A Realistic Superstring-Inspired Brane-World Cosmology

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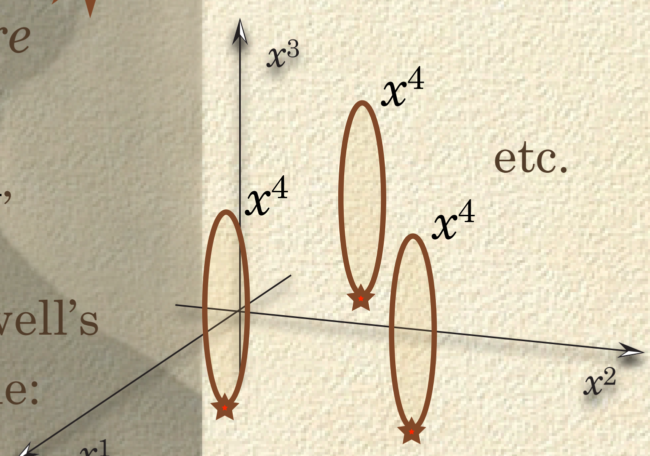
- What are Brane-Worlds?
  - *Some history, and what Brane-Worlds are (not)*
- What are Stringy Brane-Worlds?
  - *(Super)strings and Cosmology interplay*
- What are "ET"s, and Why Censor Them?
  - *A potentially disastrous observation & its cure*
- What is Dynamical Censorship?
  - *Dynamics of matter around brane-Worlds and its consequences*

What Brane-Worlds are not...

- 1914: Gunnar Nordström, (early competitor of Einstein's relativity)
- 1919: Theodor F.E. Kałuża
- 1926: Oscar Klein
- Our spacetime is a sub-spacetime of a bigger (5-dimensional) one
  - *The metric tensor* $ds^2 = g_{ij}\, dx^i\, dx^j$, *where i,j = 0,1,2,3,4*
  - *decomposes:* ($g_{\mu\nu}$, $g_{\mu 4}$, g_{44}) *where*
    - g_{04} plays the role of $\mathbf{\Phi}$,
    - (g_{14}, g_{24}, g_{34}) play the role of **A**,
    - …of electromagnetism
    - Einstein equations beget Maxwell's
    - provided x^4 curls up into a circle:



What Brane-Worlds are not…

- (Nordström)-Kałuża-Klein compactification:
  - *"extra" dim's are curled up (compact),* $\leq 10^{-33}$m*;*
  - *fields decompose into "Fourier modes" over extra dimensions*
    - Only 1 extra dimension ☞ easy: $e^{in\cdot x^4/(2\pi R)}$,
    - The mass (inertia) of the n^{th} mode = $n\cdot(\hbar/2\pi cR) \geq 10^{17}$ GeV/c^2, that is, $\Box_{(5)} \to \Box_{(4)} + [n\cdot(1/2\pi R)]^2$;
    - Plus the $g_{ij} \Rrightarrow (\, g_{\mu\nu},\, g_{\mu 4},\, g_{44}\,)$ decomposition:

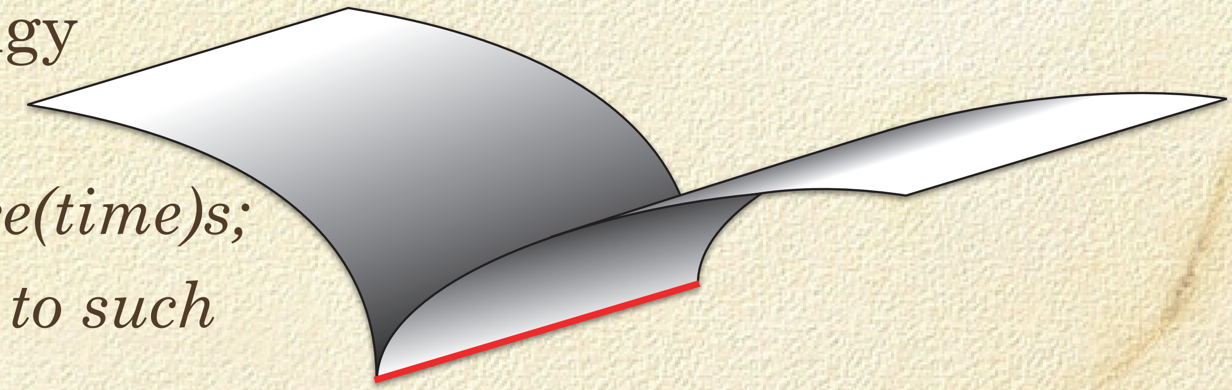
				Masses:
		⋮	⋮	⋮	
		$g^7{}_{\mu\nu}(x^0,\dots,x^3)$,	$g^7{}_{\mu 4}(x^0,\dots,x^3)$,	$g^7{}_{44}(x^0,\dots,x^3)$	$\sim 7\times10^{17}$ GeV
		⋮	⋮	⋮	
		$g^1{}_{\mu\nu}(x^0,\dots,x^3)$,	$g^1{}_{\mu 4}(x^0,\dots,x^3)$,	$g^1{}_{44}(x^0,\dots,x^3)$	$\sim 1\times10^{17}$ GeV
$g_{ij}(x^0,\dots,x^4)$	$\Rrightarrow$	$g^0{}_{\mu\nu}(x^0,\dots,x^3)$,	$g^0{}_{\mu 4}(x^0,\dots,x^3)$,	$g^0{}_{44}(x^0,\dots,x^3)$	~ 0 GeV

