^H*⁰ ^P^b ¹ *ⁿ H*⁰ *[|]n*; 0ih*n*; 0*|H*⁰ ^P^b ² *ⁿ H*⁰ *|n*; 0ih*n*; 0*|H*⁰ ī

QUANTUM MECHANICS: WARM-UP CALISTHENICS *|n*; 0ih*n*; 0*|H*⁰ *[|]n*; 0i² ⁼ ^h*n*; 0*|H*⁰ ^P^b ¹ ^P^b ² *ⁿ ^H*⁰ ^P^b ¹ *ⁿ H*⁰ *|n*; 0ih*n*; 0*|H*⁰ *[|]n*; 0i ^P^b ² *ⁿ H*⁰ *|n*; 0ih*n*; 0 ICS: WARM-UP CALISTHENICS *[|]n*; 3ⁱ ⁼ ^P^b ¹ *n* **SHANICS: WARM-UP CALIS** *n i H i A i H i H i I H i H*

• Before we introduce any diagrams, however, consider: finding all subtractions, in the form of the form of the original expression: re introduce any diagrame hourever consider: $\frac{1}{2}$ all $\frac{1}{2}$ all $\frac{1}{2}$ $\frac{1}{2}$ and $\frac{1}{2}$ from the original expression: \bullet Before we introduce any diagrams, however, consider: (*H*⁰ *^E*(1) *ⁿ*)*|n*; 2i *^E*(2) *ⁿ |n*; 1i , ^P^b ³ *|n*; 0ih*n*; 0*|H*⁰ *[|]n*; 0i² **1. PERTICULAR ORDERING INCORPORATIONS IN THE PROPERTY.** re introduce any diagrams, however, consider: α *e* any diagrams however consider: *r* any diagrams, nowever, consider. ⁼ ^P^b ¹ *ⁿ H*⁰ *[|]n*; 2i ^P^b ¹ *ⁿ [|]n*; 2ih*n*; 0 *H*⁰ *n*; 0i ^P^b ¹ *ⁿ [|]n*; 1ih*n*; 0 α e any diagrams, nowever, consider: α e any diagrams, however, consider:

*ⁿ ^H*⁰ ^P^b ¹

*ⁿ ^H*⁰ ^P^b ¹

|n; 0ih*n*; 0*|H*⁰

*ⁿ ^H*⁰ ^P^b ¹

*ⁿ H*⁰

 \mathcal{C}

*ⁿ ^H*⁰ ^P^b ¹

Perturbation Picturebook finding all subtractions, in the form of "excisions" from the original expression: and so on. The particular ordering in these expressions uncovers a simple algorithm for finding all subtractions, in the form of "excisions" from the original expression: *ⁿ H*⁰ P b 1 *ⁿ H*⁰ *|n*; 0ih*n*; 0*|H*⁰ *|n*; 0i P b 2 *ⁿ H*⁰ *|n*; 0ih*n*; 0 *H*⁰ P b 1 *ⁿ H*⁰ *n*; 0i *ⁿ H*⁰ *|n*; 0ih*n*; 0*|H*⁰ *[|]n*; 0i² , (1.89f)

Quantum Mechanics: Warm-Up Calisthenics *|* QUANTUM MECHANICS: WARM-UP CALISTHENICS

Before we introduce any diagrams, however, consider: *|n*; 3i = P b 1 *ⁿ H*⁰ P b 1 *ⁿ H*⁰ P b 1 *ⁿ H*⁰ *|n*; 0i **diagrams, nowever, con** $\overline{\text{Rof}}$

