



**EXTRA!  
EXTRA!  
EXTRADIMENSIONS**

room enough for many worlds...

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# Extra! Extra! Extra Dimensions! Room Enough for Many Worlds!

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# Extra Dimensions...

- When 3+1 just isn't enough
  - *From Maxwell, via Lorentz to Minkowski, Einstein and beyond*
- Compactification *vs.* Embedding
  - *Nordström, Kaluza and Klein vs. Randall & Sundrum*
- Brane-World Cosmologies
  - *Allegro con brio*

# When 3+1 Just Isn't Enough...

- Maxwell unified all that was known about electric and magnetic phenomena:

$$\begin{aligned}\nabla \cdot (\epsilon_0 \mathbf{E}) &= \rho_E, & \nabla \cdot (\mathbf{B} / \mu_0) &= \rho_M, \\ -\nabla \times (\epsilon_0 \mathbf{E}) &= \frac{1}{c^2} \frac{\partial (\mathbf{B} / \mu_0)}{\partial t} + \mathbf{j}_M, & \nabla \times (\mathbf{B} / \mu_0) &= \frac{\partial (\epsilon_0 \mathbf{E})}{\partial t} + \mathbf{j}_E\end{aligned}$$

- ...also written as  $\partial^\mu F_{\mu\nu} - j_\nu = 0 = \epsilon^{\mu\nu\rho\sigma} \partial_\nu F_{\rho\sigma} - j^\mu$ .

- These have the Lorentz group of symmetries:

- $F_{\mu\nu} \rightarrow \Lambda_\mu^\rho F_{\rho\sigma} \Lambda^\sigma_\nu$  (this transforms  $\mathbf{E} \leftrightarrow \mathbf{B}$ )

- and simultaneously  $x^\mu \rightarrow \Lambda^\mu_\nu x^\nu$  (!!!)

- where  $\Lambda^\mu_\nu \in SO(1,3)$  are Lorentz transformations.

# When 3+1 Just Isn't Enough...

*cont'd...*

- The  $x^\mu \rightarrow \Lambda^\mu_\nu x^\nu$  irreversibly mixes space and time...

- ...into *spacetime*.

- Well...

- what alternative do we have?

- Relaxing the Lorentz symmetry of the Maxwell eq.'s “down to” Galilean symmetry (Newtonian mechanics) ushers in “new” terms into Maxwell's eq.'s...

- ...to which Nature says: No way!

- So, Nature says: 3+1-dimensional *spacetime* it is.

*Erm... I wasn't asked! I don't want them mixed!*



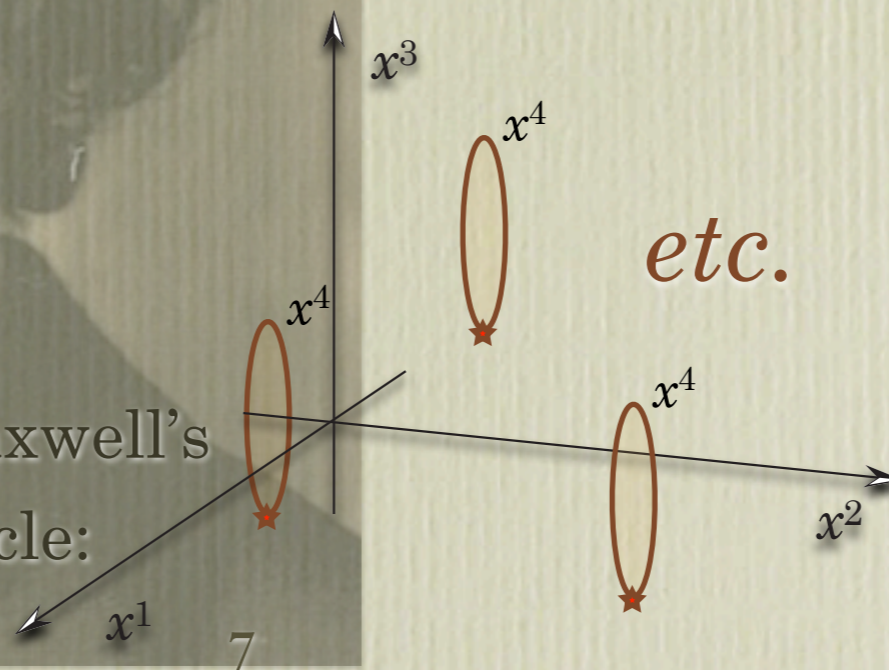
# When 3+1 Just Isn't Enough... *cont'd...*

- But, that's not all...
- First quarter of XX century:
  - Maxwell's equations = electromagnetism
  - Special, then general relativity = gravity
  - ...+ the queer quandaries of quantum mechanics & and pesky peculiarities (☞ nuclear physics)...
- So... wouldn't it be nice if EM and gravity were joined into a single theory?
- BTW: just because  $g_{\mu\nu} = +g_{\nu\mu}$  &  $F_{\mu\nu} = -F_{\nu\mu}$ ...
- ...it doesn't mean that they combine happily...