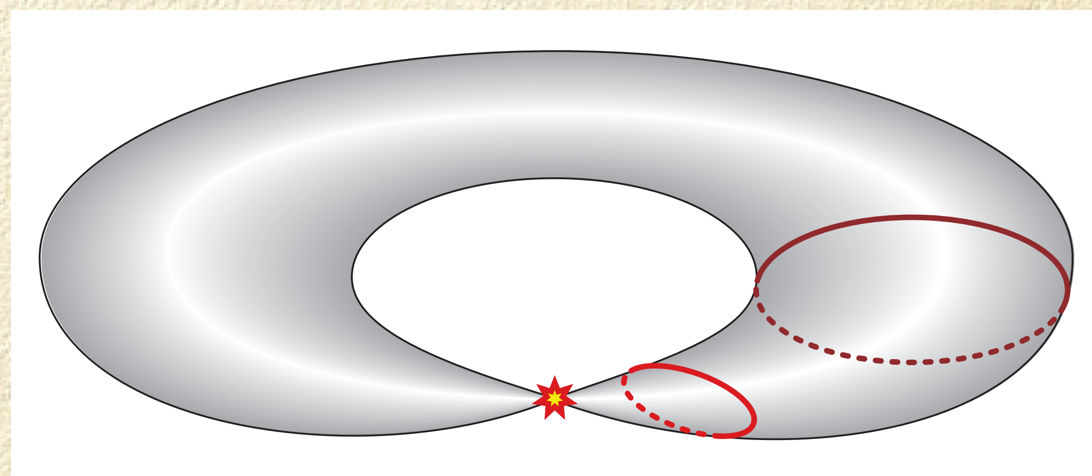
What Brane-Worlds are...not yet

- Superstrings must have 10-dimensional spacetime
- 1984: Candelas-Horowitz-Strominger-Witten
  - *Kałuża-Klein & supersymmetry* ⇛ *Calabi-Yau* ($R_{\mu\nu} \simeq 0$)
- 1986–87: Frenkel-Garland-Zuckerman, Rajeev-Bowick, Alvarez-Gaumé-Gomez-Reina, Pilch-Warner, Oh-Ramond, Harari-Hong: $R_{\mu\nu} \simeq 0$ is a (super)string condition
- "Generic/typical" stringy spacetimes are:
  - Calabi-Yau (Ricci-flat: $R_{\mu\nu} \simeq 0$, Wick-rotated complex 5-folds)
  - (Possibly, sectionally) non-compact
  - (Possibly) mildly singular
  - (Possibly) stratified

What Brane-Worlds are...

- "Generic/typical" stringy spacetimes feature*:
  - *"isolated" 4D subspace(time)s;*
  - *with matter localized to such subspace(time)s;*

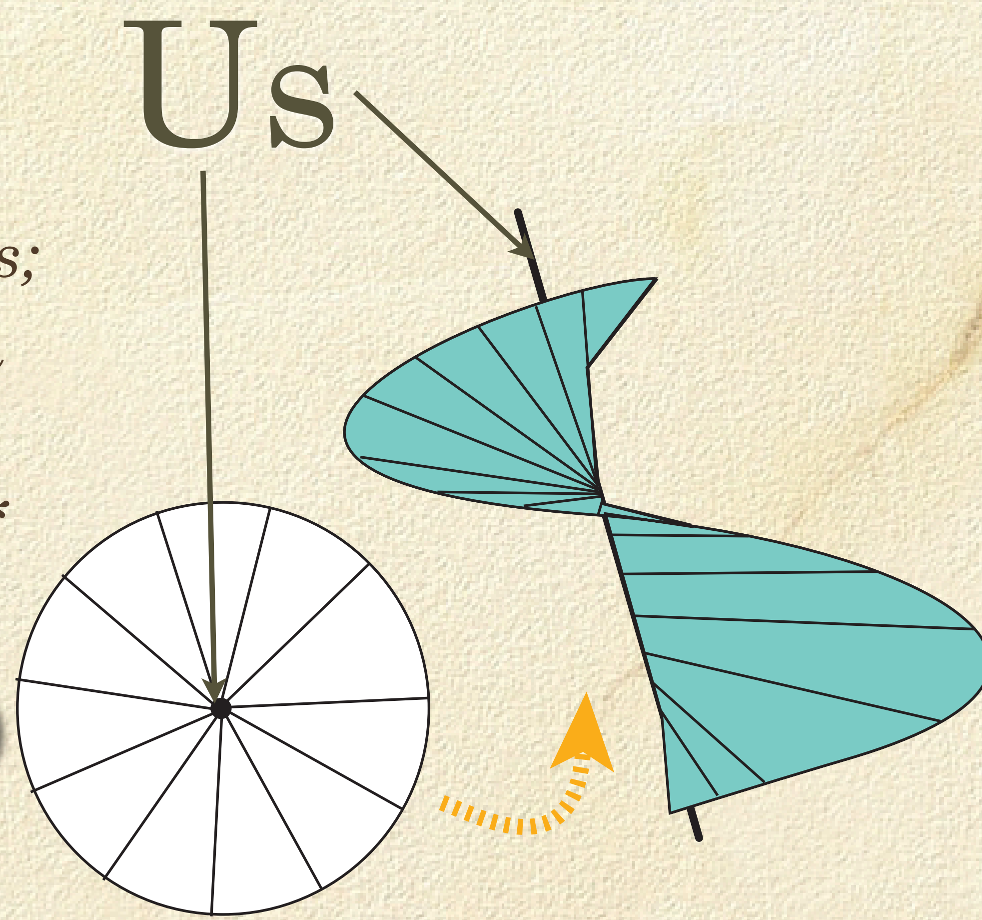




* T. Hübsch, Nucl. Phys. (Proc. Suppl.) 52A (1997) pp.347–350.

What Brane-Worlds are...

- "Generic/typical" stringy spacetimes feature*:
  - *"isolated" 4D subspace(time)s;*
  - *with matter localized to such subspace(time)s;*
  - *the first gedanken-prototype* of jump-gates and warp-drive: detouring into the "transversal" directions (hyper-space), then return*
  - *...and gravity?!*



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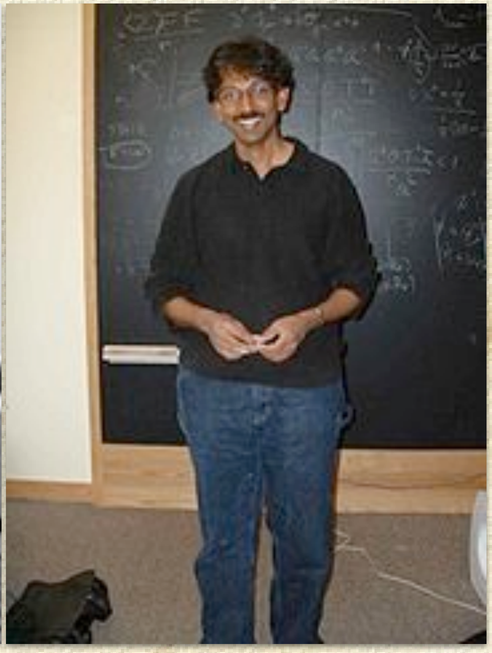
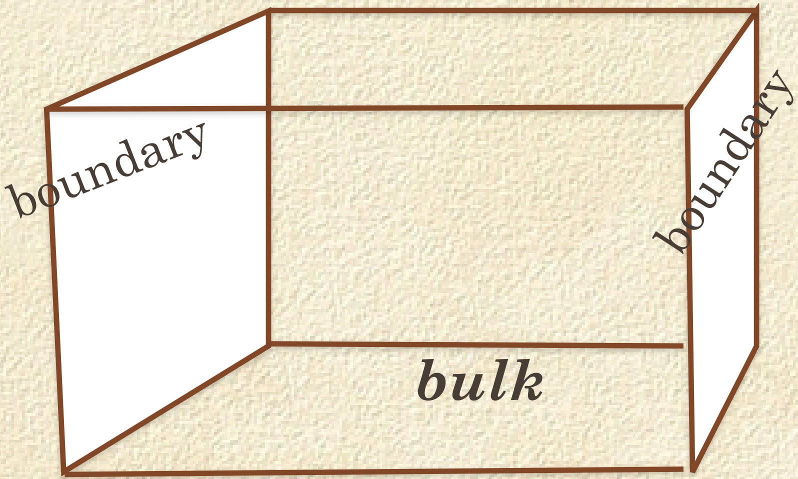
What Brane-Worlds are...

- 1999: Lisa Randal & Raman Sundrum
- Our spacetime is a sub-spacetime in a bigger (5-dimensional) one,
- ...for example, at its ***boundary***
- The “heterotic M-theory” of Hořava and Witten:
    - *two 10-dimensional boundaries in an 11-dimensional spacetime*
    - *each with an E_8 gauge group*

Fourscore years after Kałuża





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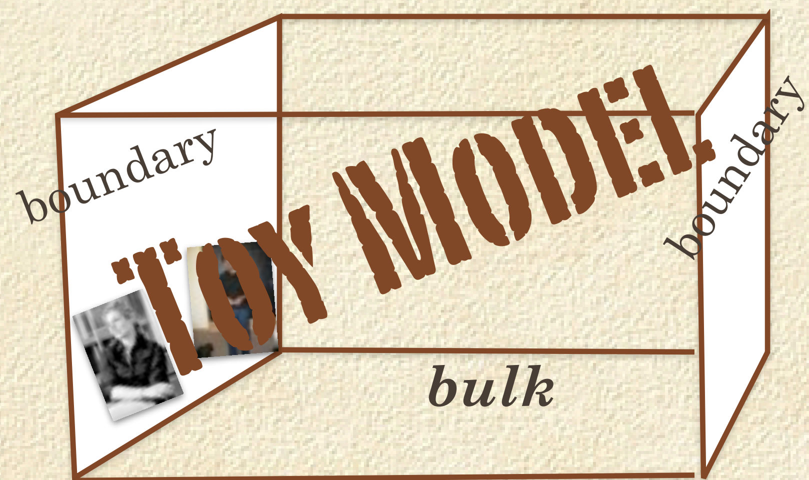
- Two most cited papers in 5 years:
  - *RS-1: exponential hierarchy (right)*
  - *RS-2: localized gravity (left)*
    - ...but not both...



- Localized matter (us)
  - *The matter particles which constitute the observable Universe must be localized to this sub-spacetime:*
    - Atiyah-Bott-Gårding-Candelas formula:
      - *For algebraic varieties defined as the* $f(z)=0$ location $Y \subset X$,

$$\mathbf{Res}_y[\cdots] := \frac{1}{2\pi i}\oint_{\gamma(Y)} \mathrm{d}z\,\frac{[\cdots]}{f(z)} = \begin{cases} [\cdots]\big|_Y & \text{for} \quad z \in Y \\ 0 & \text{for} \quad z \notin Y \end{cases}$$

      - *…generalized* for all cohomology on Y,*
      - *…in the supersymmetry limit (when* $\mathbb{C}$*-analysis works)*
    - Plus: a dynamical mechanism; see below
- Localized Yang-Mills (interaction) fields (us)
  - *EW & S: gauge quanta must be localized just like matter*
  - *And they are: in the* $A_{\parallel} - A_{\perp}$ *decomposition,*
  - *…the former always have localized harmonic representatives*

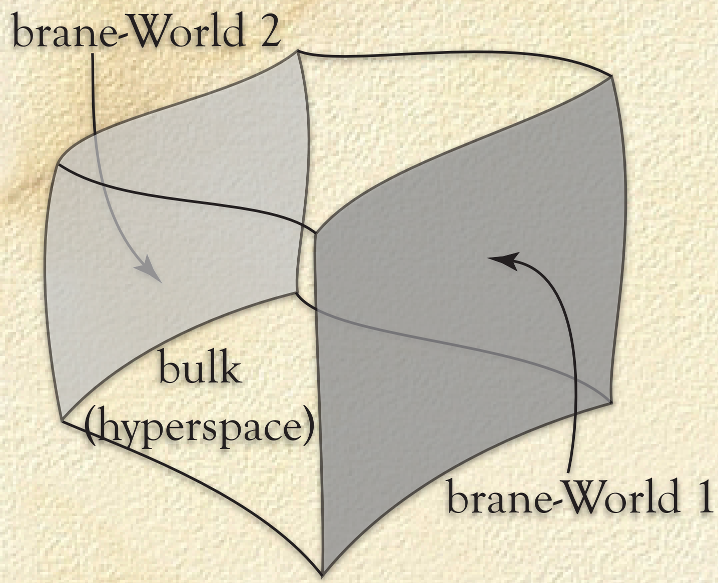
*P. Berglund & T. Hübsch, *Int. J. Mod. Phys.* **A10** (1995) 3381-3430.

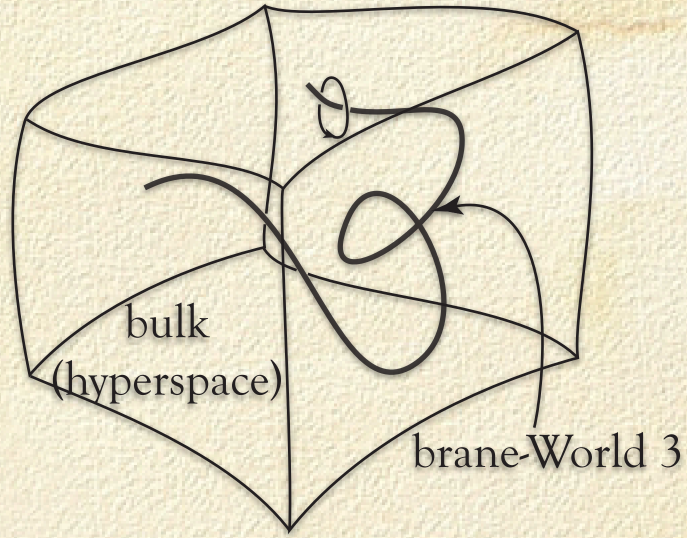
- Localized fields (us)
  - *Newtonian Gravity*[†]*: force must follow the* r^{-2}*-law*
    - RS-mechanism:
    - $|x|$-like non-analytic functional dependence…
    - …produces δ-like terms in the Riemann curvature…
    - …and δ-like effective potential for fluctuations.
    - With a –sign, δ-potential ⇛ unique localized 0-mode!
    - The continuum "above" corrects Newton's law: $G_N\left[\frac{1}{r^2}+\frac{L^k}{r^{k+2}}\right]$
    - …where L is a curvature-related length scale.
  - *Mass-scale induction — a bonus!*
    - Exponential hierarchy[†]: re-scaling of G_N and $M_P \sim 10^{19}$ GeV,
    - so that in the higher-dimensional spacetime $M_P \sim 10^2$ GeV;
    - This depends not on the extra dimensions' size…
    - …but **curvature**, near the brane-World.

[†] L. Randall & R. Sundrum, *Phys. Rev. Lett.* 83 (1999) pp.3370, and 4690.

... in pictures:

If you can walk around it,
it is codimension-2 or more.





Brane-Worlds as boundaries

Codimension = 1

Embedded brane-Worlds

Codimension > 1

Randall and Sundrum found it possible to: localize gravity in brane-World 1, and to obtain an exponential mass-hierarchy in brane-World 2 — but not together.

So, how *does* one localize gravity and induce a mass-hierarchy?

- Expand $g_{\mu\nu}(x,y) = \sum_k g^{(k)}_{\mu_x\nu_x}\phi_k(y)$ into "transversal" modes;
- prove the semi-infinity of the spectrum, $\{k \geq 0\}$;
- $g^{(0)}_{\mu_x\nu_x}$ is "our" localized graviton—*iff* $\phi_0(y)$ is bound to the brane-World;
- sum the contributions of $\phi_{k>0}(y)$ to Newton's law:

$$V(r) = M_D^{-D_B}\,\frac{m_1 m_2}{r_x}\sum_{k=0}^{\infty}\rho_k(r)\,\frac{|\phi_k(0)|^2}{\langle\phi_k|\phi_k\rangle} = G_N\,\frac{m_1 m_2}{r}\Big[1+\Big(\frac{L}{r}\Big)^n\Big]\,,$$

so…

$$G_N = M_D^{-D_B}\,\frac{|\phi_0(0)|^2}{\langle\phi_0|\phi_0\rangle} \quad \& \quad \boxed{\triangle V(r) = G_N\,\frac{m_1 m_2}{r}\,\frac{\langle\phi_0|\phi_0\rangle}{|\phi_0(0)|^2}\sum_{k>0}^{\infty}\rho_k(r)\,\frac{|\phi_k(0)|^2}{\langle\phi_k|\phi_k\rangle}}\,.$$

Here, $M_D = \sqrt{\frac{\hbar c}{G_N^{(D)}}}$ is the Planck mass in D dimensions.

(Super)strings and Brane-Worlds

- All (super)strings
  - *have a well-defined (point-field) limit, w/(super)gravity,*
  - *may be regarded as torus-compactifications of F-theory;*
  - *so must contain:*

$$S_{\text{eff}} = \frac{1}{2\kappa^2}\int \mathrm{d}^D x \sqrt{-g}\left(R - G_{\alpha\bar{\beta}} g^{\mu\nu}\partial_\mu\phi^\alpha\partial_\nu\phi^{\bar{\beta}} + \cdots\right),$$

where $2\kappa^2 = 16\pi G_N^{(D)}$ is the strength of gravity in D dimensions.

  - *and the ϕ^α are moduli fields (parametrizing the shapes and sizes of the spacetime)*

- Simplify*:
  - *consider a single modulus field, τ, — for a torus,*
  - *choose the appropriate (Teichmüller) metric,*
  - *assume the spacetime metric to:*
    - be of the "warped" direct product type,
    - depend only on a planar radius,
  - *assume τ to depend only on the planar angle:*

$$\mathrm{d}s^2 \;=\; A(z)\eta_{ab}\mathrm{d}x^a\mathrm{d}x^b + B(z)\mathrm{d}z^2 + B(z)l^2\mathrm{d}\theta^2 \;, \quad z = \log(r/l)$$

$$\tau = \tau(\theta), \quad G_{\tau\bar{\tau}} = \frac{-1}{[\Im m(\tau)]^2} \quad \textit{(the Teichmüller metric)}$$

… and then solve the coupled metric-modulus (spacetime-matter) system.

*P. Berglund, D. Minic & T. Hübsch: *see later for a complete bibliography.*

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R \;=\; T_{\mu\nu}(\tau,\bar\tau)\;, \qquad g^{\mu\nu}\big[\nabla_\mu\partial_\nu\tau - \Gamma^\tau_{\tau\tau}\partial_\mu\tau\partial_\nu\tau\big] \;=\; 0\;,$$

but, $$T_{\mu\nu}(\tau,\bar\tau) \;\stackrel{\rm def}{=}\; \partial_\mu\tau\partial_\nu\bar\tau - \tfrac{1}{2}g_{\mu\nu}(g^{\rho\sigma}\partial_\rho\tau\partial_\sigma\bar\tau)$$

$$R_{\mu\nu} \;=\; \widetilde{T}_{\mu\nu} \;\stackrel{\rm def}{=}\; \partial_\mu\tau\partial_\nu\bar\tau\;!$$ *Note: separation of variables!*

- The above simplification occurs *only* for matter that has *no potential*, which is true only of (superstrings') moduli fields!
- $$\tau'' + \frac{2}{\bar\tau - \tau} \;=\; 0$$

  is solved by:

$$\tau_I(\theta) = \mathrm{a}_0 + i\,g_s^{-1}\,e^{\omega(\theta-\theta_0)}\;, \qquad \mathrm{a}_0, g_s, \omega, \theta_0 = const$$

$$\tau_{II}(\theta) = \mathrm{a}_0 \pm g_s^{-1}\,\frac{\sinh[\omega(\theta-\theta_0)] + i}{\cosh[\omega(\theta-\theta_0)]}\;,$$

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R \;=\; T_{\mu\nu}(\tau,\bar\tau)\;, \qquad g^{\mu\nu}\big[\nabla_\mu\partial_\nu\tau - \Gamma^\tau_{\tau\tau}\partial_\mu\tau\partial_\nu\tau\big] \;=\; 0\;,$$

but,

$$T_{\mu\nu}(\tau,\bar\tau) \;\stackrel{\text{def}}{=}\; \partial_\mu\tau\partial_\nu\bar\tau - \tfrac{1}{2} g_{\mu\nu}(g^{\rho\sigma}\partial_\rho\tau\partial_\sigma\bar\tau)$$

$$R_{\mu\nu} \;=\; \widetilde{T}_{\mu\nu} \;\stackrel{\text{def}}{=}\; \partial_\mu\tau\partial_\nu\bar\tau\;!$$

Note: separation of variables!

$$-\eta_{ab}\,\ell^{-2}\Big[a_0\xi\,\mathrm{sign}^2(z)\,Z^{\frac{D-1}{D-2}}\,e^{-\frac{\xi}{a_0}(\beta-Z^2)} - \varrho\,\delta(z)\Big] = T_{ab} + T^b_{ab}\;,$$

$$-a_0\xi\,\mathrm{sign}^2(z) = T_{zz} + T^b_{zz}\;,$$

$$+a_0\xi\,\mathrm{sign}^2(z) + 2a_0\,\delta(z) = T_{\theta\theta} + T^b_{\theta\theta}\;.$$

with $\varrho := a_0 e^{-\frac{\xi}{a_0}(\beta-1)}\big[\frac{D-3}{D-2} - 2\frac{\xi}{a_0}\big]$

$$\mathrm{d}s^2 = A(z)\eta_{ab}\mathrm{d}x^a\mathrm{d}x^b + \ell^2 B(z)(\mathrm{d}z^2 + \mathrm{d}\theta^2) \qquad Z(z) := 1 + a_0|z|$$

$$A(z) = Z^{\frac{2}{D-2}} \qquad B(z) = Z^{-\frac{D-3}{D-2}}\,e^{\frac{\xi}{a_0}(\beta-Z^2)}$$

- Neither of the two (only!) solutions for the matter fields are single-valued! Instead, they "jump" across the direction θ_0:

$$\tau_1(\theta_0 + 2\pi) = \frac{a\tau_1(\theta_0) + b}{c\tau_1(\theta_0) + d} \ , \qquad \tau_2(\theta_0 + 2\pi) = \tau_2(\theta_0) \pm n \ ,$$

There is a 3-parameter space of choices for the first configuration, which for special choices of a_0*,* g_s *and* ω*, contain integral solutions for a,b,c,d.*

Both of these solutions exhibit an $SL(2,\mathbb{Z})$ monodromy,
— just as superstring moduli (and no mere matter fields) do!

Thus, we have two configurations of a matter field which could not have stemmed from anything but superstrings.

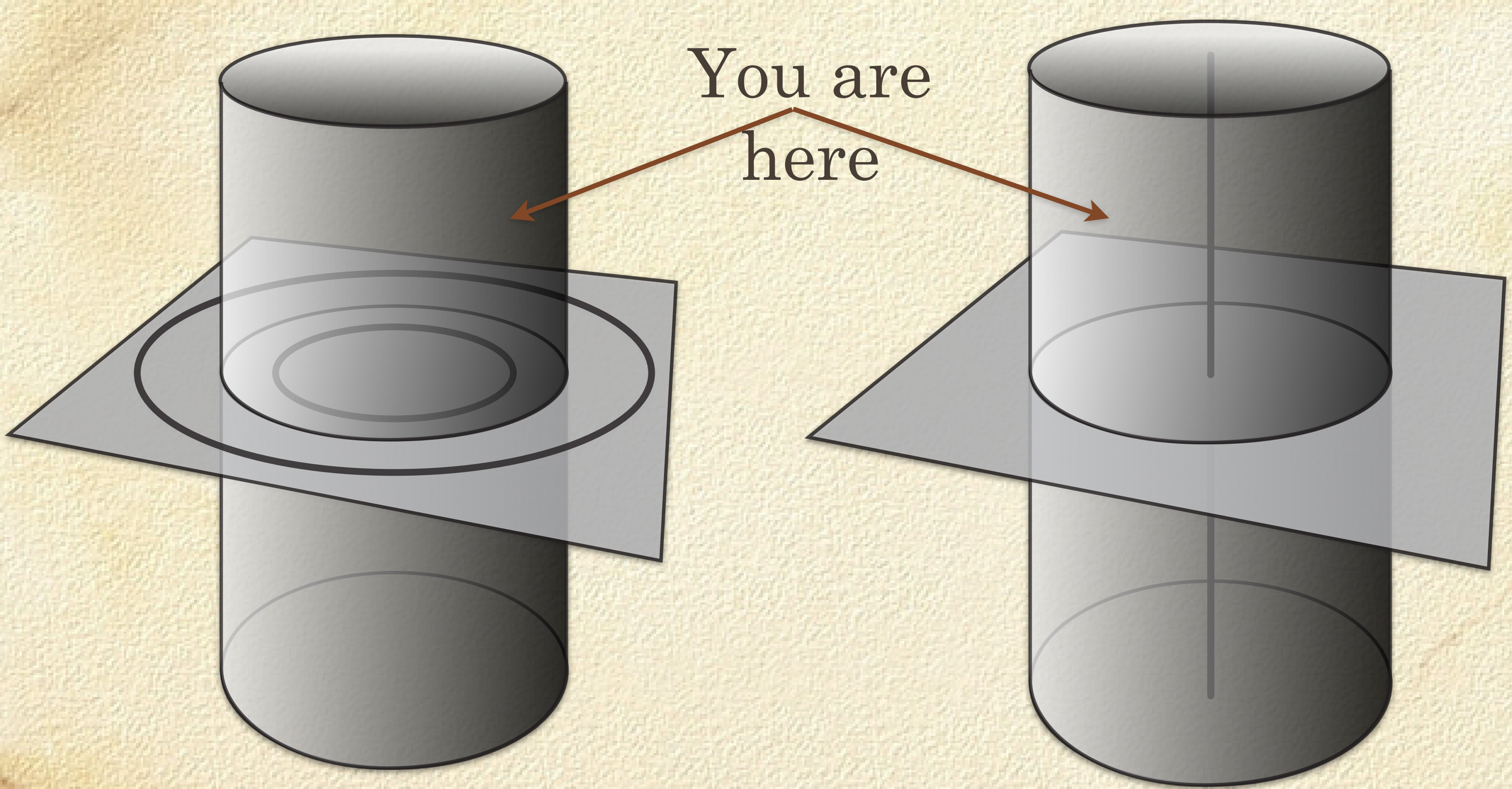
The metric ("transversally")

$$\mathrm{d}s^2 = A(z)\eta_{ab}\mathrm{d}x^a\mathrm{d}x^b + B(z)\mathrm{d}z^2 + B(z)l^2\mathrm{d}\theta^2 \ , \qquad z = \log(r/l)$$

$$A(z) = Z^{\frac{2}{D-2}} \ , \quad B(z) = Z^{-\frac{D-3}{D-2}}\, e^{\frac{\xi}{a_0}[\beta - Z^2]} \ , \qquad Z(z) \stackrel{\text{def}}{=} 1 + a_0|z|$$

- With $a_0 < 0$, this metric exhibits naked singularities:
  - *where it (and an invariant measures of curvature!) grow unbounded;*
  - *and which coincides with the horizon (place of no return for light);*
  - *beyond which, the distance-squared is complex, i.e., unphysical.*
- With $a_0 > 0$, this metric exhibits time-like singularities:
  - *at* $z = \pm\infty$ ($r = 0, \infty$), *time stops to advance.*
- In both cases, at $z = 0$ ($r = l$), there is a δ-function like source.