*ⁿ H*⁰ P

*ⁿ H*⁰ P

*ⁿ H*⁰ P

ⁿ [*H*⁰ P

ⁿ [*H*⁰ P

 $\mathbf \lambda$

b 1

 $\overline{}$

 $\overline{}$

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 \overline{a}

b 1

*ⁿ H*⁰

*ⁿ H*⁰

- $|n;3\rangle = \hat{\Pi}_{n}^{1} H' \hat{\Pi}_{n}^{1} H' \hat{\Pi}_{n}^{1} H'|n;0\rangle$
	- b 1 $\begin{matrix} \nu, & \nu \end{matrix}$ $\Bigg\}$ P $\sqrt{2}$ $-\hat{\Pi}_{n}^{1}[H']\hat{\Pi}_{n}^{1}[H'\hat{\Pi}_{n}^{1}[H'|n;0]$
	- $\langle U \rangle$ \boldsymbol{h} , $\langle \cdot, \cdot \rangle$ $-\widehat{\Pi}_{n}^{1} H^{\prime} \widehat{\Pi}_{n}^{1} [H^{\prime}] \widehat{\Pi}_{n}^{1} H^{\prime} | n; 0$
	- $\langle i:0\rangle$ $-\widehat{\Pi}_{n}^1 H^1 \widehat{\Pi}_{n}^1 H^1 \widehat{\Pi}_{n}^1 [H'] |n;0$
	- $\frac{1}{n}$. *ⁿ H*⁰ P $\ket{n;0}$ *ⁿ H*⁰ P $-\widehat{\Pi}_{n}^{1}\left[H^{\prime}\right]\widehat{\Pi}_{n}^{1}\left[H^{\prime}\right]\widehat{\Pi}_{n}^{1}H^{\prime}|n;0\rangle$
	- $-\hat{\Pi}_n^1 H^1 \hat{\Pi}_n^1 [H^1 \hat{\Pi}_n^1 H^1] |n;0\rangle$

The subtractions may in fact be generated from the original expression!] P b 1 *A*^{*h*} $\frac{1}{2}$ *ⁿ H*⁰ P *n* a *i*ⁿ **1** *n*_e subtractive *ⁿ* [*H*⁰]*|n*; 0i *n*ay in fact b *n*₁ \rightarrow *n*₂ $W_{\ell} = \widehat{\Pi}_{n}^{1}[H'\widehat{\Pi}_{n}^{1}H']\widehat{\Pi}_{n}^{1}[H' | n; 0]$ expression ℓ] P b 1 An observation, $-\hat{\Pi}_n^1[H'\hat{\Pi}_n^1H']\hat{\Pi}_n^1H'|n;0$ expression! not a proof!

 \bullet ... and so on. Now, you try for the 3rd one... $\frac{1}{2}$ $\widehat{\Pi}_{n}^{1}[H']\,\widehat{\Pi}_{n}^{1}\,H'\,\widehat{\Pi}_{n}^{1}\,H'|n;0\rangle=\widehat{\Pi}_{n}^{1}\,\widehat{\Pi}_{n}^{1}\,H'\,\widehat{\Pi}_{n}^{1}\,H'|n;0\rangle\,\langle n;0|H'|n;0\rangle,$ $\frac{1}{\sqrt{2}}$ $\widehat{\Pi}_{n}^{1}[H'\,\widehat{\Pi}_{n}^{1}H']\,\widehat{\Pi}_{n}^{1}[H'|n;0\rangle = \widehat{\Pi}_{n}^{1}\bullet\widehat{\Pi}_{n}^{1}[H'|n;0\rangle\,\langle n;0|H'\,\widehat{\Pi}_{n}^{1}[H'|n;0\rangle,$ <u>}</u> $\prod_{n=1}^{N} H'_{n} \prod_{n=1}^{N} H' \prod_{n=1}^{N} H' |n;0\rangle = \prod_{n=1}^{N} H' \prod_{n=1}^{N} H' |n;0\rangle \langle n;0|H'|n;0\rangle,$ * $\widehat{\Pi}_{n}^{1}\left[H^{\prime}\,\widehat{\Pi}_{n}^{1}\,H^{\prime}\right]\widehat{\Pi}_{n}^{1}\,H^{\prime}|n;0\rangle=\widehat{\Pi}_{n}^{1}\,\widehat{\Pi}_{n}^{1}\,H^{\prime}|n;0\rangle\,\langle n;0|H^{\prime}\,\widehat{\Pi}_{n}^{1}\,H^{\prime}|n;0\rangle,$ $\frac{1}{1}$ T_{ref} ... and so on. Thong you if y for d *n*_i, 0¹ *n*₂ *n***₂** *n***_{2**} \mathbf{u} *|n*; 0i = P *• |n*; 0i, *etc.*

*ⁿ H*⁰

*ⁿ H*⁰

*ⁿ H*⁰ P

*ⁿ H*⁰

b 1

 $\frac{1}{2}$ **b**

 \mathcal{P}

P

 $\sum_{i=1}^n$

Perturbation Picturebook **Perturbation Picturebook** $\frac{1}{2}$ $\frac{1}{2}$ *|m*; 0ih*m*; 0*|* (*E*(0) *^m*)*^a [|]n*; 0ⁱ ⁼ ^Â 1 (*E*(0) *^m*)*^a [|]m*; 0i h*m*; 0*|n*; 0ⁱ *ⁿ |n*; 0i = Â . (1.93) *|m*; 0ih*m*; 0*|* **MUNICE: WA** 1 106 *Chapter 1. Physics in Spacetime* 106 *Chapter 1. Physics in Spacetime* (*E*(0) *ⁿ ^E*(0) *^m*)*^a [|]m*; 0i h*m*; 0*|n*; 0ⁱ | {z }

Quantum Mechanics: Warm-Up Calisthenics Recall: =0 *m*6=*n ⁿ ^E*(0) *m*6=*n ⁿ ^E*(0) | {z } P_{α} that the second that the normalization (1.86) guarantees: $ENIGS$ S **Necall**: b *ⁿ ···* P b *b ⁿ H*0) have a non-vanishing expectation *m*6=*n* M MECHANICS: WAR =0 (* *m*6=*n*) MECHANICS: WARM-LIP CALISTHENICS 106 *Chapter 1. Physics in Spacetime* 106 *Chapter 1. Physics in Spacetime* 106 *Chapter 1. Physics in Spacetime*

Now that we have the computations, depict them: \prod b $\binom{\alpha}{n}|n;0\rangle = \sum_{n}$ $m{\neq}n$ $\ket{m;0}\bra{m;0}$ $\frac{1}{(E_n^{(0)} - E_m^{(0)})^{\alpha}} |n;0\rangle = \sum_{m \neq n}$ $m{\neq}n$ 1 $\langle E_n^{(0)} - E_m^{(0)} \rangle^{\alpha}$ |*m*; 0) $\langle m; 0 | n; 0 \rangle$ $=0$ (\therefore *m* \neq *n*) . (1.93) Since only factors of the form (*H*0 P b *b* $E_n^{(k)} = \langle n; 0 | H'(\hat{\Pi}_n^1 H')^{k-1} | n; 0 \rangle - \text{all "excisions".}$ $k \ge 1$, t then be written as t may then be written as t $\widehat{\Pi}_{n}^{\alpha}|n;0\rangle = \sum \frac{|m;0\rangle\langle m;0|}{\langle\alpha\rangle\langle n;0\rangle} |n;0\rangle = \sum \frac{1}{\langle\alpha\rangle\langle n;0\rangle\langle n;0|n;0\rangle}.$ t^{n+1} , t^{n} , t^{n} , t^{n} $\sum_{m \neq n} (E_n^{(0)} - I_n^{(0)})$ $|n; k\rangle = (\hat{\Pi}_n^1 H')^k |n; 0\rangle - \text{all "excisions",}$ $k \ge 0,$ $E_n^{(k)} = \langle n; 0 | H'(\widehat{\Pi}_n^1 H')^{k-1} | n; 0 \rangle -$ all "excisions". $k \ge 1$, Since only factors of the form (*H*0 P *a ⁿ ···* P b *b ⁿ H*0) have a non-vanishing expectation $\widehat{\Pi}_{n}^{\alpha}|n;0\rangle = \sum \frac{|n\nu v_{\perp}^{\alpha}|}{\Gamma^{(0)}\Gamma^{(0)}\Gamma^{(0)}\Gamma^{(0)}}|n;0\rangle = \sum \frac{1}{\Gamma^{(0)}\$ tions $m \neq n$ ($E_n^{(8)} - E_n$ $\langle n; k \rangle = (\widehat{\Pi}_n^1 H')^k |n; 0 \rangle - \text{all "excisions",}$ $k \ge 0,$ $\frac{1}{\Gamma(0)}$ $s_n = \mathbf{L}_m$ is $m \neq n$ (\mathbf{L}_m instance) $k \geqslant 0$, = h*n*; 0*|* P b 1 *n* s ". k b 1 = h*n*; 0*|* P b 1 *n* $\sum_{m_{\tilde{z}}}\left(\frac{1}{r}\right)$ $\frac{1}{\Gamma(0)}$ $s_n = L_m$) $m \neq n$ ($L_n = L_m$) S ". k b 1
1
1 $\sum_{m=1}^{\infty}$ Using this "excising" notation, *e.g.*, the expression (1.89e) becomes: $\frac{1}{m} \lim_{m \to n} \frac{1}{(E_n^{(0)} - E_m^{(0)})^{\alpha}}$ $\frac{1}{m} \lim_{m \to n} \frac{1}{(E_n^{(0)} - E_m^{(0)})^{\alpha}}$ $\frac{1}{m} \lim_{m \to n} \frac{1}{(E_n^{(0)} - E_m^{(0)})^{\alpha}}$ sions." Take, for instance, the candidate $\mathcal{G}% _{M_{1},M_{2}}^{\alpha,\beta}(\omega)$ *ⁿ H*⁰ $E_n^{(n)} = \langle n; 0 | H'(11^n H')^{n-1} | n; 0 \rangle -$ all "excis *** *r* that we have the computations, depict them: Using this "excising" notation, *e.g.*, the expression (1.89e) becomes: $\frac{1}{m+1}$ $\frac{1}{m+1}$ $s \mapsto \frac{1}{2}$ *ⁿ H*⁰ P $\mathcal Y$ *ⁿ H*⁰ that we have the computations, depict them: Using this "excising" notation, *e.g.*, the expression (1.89e) becomes: $\frac{1}{m+1}$ $\frac{1}{m+1}$ $s \mapsto \frac{1}{2}$ *ⁿ H*⁰ P b 1 *ⁿ H*⁰ *** hat we have the computations, depict them:

[|]n; 0ⁱ (1.89a) 7! *^E*(1) *[|]n*; 0ⁱ (1.89a) 7! *^E*(1) *[|]n*; 0ⁱ (1.89a) 7! *^E*(1) Tuesday, November 1, 11

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¨ubsch, thubsch@howard.edu, with any comments / suggestions / corrections; thank you! — DRAFT ¨ubsch, thubsch@howard.edu, with any comments / suggestions / corrections; thank you! — DRAFT and so on, where the subtractions are shown as stacks of diagrams, and represent a product a product a product ¨ubsch, thubsch@howard.edu, with any comments / suggestions / corrections; thank you! — DRAFT Tuesday, November 1, 11

Perturbation Picturebook *1.3.1 Diagrams* Pertiliraation Pictureacok agrams*12*. It is important to understand that these diagrams must not be taken as a literal *1.3.1 Diagrams* Processes between particles are naturally represented graphically, by so-called Feynman diagrams 122. It is in the understand that the understand that the understand that the understand that the understand
It is in the understanding the understanding the understanding the understanding the understanding the und *1.3.1 Diagrams* Processes between particles are naturally represented graphically, by so-called Feynman diagrams*12*. It is important to understand that these diagrams must not be taken as a literal

SPACETIME PROCESSES (QM & QFT)

The general idea is to depict physical processes, in a 1–1 unambiguous way: to the conoralides is to depict physical processes in a 1 Fine Senerui rueu le ce aepre $F = \frac{1}{2}$ File $\frac{1}{2}$ $e^{-\int t}$ τ help estimated idea is to depict physical processes in a 1 1 e^-

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SPACETIME PROCESSES (QM & QFT)

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The "process" is whatever takes "input" → "output": e^{u} The "process" is urbetause to kee "input" e^{u} tile process to where

- In this case, it is a 1-photon electromagnetic interaction (scattering) of an electron with another electron. where the state \mathcal{L} is virtual; particles and processes that \mathcal{L} is virtual; particles and processes that \mathcal{L} are entirely with this case, it is a r-photon electromagnetic interaction
- What <u>is</u> observed are the incoming two particles in the incoming 2-particle state, and the outgoing two particles in the outgoing 2-particle state. What is observed are the incoming two particles in the in the outgoing 2-particle state. \mathbf{W}_{α} the exchanged γ *Not* the exchanged *γ*!

Perturbation Picturebook a photon, which then the right-hand side particle absorbs; according to the interpretation \mathcal{L} on the right-hand side particle emitted a photon side particle emitted particle emitted by a photon side of a photon sid hand side particle absorbs. Thus we simply speak of an "exchanged" photon, and a diagram

SPACETIME PROCESSES (QM ► QFT)

The un-observable exchange photon is thus virtual! Indeed, the two 3-particle processes *separately*... real depends of the process in spacetime. The an observable exemiting photon to thus virtual.

are kinematically forbidden. $Q: Why?$ A.: Conservation of 4-momentum. Δ . Concorration of Δ man

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Perturbation Picturebook where the schematic region in the shaded ellipse is virtual; particles and processes that processes th are entirely within this region can neither be observed nor measured directly as a matter of (Heisenberg's indeterminacy) principle. On the other hand, that also means that $\overline{}$ where the schematic region in the schematic region in the shaded ellipse is virtual; particles and processes that μ Parturbation Dieturahool ter of (Heisenberg's indeterminacy) principle. On the other hand, that also means that \mathcal{L} , and the contact direction \mathcal{L} where the schematic region in the schematic region in the shaded ellipse is virtual; particles and processes that are entirely within the region can neither be observed not measured the can neither be observed not measured n ter of (Heisenberg's indeterminacy) principle. On the other hand, that also means that Γ where the schematic region in the schematic region in the shaded ellipse is virtual; particles and processes that are entirely within the region can neither be observed not measured the can neither be observed in the can nei ter of (Heisenberg's indeterminacy) principle. On the other hand, that also means that DRAFT — contact directly Tristan H

SPACETIME PROCESSES (QM & QFT) within the shaded region of indeterminacy, many other sub-processes—*i.e.*, *all* possible sub-

The "process" is whatever takes "input" → "output": $m = 1$ of the contributions to the physical quantity being computed for the physical quantity bein **c** ine process is whatever takes input \rightarrow output : mine the hierarchy of their contributions to the physical quantity being computed for the The process is whatever takes input \rightarrow output: mine the hierarchy of their contributions to the physical quantity being computed for the The process is whatever takes input \rightarrow output: mine the hierarchy of their contributions to the physical quantity being computed for the The process is whatever takes input \rightarrow output:

Whatever is not forbidden, is mandatory. –R. Feynman Notice, these sub-processes can be ordered:

- by counting the "elementary" interaction vertices
- \bullet by counting loops
- by (non-)planarity
- …which are qualitative—**& depicted**—characteristics

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SPACETIME PROCESSES (QF

• A 1-1 correspondence:

- the fundamental theory that designs the considered process,
- diagram elements depicting terms from a Lagrangian,
- rules of linking graphical elements, depicting a computation with the individual terms from the specified Lagrangian,
- rules of listing all possible—and needed—Feynman diagrams,
• the final mathematical expression for the matrix element
- (amplitude of probability) for the considered process, as a weighted sum of sub-processes,
- the computation (estimate) of this mathematical expression. \bullet This would be the goal of a Quantum Field Theory course…
- …which this is not.

☺

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Spacetime Processes (QFT)

 \bullet Back in the *A*-*B*-*C* toy model, the *A* \rightarrow *B*+*C* decay.

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Perturbation Picturebook *A C B* and so on. The lowest order contribution (1.127) is depicted by a tree-graph (with no closed by a tree-graph (with no closed by a tree-graph (with no closed) is depicted by a tree-graph (with no closed by a tree-graph (wit *C B* and so on. The lowest order contribution (1.127) is depicted by a tree-graph (with no closed by a tree-graph (with no closed by a tree-graph (with no closed) is depicted by a tree-graph (with no closed by a tree-graph (wit

Spacetime Processes (QFT) loop). The subsequent contributions (1.128) all have precisely one closed loop and are of closed loop and are o

And from *O*(*g*5) onward, there are also complications: \bullet And from $O(\sigma^2)$ onward, there are also complications. $\frac{1}{2}$ order and $\frac{1}{2}$ order and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ are following to $\frac{1}{2}$ \bullet And from $O(\sigma^2)$ onward, there are also complications. $\frac{1}{2}$ order and $\frac{1}{2}$ order and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ are seen promotions:

A Few Stray Comments

OVERVIEW

Notice the extraordinary double-duty done by the physics discipline of Elementary Particle Physics—possibly including also a third tier, as professed to exists by such as string theory.

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Conservation Laws

- Strict Conservation Laws
	- Spacetime
		- Continuous (4-momentum & angular momentum)
		- \bullet Discrete (P, T, C)
	- Not Spacetime
		- EM charge
		- Chromodynamic Color
		- Weak Isospin
- Phenomenological Conservation Laws
	- lepton number(s, incl. e^- , μ [–] & τ [–] separately)
	- baryon number(s, incl. strangeness, charm, …)

Thanks!

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<http://homepage.mac.com/thubsch/>

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