# When 3+1 Just Isn't Enough...

*cont'd...*

- 1914: Gunnar Nordström, (*early competitor of Einstein's*)
- 1919: Theodor F.E. Kaluza
- 1926: Oscar Klein
- Our spacetime is a sub-spacetime of a bigger (*5-dimensional!*) one:
- The metric tensor  $ds^2 = g_{ij} dx^i dx^j$ , where  $i, j = 0, 1, 2, 3, 4$ 
  - decomposes:  $(g_{\mu\nu}, g_{\mu 4}, g_{44})$
  - $g_{04}$  plays the role of  $\Phi$ ,
  - $g_{i4}$  play the role of  $A_i$ ,
  - ...of *electromagnetism!*
- Einstein's equations beget Maxwell's
- ...provided  $x^4$  curls up in a circle:



# When 3+1 Just Isn't Enough...

*cont'd...*

## ● (Nordström)-Kaluza-Klein *compactification*:

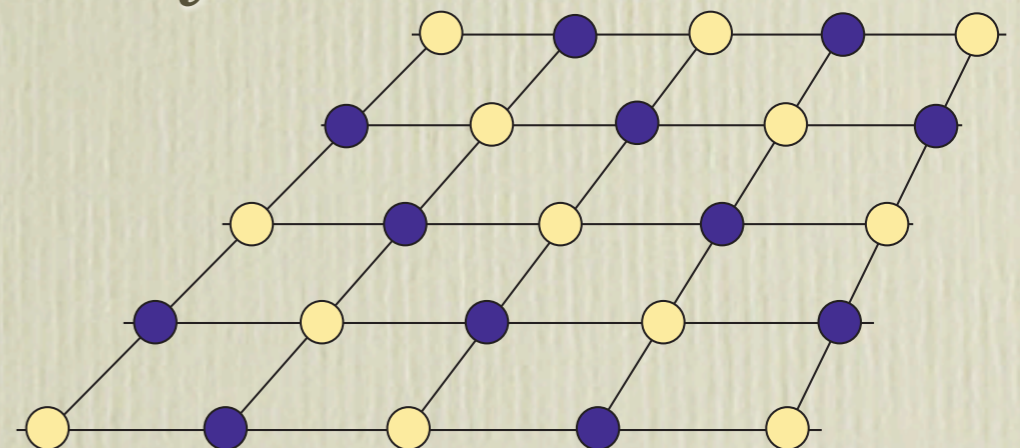
- extends spacetime from 3+1 to 4+1, to a total of 5
- the “*extra*” 5<sup>th</sup> *dim* is curled up into a circle
- the circumference of this circle must be small enough not to be noticed directly

*Hrrmpff...  
...I s'pose, if you're  
really small...*



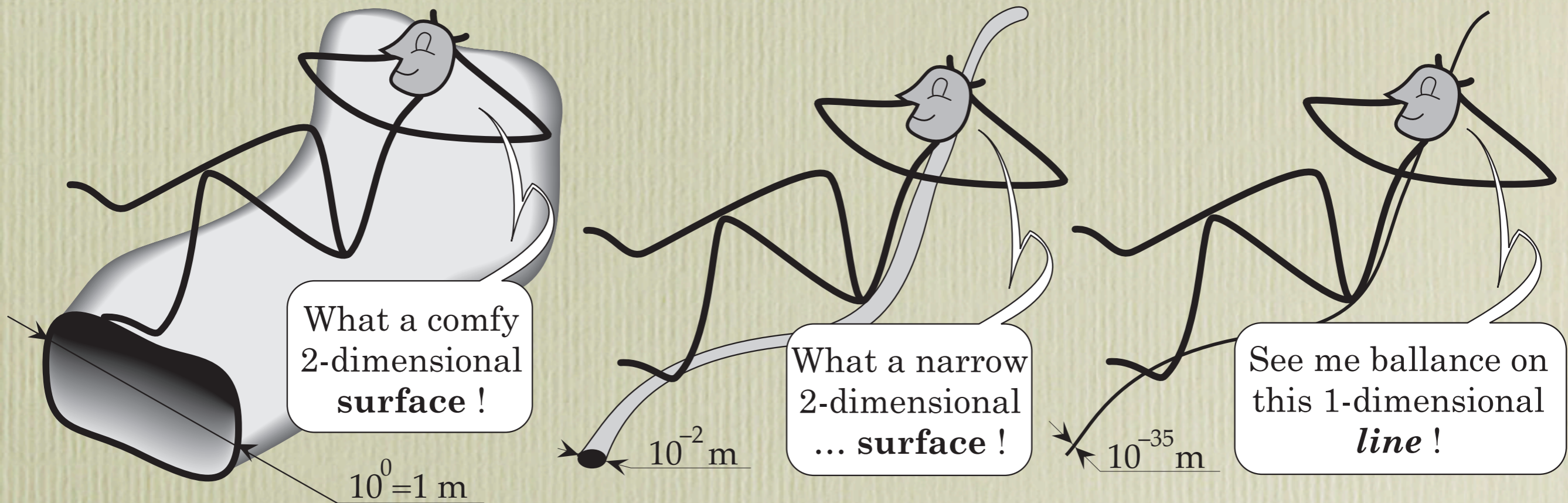
## ● This compactification still raises eye-brows