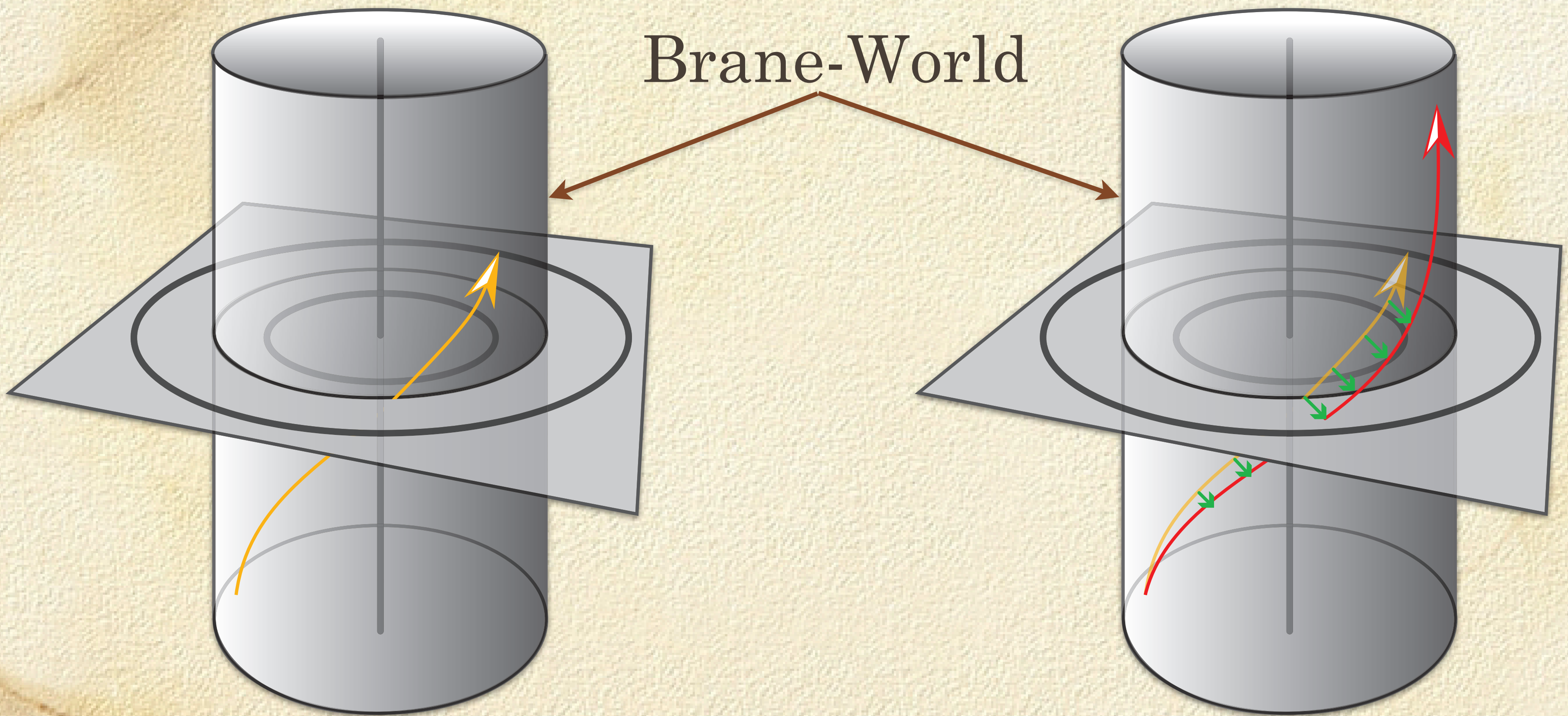
The metric – in pictures



a_0<0, the δ-function brane is sheathed by singularities

a_0>0, the δ-function brane is coaxial with a singularity

Oh, by the way...



a_0<0, the δ-function brane-World is sheathed by singularities; the **yellow arrow** is a traveler

By moving off of the brane-World, the **red traveler** can move faster*, ahead of the Brane-World **yellow**

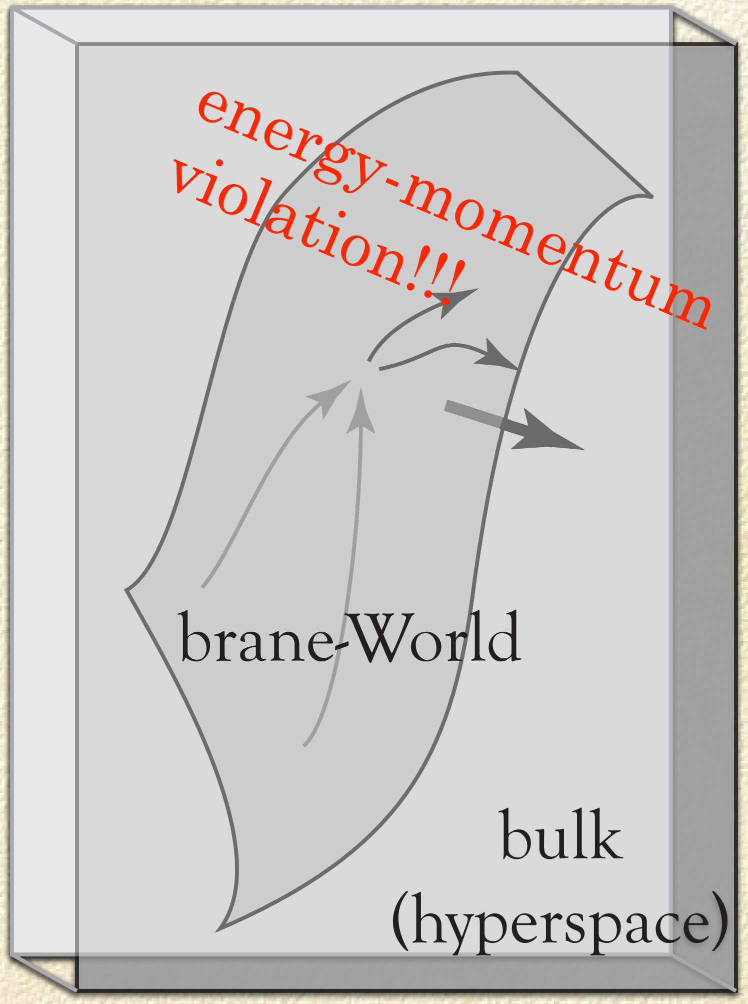
*A.F. Roane, *Some Brane-World Cosmological Models*, PhD dissertation

“ET”s, and Why Censor Them

- Bulk-roaming modes are unobservable…
- …most of the time.

- But, when they pass through “our” spacetime…
- …and interact with us,
- …they invalidate all our (local) conservation laws:

- Thus we must:
  - *either ban all ET’s*
  - *or localize them all!*



A Dynamical "ET" Censoring Mechanism

- Matter localized to the δ-function brane-World
  - *with exact supersymmetry, may be represented by residues*;*
  - *upon supersymmetry breaking, become Gaussian distributions.*
- Bulk-roaming matter (the "ET"s)
  - *represented by p-brane-probes of various spatial extension, p;*
  - *the dynamics of which is "pseudo-relativistic":*

$$S_p \;=\; -\,\tau_p \int \mathrm{d}^{p+1}\xi\; e^{-\Phi}\sqrt{\det[G^s_{ab}+B_{ab}+2\pi\alpha' F_{ab}]} + \mu_p \int \mathrm{d}^{p+1}\xi\; C_{p+1}\,,$$

$$[G^s_{ab}] = \mathrm{diag}[-(A-Bv^2)e^{\Phi/2}, \underbrace{Ae^{\Phi/2},\cdots,Ae^{\Phi/2}}_{p}]\,, \qquad \text{set } B_{ab}=0=F_{ab}\;.$$

*P. Berglund & T. Hübsch, *Int. J. Mod. Phys.* **A10** (1995) 3381-3430.

Effectively, this is the dynamics of a relativistic particle:

$$S_p = \int \mathrm{d}^{p+1}\xi \; \mathcal{L} \,, \qquad \mathcal{L} = -m\,c^2\sqrt{1 - \frac{v^2}{c^2}} - \mathcal{V} \,,$$

$$c = \frac{A(z)}{B(z)} \,, \quad m = \tau_p e^{\frac{(p-3)}{4}\Phi}\big(A(z)B^2(z)\big)^{p-1} \,, \quad \mathcal{V} = -\mu_p C_{p+1} \,.$$

As these vary radially, so does the brane-probe's dynamics.

Since

$$\mathcal{E} = \frac{mc^2}{\sqrt{1 - v^2/c^2}} + \mathcal{V} \quad \Rightarrow \quad v^2 = c^2\left[1 - \frac{m^2c^4}{(\mathcal{E} - \mathcal{V})^2}\right]$$

$$\mathcal{E} \geq mc^2 + \mathcal{V} = \tau_p e^{\frac{(p-3)}{4}\Phi} A^{p+1} - \mu_p C_{p+1}$$

A Conundrum

- However, this inequality pertains to the total, (Hamiltonian), *conserved* energy.
- Yet, the brane-probes will radiate away their energy, through EM, gravitational, *etc.* waves.
- But, without a detailed knowledge of the brane-probes' interactions with other matter,
  - *How can one estimate the radiation loss?*
  - *How can one integrate its "deduction" from energy?*

Easy:

The heart is a-start,
And this crown comment, the function I meant
To use but could not, now spry upon foment!
— Shawn Benedict Jade

Coupling to radiation –*of any kind*– modifies the momentum, and so also the velocity, to a "covariant" momentum:

$$\vec{v} \to \vec{v} + c\vec{\Gamma}~, \qquad (\vec{v} + c\vec{\Gamma})^2 = v^2 + c^2\Gamma^2~, \qquad \vec{v}\cdot\vec{\Gamma} = 0~.$$

This "connection" represents an average, combined connection, *including all interactions* of the brane-probe.

$$\mathcal{E}' \;=\; \mathcal{V} + mc^2 A_{d-2} N \int_0^{\sqrt{1-v^2/c^2}} \frac{\Gamma^{d-3}\mathrm{d}\Gamma}{\sqrt{1-\frac{v^2+c^2\Gamma^2}{c^2}}}~,$$

$$\mathcal{E}'|_{v=0} = \mathcal{E}|_{v=0}, \qquad \mathcal{E}' = \mathcal{V} + mc^2\Big(1-\frac{v^2}{c^2}\Big)^{\frac{d-3}{2}}$$

$$\mathcal{E}'_{d=4} = \mathcal{V} + mc^2\sqrt{1-v^2/c^2}~, \text{ not conserved!}$$

The dynamics of $\mathcal{E}'$ being governed by $\dfrac{\mathrm{d}\mathcal{E}'}{\mathrm{d}t} = \{\,\mathcal{E}\,,\,\mathcal{E}'\,\}_{\mathrm{PB}}$,

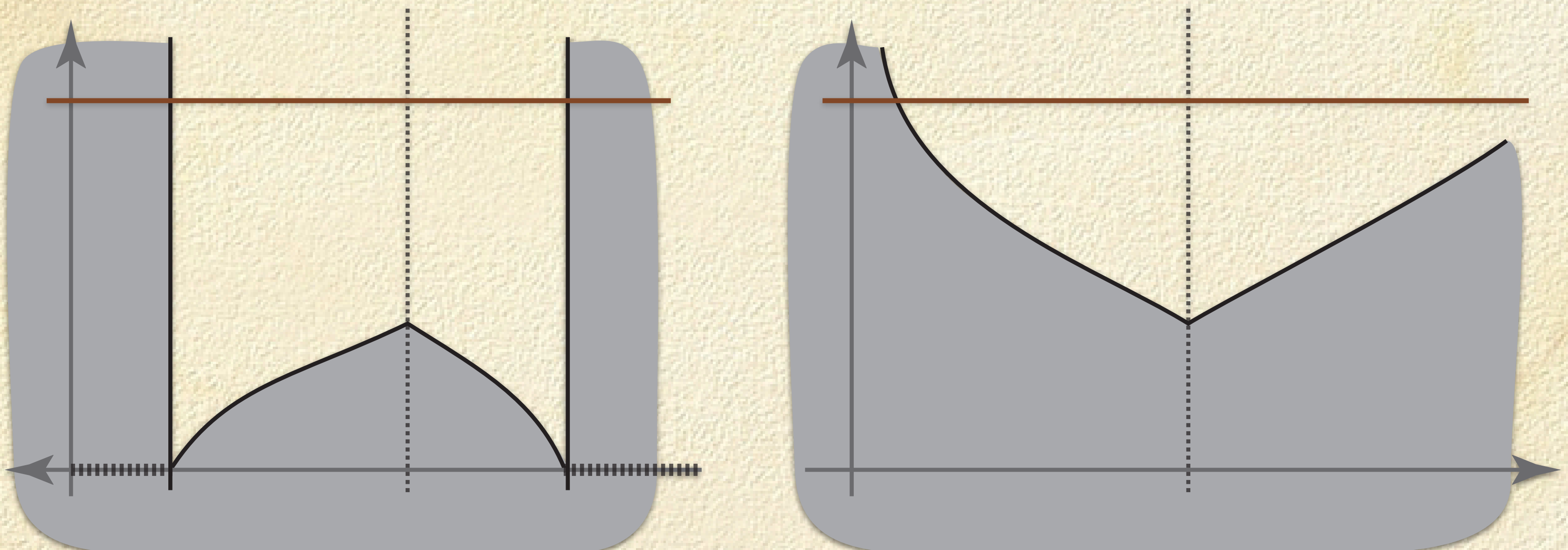
- it is now possible to calculate:
  - *the total radiation loss,*
  - *the rate of radiation loss,*
  - *a life-time of the brane-probe's motility.*

- This induces a dynamical mechanism for
  - ***banning*** *the brane-probes from the* δ*-function brane-World when* $a_0<0$*, where the brane-World is sheathed by singularities,*
  - ***localizing*** *the brane-probes to the* δ*-function brane-World when* $a_0>0$*, where the brane-World is skirted by smooth space.*

– a "latch-on, or be gone" mechanism.

Recall that $c = \frac{A(z)}{B(z)}$, $m = \tau_p e^{\frac{(p-3)}{4}\Phi} A^{p-1}(z) B^2(z)$,

so $mc^2 + \mathcal{V} = \tau_p e^{\frac{(p-3)}{4}\Phi} A^{p+1} - \mu_p C_{p+1}$ is an effective potential.



$a_0<0$ the brane-probe is free to roam

$a_0>0$ the brane-probe is free to roam

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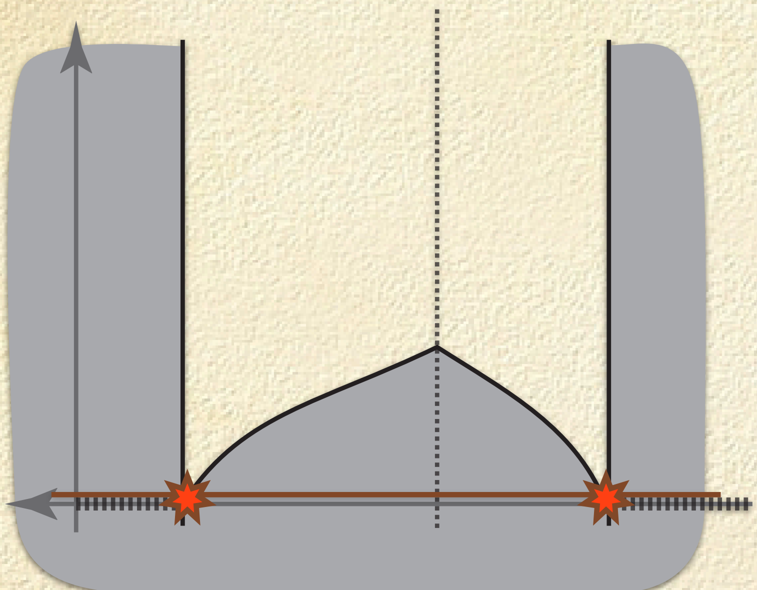
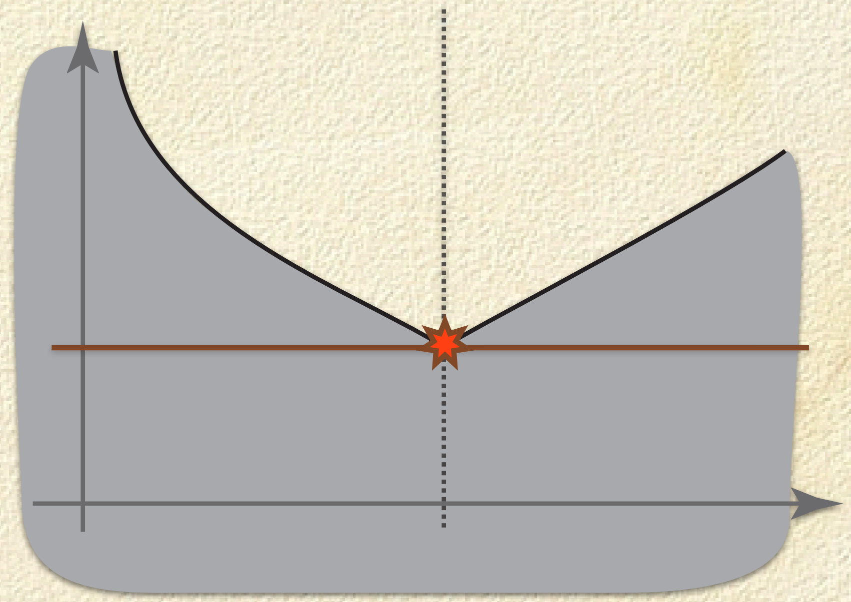


$a_0 < 0$

the brane-probe is **banned**!



$a_0 > 0$

the brane-probe is **trapped**!

Further reading

- P. Green & T. Hübsch, *Int. J. Mod. Phys.* **A9** (1994) 3203–3228 (stringy cosmic strings, generalized later to stringy cosmic branes)
- T. Hübsch, *Nucl. Phys. (Proc. Suppl.)* **52A** (1997) 347–351 (the "generic/typical" stringy spacetime, jump-gates, warp-drives, etc.)
- P. Berglund, D. Minic & T. Hübsch, *JHEP* **09** (2000) 015
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- A.F. Roane, *Some Brane-World Cosmological Models*, PhD dissertation (physics, Howard University: a gedanken prototype of hyperspace jump-

The End

or, more likely,

...to be continued...