1 The Two-Capacitor Conundrum

Consider two capacitors, connected and charged initially as indicated at left in Figure 1. Long after the

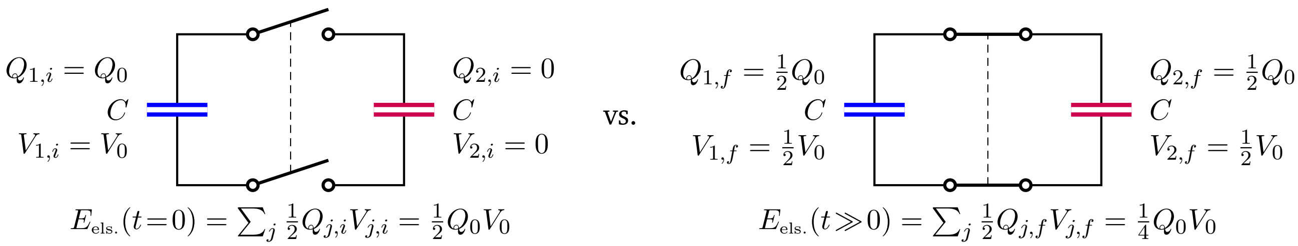

Figure 1: The two-capacitor circuit: at $t\!=\!0$ at left, and (much) later, $t\!\gg\!0$, at right

switches have connected the circuit loop at $t\!=\!0$, the electric charge is expected to have equilibrated, $\frac{1}{2}Q_0$ on the left-hand side capacitor, and $\frac{1}{2}Q_0$ on the right-hand side capacitor, as shown on the right of Figure 1. In doing so, however, the total electrostatic energy stored in the capacitors halves, from $E_{\text{els.}}(t\!=\!0)\!=\!\frac{1}{2}Q_0V_0 = \frac{1}{2}C{V_0}^2$, down to, *much* later, $E_{\text{els.}}(t\!\gg\!0)\!=\!2\!\times\!\frac{1}{2}(\frac{1}{2}Q_0)(\frac{1}{2}V_0) = \frac{1}{4}C{V_0}^2$.

Conundrum: *Where did half the energy go?*

This well-trodden question of course is itself both perfectly reasonable (as asked) and has a perfectly reasonable answer, one which *realistically* in fact involves *all of the resolving details* brought up over the (at least!) 68-year age of this conundrum, as mentioned in the Wikipedia article[1] — in spite of being called a "*paradox*" there and elsewhere in the literature and media. Suffice it for now just to say that the conundrum stems from a very incomplete view of the arrangement and process as depicted in Figure 1.

1.1 A Mechanical Equivalent

The above electric circuit and process has a rather analogous mechanical equivalent, where the above question would never come up, and where the "common wisdom" intuition would immediately point to the correct analysis and assessment of the mechanical process — thereby indicating a hint how to re-analyze the above electric circuit process more correctly.

Consider the arrangement of two linked pendula depicted in Figure 2 in two of its possible positions. Both beads have mass m, and the link is rigid but assumed to have negligible mass. Were one to start from

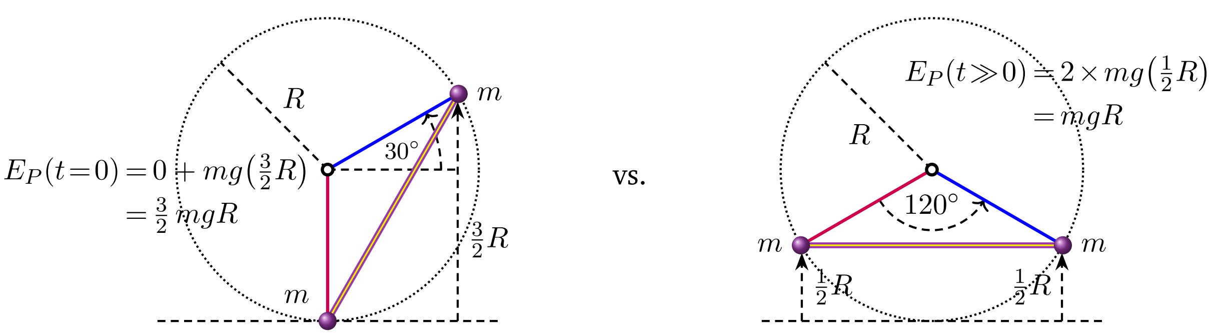

Figure 2: Two linked pendula: the initial position at left, and another position at right

[1] https://en.wikipedia.org/wiki/Two_capacitor_paradox

the position depicted at left in Figure 2 and let go from rest, the linked pendula will surely start swinging in the clockwise direction: the upper bead downward, and the lower bead upward, reaching at some points the position depicted in Figure 2 at right. At the initial (left) position, the system has potential energy $\frac{3}{2}mgR$, while in the position depicted a right, it's potential energy dropped to mgR — having lost $\frac{1}{2}mgR$ of energy in the transition.

The question "where did 1/3 of the potential energy go" would be immediately answered by anyone who has ever in their life seen a swing (even just on TV): "into *kinetic* energy!"

While this mechanical contraption is quite a reasonable analogue of the above electronic circuit, it clearly presents no conundrum at all! Presumably, our daily experiences have prepared us for a much better understanding of mechanical arrangements and processes than of electromagnetic ones [2].

1.2 Another Mechanical Equivalent

A persnickety student might complain that the system of linked pendula is not a good analogue, since the potential energy depends linearly with the variable height, whereas the electrostatic energy of the two-capacitor circuit depends quadratically with the measured voltage.

Let such a student contemplate the example of a two-spring system depicted in Figure 3, with the "initial" of its positions at left and the equilibrium position depicted at right. The potential energy stored in

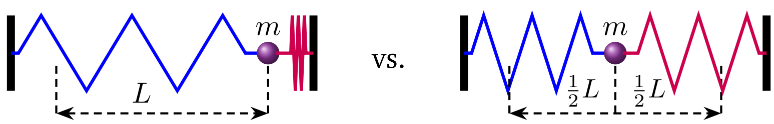

Figure 3: The spring-sandwich: In the initial position at left, the left-hand side spring is at its equilibrium length, L, while the right-hand side spring (physically identical to the left-hand side one) is maximally compressed. In the position at right, both springs are compressed to half of their equilibrium length.

the spring-sandwich in the configuration at the left equals $E_S(t=0) = \frac{1}{2}k(0)^2 + \frac{1}{2}k(-L)^2 = \frac{1}{2}kL^2$. In the position at right, the potential energy stored in the spring-sandwich equals $E_S(t \gg 0) = 2 \times \frac{1}{2}k(-\frac{1}{2}L)^2 = \frac{1}{4}kL^2$. This mechanical system indeed loses half its potential energy in the transition from the left-hand side position into the right-hand side position, and so is numerically a better analogue of the two-capacitor circuit, although perhaps not quite as universally intuitive as a swing.

Whereas this right-hand side position is indeed the equilibrium position, as is the position depicted at right in Figure 2 for the two linked pendula and the configuration depicted at right in Figure 1 for the two-capacitor circuit, it is perfectly clear that: (**1**) both mechanical systems undergo oscillations ***through*** the equilibrium position; (**2**) this oscillation is dampened by friction (and possibly other) losses; (**3**) the system settles into the equilibrium position only once — *and because* — $\frac{1}{2}mgR$, i.e., $\frac{1}{4}kL^2$ of energy is transformed into these irretrievable "losses."

So, *why is it not equally obvious* that the same fate befalls the two-capacitor circuit?

References

[1] A. Bierce, *The Unabridged Devil's Dictionary*. University of Georgia Press, revised ed., January, 2002.

[2] Indeed, a *magnet* (n.) is something acted upon by magnetism, and *magnetism* (n.) is something acting upon a magnet, whereas *electricity* (n.) is the power that causes all natural phenomena not known to be caused by something else [1].