- ...but is not *that* weird:
- The crystal of NaCl is...
- ...just as periodic:



# When 3+1 Just Isn't Enough... *cont'd...*

- The physical concept of *dimension* is relative:

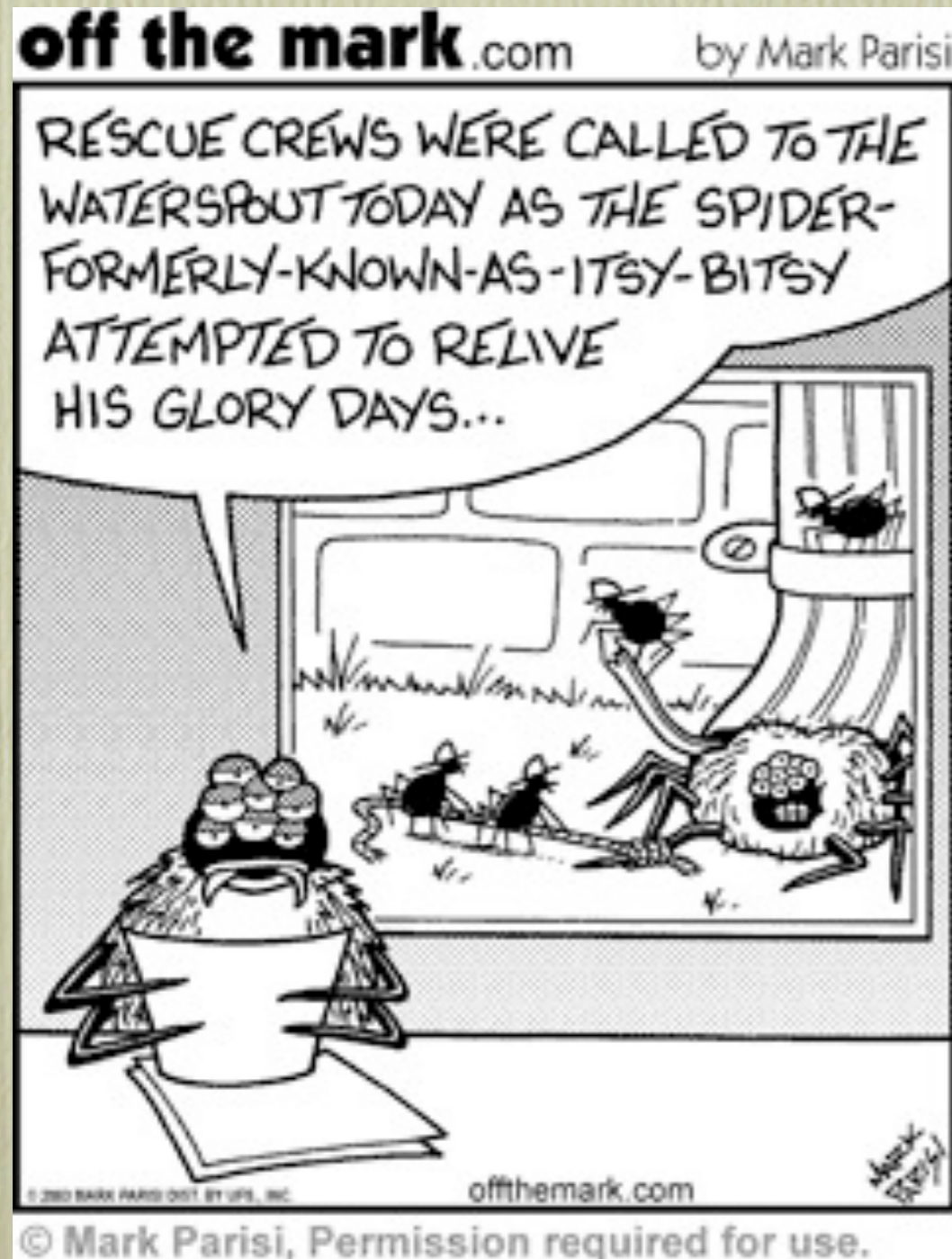


- ...so, 4+1-dimensional space, with the 4<sup>th</sup> spatial dimension being curled up too small, will appear as if 3+1-dimensional.

# When 3+1 Just Isn't Enough... *cont'd...*

- (Nordström)-Kaluza-Klein compactification:
  - “*extra*” 5<sup>th</sup> dim is curled up (compact),  $\leq 10^{-33}$  m;
  - fields decompose into “Fourier modes” over the extra dimensions
    - in only 1 extra dimension  $\Rightarrow$  easy:  $e^{in \cdot x^4 / (2\pi R)}$
  - The rest-mass of the  $n^{\text{th}}$  mode  $\sim n \cdot \hbar / R \geq 10^{17}$  GeV/ $c^2$ :
    - (particles need  $\geq 10^{17}$  GeV to swerve ‘round  $\leq 10^{-33}$  m)
  - Plus the  $g_{ij} \Rightarrow (g_{\mu\nu}, g_{\mu 4}, g_{44})$  decomposition...
  - ...creates indefinite “towers” of particles and fields:
    - the 0- and all the higher- $n$ -modes (mass  $\sim n \cdot 10^{17}$  GeV/ $c^2$ )

# When 3+1 Just Isn't Enough... *cont'd...*



Are you kidding me?!

We *need* extra dimensions!!!



# When 3+1 Just Isn't Enough... *cont'd...*

- Superstrings must have 10-dimensional spacetime

- 1984: Candelas-Horowitz-Strominger-Witten

- Kaluza-Klein & supersymmetry  $\Rightarrow$  Calabi-Yau ( $R_{\mu\nu} \simeq 0$ )

- 1986–87: Frenkel-Garland-Zuckerman, Rajeev-Bowick, Alvarez-Gaumé-Gomez-Reina, Pilch-Warner, Oh-Ramond, Harari-Hong:  $R_{\mu\nu} \simeq 0$  is a (super)string condition

- “Generic/typical” stringy spacetimes\* are:

- Calabi-Yau (Ricci-flat:  $R_{\mu\nu} \simeq 0$ ...

- ...Wick-rotated complex 5-folds)

- (Possibly) stratified

- (Possibly, sectionally) non-compact

*Erm, what next?  
Spike-studded  
spacetime?*

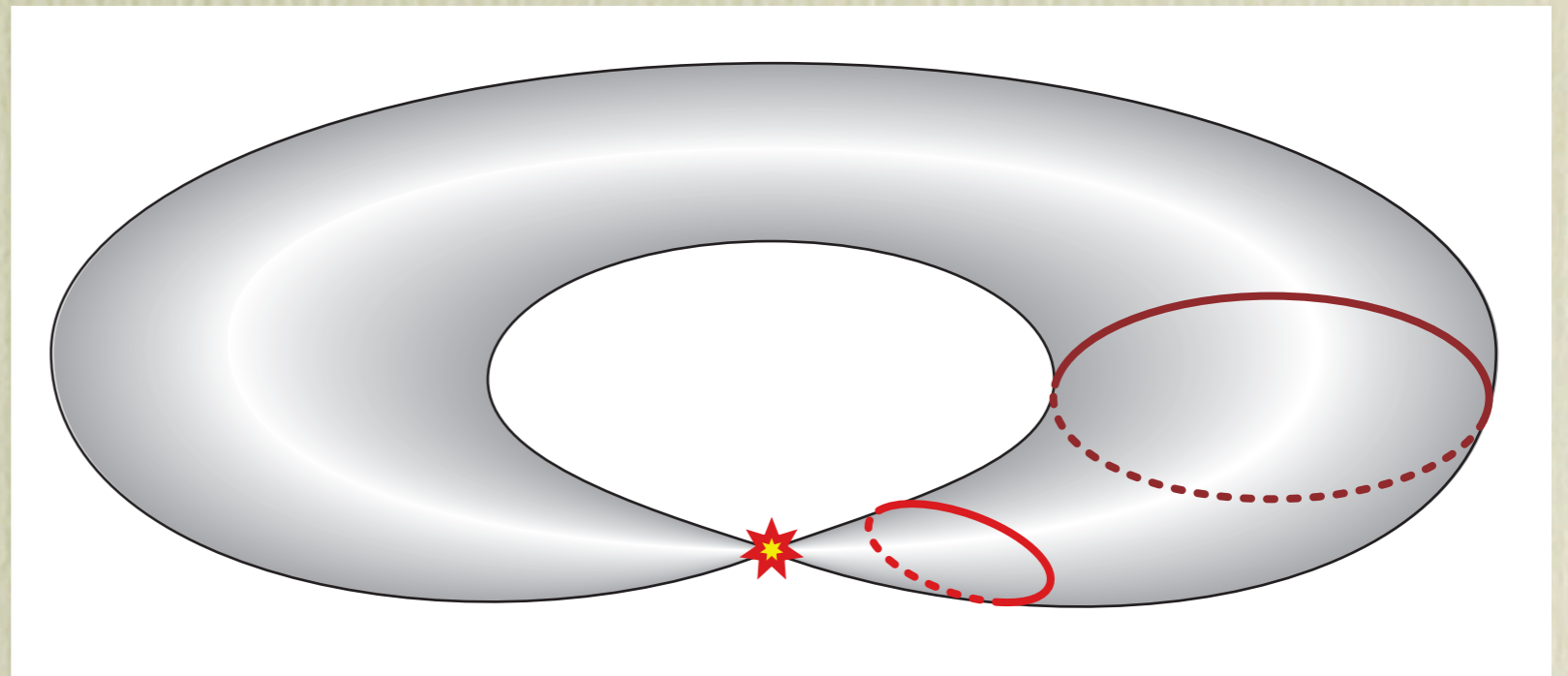
\* T. Hübsch, Nucl. Phys. (Proc. Suppl.) 52A (1997) pp.347–350.



# When 3+1 Just Isn't Enough...

*cont'd...*

- “Generic/typical” stringy spacetimes feature\*:
- “isolated” 4D subspace(time)s;
- with matter localized to such subspace(time)s;

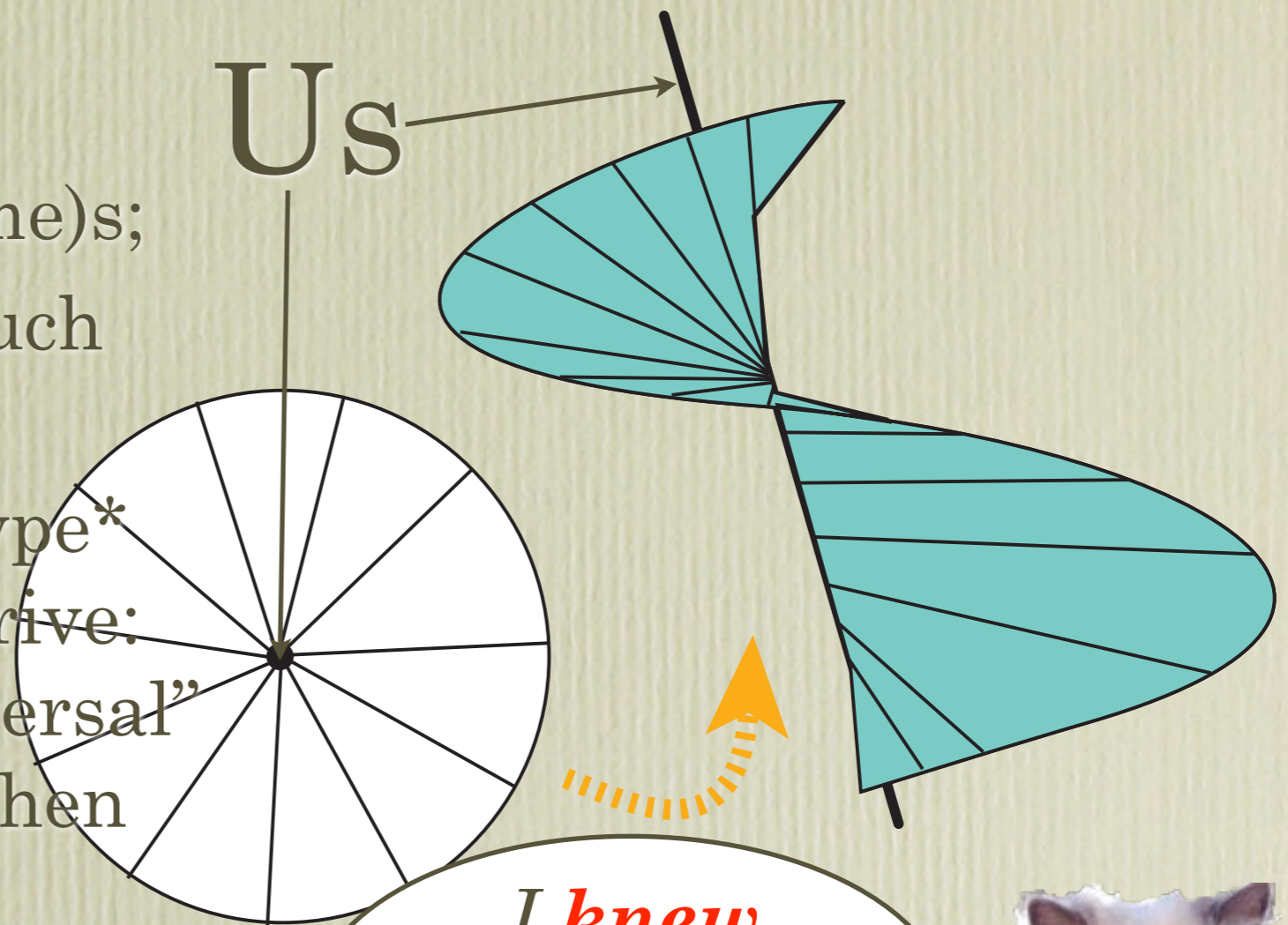


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# When 3+1 Just Isn't Enough...

*cont'd...*

- “Generic/typical” stringy spacetimes feature\*:
- “isolated” 4D subspace(time)s;
- with matter localized to such subspace(time)s;
- the first gedanken-prototype\* of jump-gates and warp-drive: detouring into the “transversal” directions (hyper-space), then return 😊
- ...and gravity?! 😞



*I knew  
this could never  
work out!*

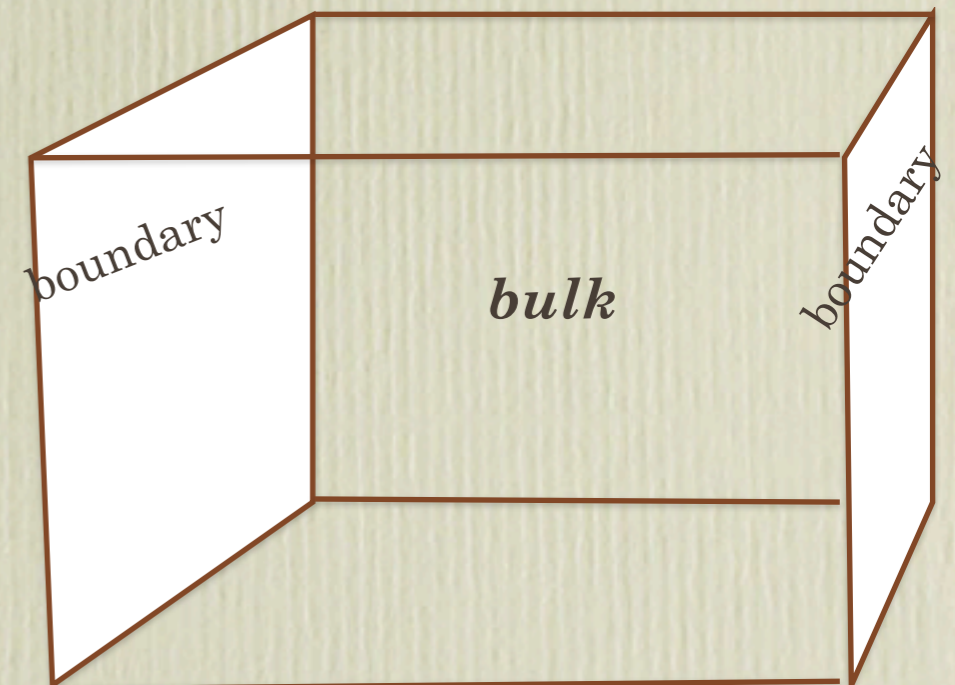


\* T. Hübsch, Nucl. Phys. (Proc. Suppl.) 52A (1997) pp.347-350.

# Surprise: (Mem)Brane-Worlds

- 1999: Lisa Randal
- & Raman Sundrum
- Our spacetime is a sub-spacetime in a bigger (5-dimensional) one,
- ...for example, at its *boundary*
- The “heterotic M-theory” of Hořava and Witten:
- two 10-dimensional boundaries in an 11-dimensional spacetime
- each with an  $E_8$  gauge group
- ...room enough for us, right?

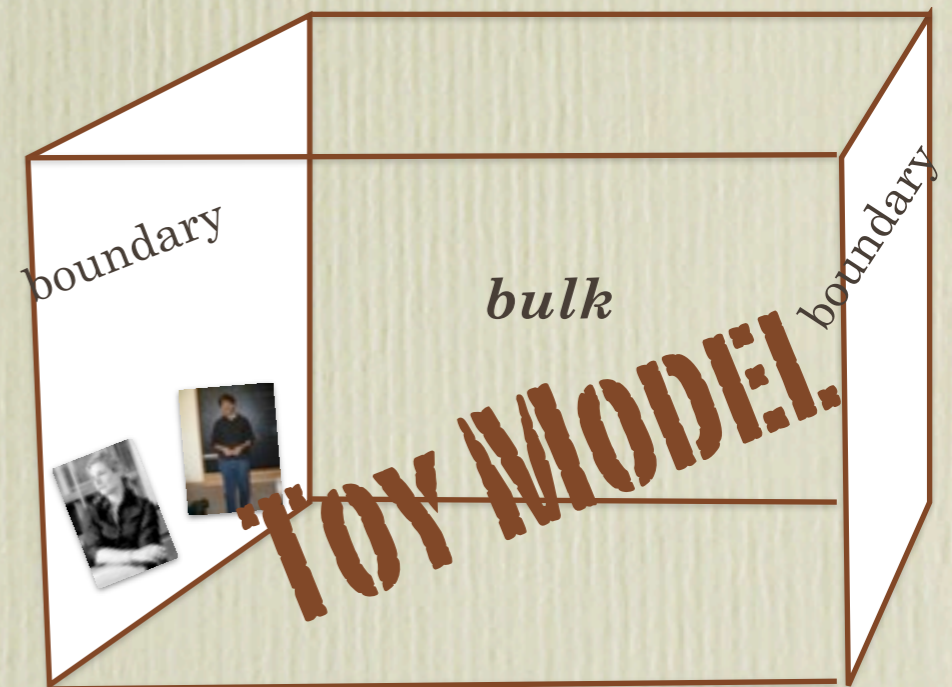
Four score years  
after Kakuza



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Four score years  
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# Surprise: (Mem)Brane-Worlds

*cont'd...*

● (Party like it's) 1999:

the two most-cited papers in the past decade

● RS-1: exponential hierarchy

●  $M_P \sim 10^{19}$  GeV (gravity becomes confiningly strong)

●  $M_W \sim 10^2$  GeV (masses of  $W^\pm, Z^0$ )

● ...exponentially related through  $x^4$  curvature 😊

● RS-2: localized gravity

● bulk gravity  $\sim 1/r^8$  force

● “boundary” gravity  $\sim 1/r^2$  force 😊

● But, not in one and the same  
end-of-the-world World! 😞

*Hah! I knew  
it was not going  
to work!*



# Surprise: Stringy Brane-Worlds

- All (super)strings

- have a well-defined (point-field) limit, w/(super)gravity,
- may be regarded as torus-compactifications of F-theory;
- so must contain:

$$S_{\text{eff}} = \frac{1}{2\kappa^2} \int d^D x \sqrt{-g} \left( R - G_{\alpha\bar{\beta}} g^{\mu\nu} \partial_\mu \phi^\alpha \partial_\nu \phi^{\bar{\beta}} + \dots \right),$$

where  $2\kappa^2 = 16\pi G_N^{(D)}$  is the strength of gravity in D dimensions.

- and the  $\phi^\alpha$  are moduli fields (parametrizing the shapes and sizes of the spacetime)

# Surprise: Stringy Brane-Worlds *cont'd...*

- We\* considered:
  - a universally present, *axion-dilaton* field,  $\tau$
  - it's standard dynamics (Teichmüller metric)
  - the  $\tau$  depend on only 2 spatial dimensions
- We found two classes of solutions,
  - with  $SL(2, \mathbb{Z})$  monodromy...
  - ...which only stringy moduli are known to have!

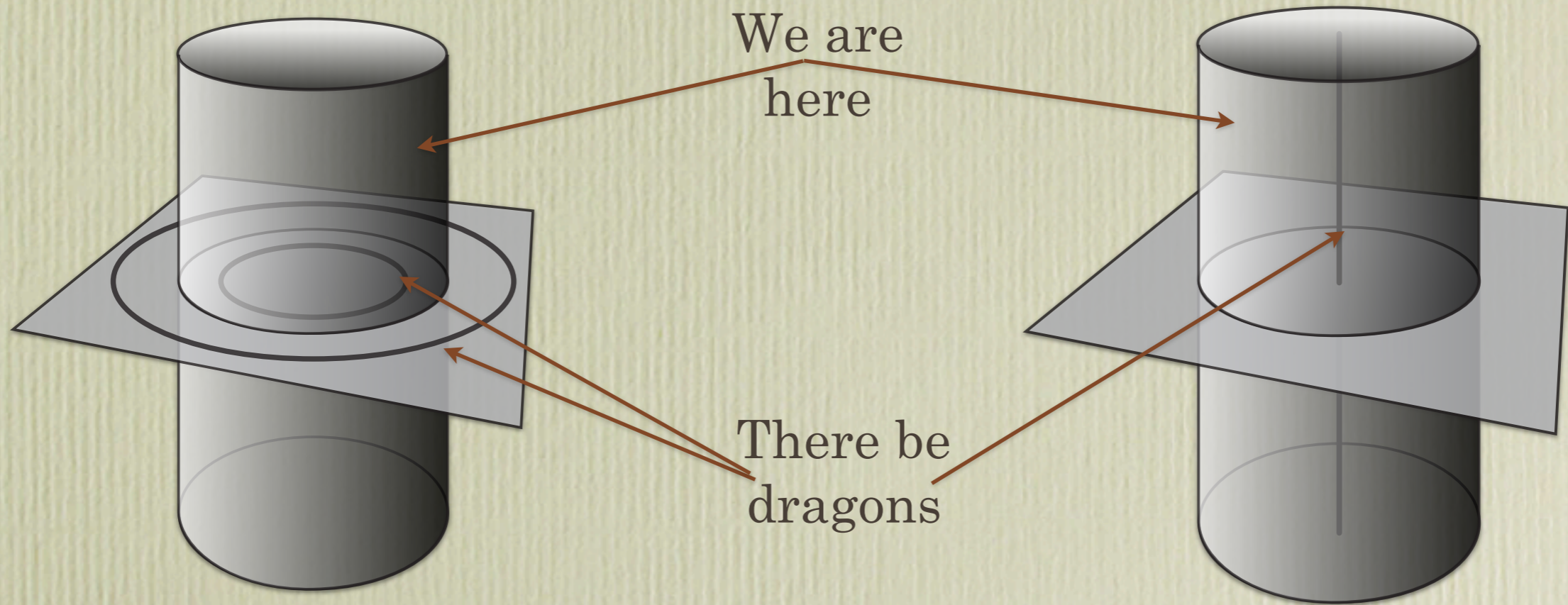
Thus, we have two configurations of a matter field which could not have stemmed from anything but superstrings.

\*P. Berglund, D. Minic & T. Hübsch: see later for a complete bibliography.

# Surprise: Stringy Brane-Worlds

*cont'd...*

● We\* find:



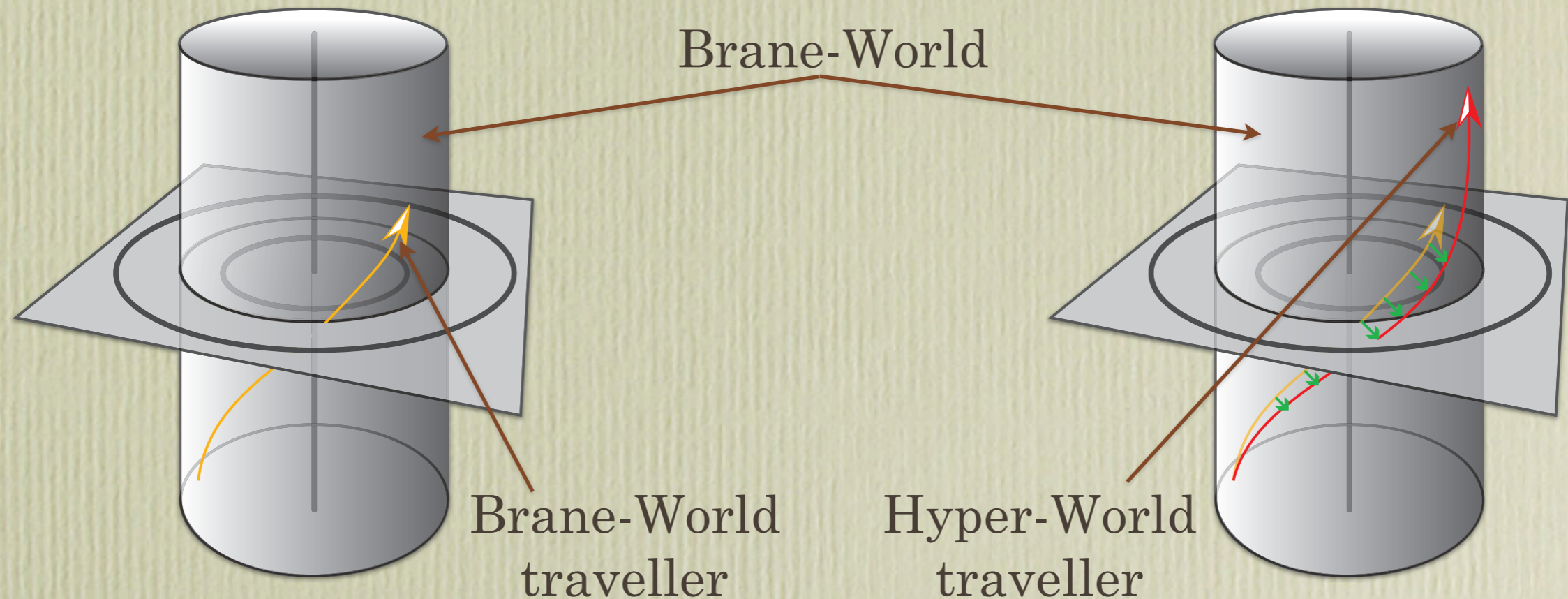
$a_0 < 0$ , the  $\delta$ -function brane is sheathed by singularities

$a_0 > 0$ , the  $\delta$ -function brane is coaxial with a singularity

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# Surprise: Stringy Brane-Worlds

cont'd...



$\alpha_0 < 0$ , the  $\delta$ -function brane-World is sheathed by singularities; the **yellow arrow** is a traveler

By moving off of the brane-World, the **red traveler** can move faster\*, ahead of the brane-World **yellow**

\*A.F. Roane, *Some Brane-World Cosmological Models*, PhD dissertation

# ETs: Menace to Conservation Laws

- Bulk-roaming modes are unobservable...
- ...most of the time.
- But, when they pass through “our” spacetime...
- and interact with us,
- ...they invalidate all our (local) conservation laws:
- Thus we must:
  - either ban all ET’s
  - or localize them all!



# Surprise: Stringy Brane-Worlds

*cont'd...*

- The  $SL(2, \mathbb{Z})$  monodromy  $\tau$ -driven brane-Worlds:
  - have exponential  $M_P : M_W$  hierarchy
  - localized gravity
  - an induced de Sitter metric in the brane-World
  - an acceptable value of the cosmological constant...
  - ...related to supersymmetry breaking...
  - ...which is geometry-induced.
- The energy-loss in bulk-roaming fields
  - either traps them into the brane-World,
  - or bans them from it.

# Further reading...

- P. Green & T. