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Chameleonic σ-Models

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ABSTRACT

1+1-dimensional, non-linear and (2,2)-supersymmetric σ-models are constructed in which the target space changes topology at distinguished regions of the parameter space. In particular, a σ-model formulation is provided for the recently discovered topological transitions among many of the Calabi-Yau manifolds.

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1. Does Topology Fluctuate ?

Certain physical observables are very robust with respect to quantum corrections; loosely speaking, they are called 'topological'. A well-known example is the Witten index, $(-)^F$, which equals the Euler characteristic of the configuration space [1] and often counts the difference between the number of left- and right-handed massless fermions. This indeed is a constant as long as the configuration space is not (de)singularized : in general, singularities contribute to $(-)^F$ just as boundaries do (for the latter, see section 8 of Ref. [2]). Whether such topology changing processes are actually encountered by quantum fluctuations and how $(-)^F$ gets corrected is determined by the dynamics of the particular model. For cases when the complete quantum field theory is not known, we find it worthwhile to contemplate a 'phenomenological' description of topology fluctuation.

It was recently discovered that conifolds [3,4] interpolate between topologically distinct smooth Calabi-Yau manifolds, connecting their moduli spaces just as moduli spaces of Riemann surfaces of different genera are connected into the *universal moduli space* [5]. The paths in the Calabi-Yau moduli space which connect topologically distinct manifolds are continuous and of finite length in the Weil-Petersson metric [6]. Whether the corresponding paths in the space of (super)string vacuua are of finite length, or even continuous, cannot be established as yet. A better understanding of nodal (and also smooth) compactifications is desired.

Here we construct 1+1-dimensional, (2,2)-supersymmetric, non-linear σ-models the target space of which undergoes the corresponding singularizations (sections 2 and 4). Special care is taken in the analysis of the interfacing nodal σ-model (section 3) which appears as a common limit of two topologically distinct σ-models. Beside providing a σ-model interpretation for the topology change of Ref. [3,4], we derive the following : (1) The (2,2)-supersymmetry of the constrained σ-model is inherited from the ambient σ-model as long as the second order Taylor expansion of the constraint(s) does not vanish. (2) At an isolated singular point in the target space, the σ-model couples to the (non-divergent) ambient space curvature, so that the singularity appears innocuous.

While in this paper we concentrate on Calabi-Yau compactifications, it is not difficult to adapt our analysis to other situations. For example, the requirement of (super)conformal invariance may be relaxed in part. It is crucial, however, that the target spaces of our σ-models can be realized through systems of local constraint equations. Also, for systems without supersymmetry, the Hamilton-Dirac treatment of the (second class) constraints provides all the information which is here obtained by using supersymmetry.

2. Constrained σ-Models

To begin with, consider a (2,2)-supersymmetric non-linear σ-model describing strings in a hypersurface in $\mathscr{X}$, defined by means of a single constraint

$$\mathcal{M}^\flat \;:\; C(x) = 0 \;. \tag{1}$$

The corresponding action

$$S^\flat = \int_\Sigma \mathrm{d}^2\sigma\,\mathrm{d}^2\varsigma\,\mathrm{d}^2\bar{\varsigma}\;\mathring{\mathbf{K}}(\boldsymbol{X},\overline{\boldsymbol{X}}) \;+\; \Big[\int_\Sigma \mathrm{d}^2\sigma\,\mathrm{d}^2\varsigma\;\boldsymbol{\Lambda}\,C(\boldsymbol{X}) \;+\; \text{h.c.}\;\Big] \tag{2}$$

governs the dynamics of our σ-model. The

$$\boldsymbol{X}^\mu \;=\; X^\mu \;+\; \varsigma^\pm \xi_\pm{}^\mu \;-\; \varsigma^+\varsigma^-\mathsf{X}^\mu \tag{3}$$

are chiral (2,2)-superfields. Here X^μ are the usual σ-model maps that embed the world sheet into the ambient space $\mathscr{X}$, but are subject to the constraint so that the true target space is $\mathcal{M}^\flat$.

$$\boldsymbol{\Lambda} \;=\; \Lambda \;+\; \varsigma^\pm\lambda_\pm \;-\; \varsigma^+\varsigma^-\mathsf{L} \tag{4}$$

is a Lagrange chiral (2,2)-superfield.

The Kähler potential $\mathring{\mathbf{K}}(\boldsymbol{X},\overline{\boldsymbol{X}})$ is chosen so that $S^\flat$ is a superconformally invariant (effective) action, i.e., $\mathring{\mathbf{K}}(\boldsymbol{X},\overline{\boldsymbol{X}})$ corresponds to a fixed point of the renormalization flow. To obtain this in practice, when $\mathscr{X}$ is a product of (weighted) complex projective spaces, we may start with the linear combination $w^A\mathbf{K}_A$ of Fubini-Study Kähler potentials; quantum corrections will renormalize this to $\mathring{\mathbf{K}}$ [7,8]. Recall now that the Fubini-Study Kähler forms always span (at least a part of) the (1,1)-cohomology of the constrained subspace [9]. Harmonic (1,1)-forms correspond to certain massless (moduli) fields in the low-energy effective model, which in turn correspond to exactly marginal perturbations of the superconformal world sheet model [10]. It follows that the coefficients w^A accompany these exactly marginal perturbations,

$$\mathring{\mathbf{K}}(\boldsymbol{X},\overline{\boldsymbol{X}}) \;=\; w^A\,\mathring{\mathbf{K}}_A(\boldsymbol{X},\overline{\boldsymbol{X}}) \;. \tag{5}$$

After all, the effective action of each $\mathbb{CP}^n$ model [11] contains the w^As in twisted chiral superpotential terms, to which the usual non-renormalization theorems apply.

In an intrinsic formulation, $\mathring{\mathbf{K}}$ would be a suitable Kähler potential on $\mathcal{M}^\flat$ and the coordinate superfields $\boldsymbol{X}^\mu$ would span $\mathcal{M}^\flat$ and its tangent bundle. In the embedded formulation, we need to constrain the bosonic component fields of $\boldsymbol{X}^\mu$ from $\mathscr{X}$ to $\mathcal{M}^\flat$ and the fermionic component fields from $T_X(\mathscr{X})$ to $T_X(\mathcal{M}^\flat)$. To this end, we use a Lagrange multiplier and constrain all modes of $\boldsymbol{X}^\mu$. A superpotential term ($\boldsymbol{\Lambda} \to$ const.) would constrain only the 0-modes and is reliable for the IR regime only [7].

Path-integration over each component field of $\mathbf{\Lambda}$ and also over X^μ yields a delta-functional, each one enforcing an algebraic field equation. These are :

$$\int D[\Lambda] \quad\Rightarrow\quad 0 = \mathsf{X}^\mu C_{,\mu}(X) \;-\; C_{,\mu\nu}(X)\xi_+{}^\mu\xi_-{}^\nu \;, \tag{6}$$

$$\int D[\lambda_\pm] \quad\Rightarrow\quad 0 = \xi_\pm^\mu C_{,\mu}(X) \;, \tag{7}$$

$$\int D[\mathsf{L}] \quad\Rightarrow\quad 0 = C(X) \;, \tag{8}$$

$$\int D[\mathsf{X}^\mu] \quad\Rightarrow\quad 0 = \Lambda\, C_{,\mu} \;-\; \Big(\mathbf{K}_{,\mu\overline{\nu}}\, \mathsf{X}^{\overline{\nu}} - \mathbf{K}_{,\mu\overline{\nu}\,\overline{\sigma}}\, \xi_+{}^{\overline{\nu}}\xi_-{}^{\overline{\sigma}}\Big)\;. \tag{9}$$

Eq. (8) restricts the X^μ from $\mathscr{X}$ to $\mathcal{M}^\flat$. The gradient $C_{,\mu}$ represents the normal[♯1] to the constrained hypersurface and does not vanish where $\mathcal{M}^\flat$ is smooth. Eq. (7) restricts the $\xi_\pm{}^\mu$ to be orthogonal to $C_{,\mu}$, i.e., tangent to the constrained hypersurface, as should be the case.

Using Eqs. (6) and (9), the four-fermion interaction term

$$R_{\mu\overline{\nu}\rho\overline{\sigma}}\; \xi_+{}^\mu\, \xi_-{}^\rho\, \xi_+{}^{\overline{\nu}}\, \xi_-{}^{\overline{\sigma}} \tag{10}$$

appears, involving the induced Riemann tensor on the constrained subspace

$$R_{\mu\overline{\nu}\rho\overline{\sigma}} \;\stackrel{\rm def}{=}\; \mathbf{K}_{,\mu\overline{\nu}\rho\overline{\sigma}} - \Gamma^\lambda_{\mu\rho}\, \mathbf{K}_{,\lambda\overline{\kappa}}\, \Gamma^{\overline{\kappa}}_{\overline{\nu}\,\overline{\sigma}} \;-\; C_{,\mu;\rho}\, (C_{,\lambda}\, G^{\lambda\overline{\kappa}}\, \overline{C}_{,\overline{\kappa}})^{-1}\, \overline{C}_{,\overline{\nu};\overline{\sigma}}\;. \tag{11}$$

Here $\Gamma^\lambda_{\mu\rho} \stackrel{\rm def}{=} G^{\lambda\overline{\kappa}}\, \mathbf{K}_{,\mu\rho\overline{\kappa}}$ is the usual Cristoffel symbol, $G^{\mu\overline{\nu}}$ is the matrix-inverse of $\mathbf{K}_{,\mu\overline{\nu}}$ and the extrinsic curvature is $C_{,\mu;\rho} \stackrel{\rm def}{=} C_{,\mu\rho} - \Gamma^\lambda_{\mu\rho}\, C_{,\lambda}$. The particular choices of $\mathring{\mathbf{K}}$ and $C(\boldsymbol{X})$ determine the fixed point to which renormalization will drive the system.

3. Nodal σ-Models

We have so far assumed $C(\boldsymbol{X})$ to be generic; now we wish to parametrize a suitable singularization. To that end, we write

$$C(\boldsymbol{X}) \;=\; S(\boldsymbol{X})\, P(\boldsymbol{X}) \;-\; Q(\boldsymbol{X})\, R(\boldsymbol{X}) \;+\; t^\alpha \mathcal{O}_\alpha(\boldsymbol{X})\;. \tag{12}$$

P, Q, S, R are generic polynomials of appropriate degree and the $\mathcal{O}^\flat{}_\alpha$ form a complete set of polynomials of homogeneity $\deg(C)$ which cannot however be factorized as $SP - QR$.

At a special region in the t^α-space, such as $t^\alpha = 0$, we obtain the action

$$S^\sharp = \int_\Sigma {\rm d}^2\sigma\, {\rm d}^2\varsigma\, {\rm d}^2\overline{\varsigma}\; \mathring{\mathbf{K}}(\boldsymbol{X}, \overline{\boldsymbol{X}}) \;+\; \Big[\, \int_\Sigma {\rm d}^2\sigma\, {\rm d}^2\varsigma\; \mathbf{\Lambda}\, C^\sharp(\boldsymbol{X}) \;+\; \text{h.c.} \Big]\,, \tag{13}$$

[♯1] Note : $C_{,\mu}$ is independent of any connection on the constrained subspace, where $C(x) = 0$.

with $C^\sharp(\boldsymbol{X}) = C(\boldsymbol{X})|_{t^\alpha=0}$; the behaviour of $S^\sharp$ differs markedly from that of $S^\flat$. In particular, now we have

$$C^\sharp_{,\mu} \;=\; S\,P_{,\mu} + S_{,\mu}\,P - Q\,R_{,\mu} - Q_{,\mu}\,R\;, \tag{14}$$

so both $C^\sharp$ and $C^\sharp_{,\mu}$ vanish where $S, P, Q, R = 0$. For a generic choice of these four polynomials, this is bound to happen at isolated points since $\mathcal{X}$ is compact and complex four-dimensional. These singular points are nodes [12] and S, P, Q, R can be used as local coordinates on the $\mathbb{C}^4$-like neighbourhood in $\mathcal{X}$, with $S, P, Q, R = 0$ at the origin.

Eq. (7) is vacuous at the nodes ($C^\sharp_{,\mu} = 0$) and the fermions seem not to be restricted from $T_X(\mathcal{X})$ to $T_X(\mathcal{M}^\sharp)$. However, Eqs. (6) and (9) have also changed; Eq. (6) now is

$$C^\sharp_{,\mu\nu}(X)\,\xi_+{}^\mu\xi_-{}^\nu = 0\;. \tag{6$^\sharp$}$$

Since the matrix of second derivatives $C_{,\mu\nu}$ does not vanish at nodes[♯2], the quadratic field equation (6$^\sharp$) is non-empty and indeed restricts the fermions, at each node, to span the conical tangent space—as they should. (In fact, the same is true for all modality ≤ 2 and also some higher modality singular polynomials [13], upon proper Morsification.)

The induced Riemann tensor (11) on $\mathcal{M}^\sharp$ is divergent at the nodes, since it contains $(C^\sharp_{,\mu}G^{\mu\overline{\nu}}\overline{C}{}^\sharp_{,\overline{\nu}})^{-1}$ and $C^\sharp_{,\mu}$ vanishes. However, to obtain the expression (11), we have used the field equation (9) which becomes

$$\int D[\mathsf{X}^\mu]\;:\;0 = \Big(\mathbf{K}_{,\mu\overline{\nu}}\,\mathsf{X}^{\overline{\nu}} - \mathbf{K}_{,\mu\overline{\nu}\,\overline{\sigma}}\,\xi_+{}^{\overline{\nu}}\xi_-{}^{\overline{\sigma}}\Big) \tag{9$^\sharp$}$$

at the nodes and decouples from the other three field equations (6)–(8). Using this and the complex conjugate to eliminate $\mathsf{X}^{\overline{\nu}}$ and X^μ, we find that the four-fermion term couples to the ambient space Riemann tensor

$$R_{\mu\overline{\nu}\rho\overline{\sigma}} \stackrel{\text{def}}{=} \mathbf{K}_{,\mu\overline{\nu}\rho\overline{\sigma}} - \Gamma^\lambda_{\mu\rho}\,G_{\lambda\overline{\kappa}}\,\Gamma^{\overline{\kappa}}_{\overline{\nu}\,\overline{\sigma}} \tag{15}$$

at the nodes. Unlike (11), this does not diverge, indicating that nodal singularities are innocuous for the string, very much like the singular points of an orbifold [14]. (In fact, the same is true for any isolated singularity which is the 0-locus of a system of local algebraic equations.)

Unlike C (12) for $t^\alpha \neq 0$, $C^\sharp$ may be written as a sum of squares through a holomorphic change of variables, i.e., a node is an A_1-singularity [13]. The Euler characteristic of the conifold $C^\sharp = 0$ picks up a correction : $\chi^\sharp_{_E} = \chi^\flat_{_E} + N$ where N is the number of nodes. This suggests that singularities induce a correction to the Witten index also,

[♯2]Note that $C^\sharp_{,\mu\nu}$ is independent of any connection at the singular points of the constrained subspace, since there $C^\sharp_{,\mu}$ and $C^\sharp$ vanish.

somewhat like the well known boundary corrections to the Atiyah-Patodi-Singer index [2]. A Landau-Ginzburg type analysis is hampered because the constraint polynomial is singular not only at the isolated point $\boldsymbol{X}^\mu = 0$ as required in Ref. [7] : following their analysis, the superpotential $C^\sharp$ is a quintic in $\mathbb{C}^5$, singular at a bouquet of $\mathbb{C}^1$-rays. (The projectivization $\mathbb{C}^5 \to \mathbb{P}^4$ is caused by the kinetic term.) Adding a small multiple of a smooth polynomial to resolve this degeneracy would also change the asymptotic behaviour of the potential and would therefore say nothing about $S^\sharp$.

Nevertheless, the Hamilton-Dirac quantization can be carried out, starting with the second class constraint $C^\sharp(X) = 0$. Commutation with the Hamiltonian yields secondary constraints, of the form of Eq. (7) and (6), with $\xi_\pm{}^\mu$ replaced by the canonical momenta. Where both $C^\sharp(X)$ and $C^\sharp_{,\mu}(X)$ vanish, further constraints are obtained by iteration until the Dirac brackets and thereby the quantum theory are well defined.

4. Resolved σ-Models

Finally, consider a seemingly unrelated non-linear σ-model action

$$\check{S} \;=\; \int_\Sigma \mathrm{d}^2\sigma\,\mathrm{d}^2\varsigma\,\mathrm{d}^2\bar{\varsigma}\;\Big(\mathring{\mathbf{K}}(\boldsymbol{X},\overline{\boldsymbol{X}}) \;+\; w^y\mathring{\mathbf{K}}_y(\boldsymbol{Y},\overline{\boldsymbol{Y}})\Big) \tag{16}$$

$$+\Big[\int_\Sigma \mathrm{d}^2\sigma\,\mathrm{d}^2\varsigma\;\boldsymbol{\Lambda}^1\Big(P(\boldsymbol{X})\,\boldsymbol{Y}^1 + Q(\boldsymbol{X})\,\boldsymbol{Y}^2\Big) + \boldsymbol{\Lambda}^2\Big(R(\boldsymbol{X})\,\boldsymbol{Y}^1 + S(\boldsymbol{X})\,\boldsymbol{Y}^2\Big) \;+\; \text{h.c.}\;\Big] \tag{17}$$

for strings in a Calabi-Yau manifold defined by

$$\check{\mathcal{M}}\;:\qquad \begin{aligned} P(x)\,y^1 + Q(x)\,y^2 &= 0\;,\\ R(x)\,y^1 + S(x)\,y^2 &= 0\;. \end{aligned} \tag{18}$$

Here y^i are homogeneous coordinates on $\mathbb{P}^1$ and x^μ are homogeneous coordinates on some compact complex four-fold $\mathcal{X}$. $\boldsymbol{X}^\mu$ and $\boldsymbol{Y}^i$ are homogeneous coordinate (2,2)-superfields on the $\mathcal{X}$ and $\mathbb{P}^1$ factors of the embedding space, respectively, and $\mathring{\mathbf{K}}$ and $\mathring{\mathbf{K}}_y$ are suitably chosen respective Kähler potentials.

S, P, Q, R are generic homogeneous polynomials with the degrees chosen so that the target space of the σ-model is a smooth Calabi-Yau manifold. The (possibly incomplete [9,15]) parameter space of this model is spanned by w^y, the w^As in $\mathring{\mathbf{K}}(\boldsymbol{X},\overline{\boldsymbol{X}})$ and by the parameters in the polynomials S, P, Q, R.

In the limit $w^y \to 0$, all the components of the two $\boldsymbol{Y}$ superfields become non-propagating and their equations of motion are algebraic. Therefore, $\boldsymbol{Y}$ may be integrated out and we can rewrite the above constraint part of the action (17) as

$$S'_{con.} \;=\; \int_\Sigma \mathrm{d}^2\sigma\,\mathrm{d}^2\varsigma\;\Big((\boldsymbol{\Lambda}^1\boldsymbol{Y}^1)\,P(\boldsymbol{X}) \;+\; (\boldsymbol{\Lambda}^1\boldsymbol{Y}^2)\,Q(\boldsymbol{X}) \;+\; (\boldsymbol{\Lambda}^2\boldsymbol{Y}^1)\,R(\boldsymbol{X}) \;+\; (\boldsymbol{\Lambda}^2\boldsymbol{Y}^2)\,S(\boldsymbol{X})\Big)\;. \tag{19}$$

Because of the identity

$$(\mathbf{\Lambda}^1\boldsymbol{Y}^1)(\mathbf{\Lambda}^2\boldsymbol{Y}^2) \;\equiv\; (\mathbf{\Lambda}^1\boldsymbol{Y}^2)(\mathbf{\Lambda}^2\boldsymbol{Y}^1)\;, \tag{20}$$

path-integration over the two $\mathbf{\Lambda}$ and the two $\boldsymbol{Y}$ superfields has the same effect as path-integration over a single superfield $\mathbf{\Lambda}$ in

$$S^\sharp_{con.} \;\stackrel{\rm def}{=}\; \int_\Sigma \mathrm{d}^2\sigma\,\mathrm{d}^2\varsigma\;\mathbf{\Lambda}\Big[\,S(\boldsymbol{X})P(\boldsymbol{X}) - Q(\boldsymbol{X})R(\boldsymbol{X})\,\Big]\;. \tag{21}$$

Therefore, at $w^y = 0$, we might as well write

$$S^\sharp \;=\; \int_\Sigma \mathrm{d}^2\sigma\,\mathrm{d}^2\varsigma\,\mathrm{d}^2\bar\varsigma\;\mathring{\mathbf{K}}(\boldsymbol{X},\overline{\boldsymbol{X}}) \;+\Big[\;S^\sharp_{con.} +\;\text{h.c.}\;\Big] \tag{22}$$

in place of $\check S$ of Eqs. (16)–(17). The σ-model actions (22) and (13) are *identical*.

The Euler characteristic $\check\chi_{_E}$ of $\check{\mathcal{M}}$ (18) receives a correction at $w^y = 0:\ \chi^\sharp_{_E} = \check\chi_{_E} - N$, in agreement with the fact that $\chi^\flat_{_E} + 2N = \chi^\sharp_{_E} + N = \check\chi_{_E}$ [3,4]. An application of a Landau-Ginzburg analysis is again problematic. To see this, suffice it here to consider the example where P and Q are quartic while R and S are linear in $\mathbb{P}^4$. Whatever the scaling weights of $\boldsymbol{X}$s and $\boldsymbol{Y}$s, the two polynomials

$$W_1 \;=\; P(\boldsymbol{X})\,\boldsymbol{Y}^1 + Q(\boldsymbol{X})\,\boldsymbol{Y}^2\;, \qquad W_2 \;=\; R(\boldsymbol{X})\,\boldsymbol{Y}^1 + S(\boldsymbol{X})\,\boldsymbol{Y}^2 \tag{23}$$

scale differently and only one will dominate the superpotential $W_1 + W_2$. This contradicts the fact that the explicit choice of both W_1 and W_2 determines the complex structure on $\check{\mathcal{M}}$ and thus (in part) the fixed point to which the renormalization flow will take $\check S$. It is easy to see that this holds for all possible processes $S^\flat \to S^\sharp \to \check S$ among all odd dimensional Calabi-Yau complete intersections in products of complex projective spaces. The use of Lagrange multipliers is inevitable.

5. Chameleonic σ-Models

The two σ-model actions, one in Eqs. (16)–(17) and the other in Eq. (2), define consistent *families* of compactifications. In both cases, the families are swept out by marginal perturbations corresponding to (2,1)- and (1,1)-forms. For certain finite marginal perturbations (corresponding to $w^y \to 0$ in $\check S$ and $t^a \to 0$ in $S^\flat$), the interfacing σ-model action $S^\sharp$ (22) is reached as a common limit. The doubting reader may care to check that the same metric is indeed obtained for the nodal σ-model action $S^\sharp$ in both limiting procedures [16].

Since topological physical observables on one and the other 'side' of $S^\sharp$ are different [3,4], this connecting process describes a topology changing transition. It is however not clear if the classical action $S^\sharp$ defines a single 1+1-dimensional quantum field theory and so corresponds to a single vacuum[♯3]. Starting from $S^\sharp$, the renormalization flow might bifurcate and lead to two distinct vacuua, one of which is the $t^\alpha{\to}0$-limit of the vacuum defined by $S^\flat$ and another the $w^y{\to}0$-limit of the vacuum defined by $\check{S}$. If there is no bifurcation, the vacuua corresponding to $S^\flat$ and $\check{S}$ do have a common limit. In an analogy to the *universal moduli space* for Riemann surfaces Ref. [5], we should then construct the 'universal Calabi-Yau moduli space' by forming a union of all Calabi-Yau moduli spaces, interfaced by the moduli spaces of nodal or worse singularizations.

The finiteness of the *exact* Zamolodchikov distance between $S^\flat$ and $\check{S}$ is again an important but separate issue, perturbatively addressed in Ref. [6,17]. In Ref. [19], it is argued that the Zamolodchikov distance between topologically distinct models is infinite, opposing the expectation based on the finiteness of the Weil-Petersson distance [6,4]. Recall that the Weil-Petersson distance between the sphere and any torus is infinite; according to Ref. [19], the same is true of the Zamolodchikov distance. Yet, the moduli space of tori is compactified with the moduli space of a sphere (a point), in the *universal moduli space* for Riemann surfaces. With Calabi-Yau spaces, at least the Weil-Petersson distance has been proven to be finite; this 'universal moduli space' appears better behaved.

At this stage, all the vacuua encountered in this σ-model on the connected moduli space are degenerate because of the local N=1 supersymmetry in 3+1-dimensional space-time and no 'minimizing principle' can possibly choose between them. Promoting the parameters such as t^α and w^A into space-time dependent (moduli) fields of which t^α and w^A are the vacuum expectation values, one constructs a spacetime σ-model[♯4] in which the couplings are n-point functions computed from $S^\flat$, $S^\sharp$ or $\check{S}$. For example, the 2-point function is the Zamolodchikov metric and occurs in the kinetic terms for the moduli fields. The minimization of the free energy in this spacetime σ-model should choose the vacuum, possibly hovering about in the 'universal Calabi-Yau moduli space', but only upon supersymmetry breaking.

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[♯3] I thank E. Martinec and V. Kaplunovski for discussions on this point.

[♯4] Very similar ideas on topology change are seen at work in a recent study where the dynamics of this spacetime σ-model produces stringy cosmic strings [18].

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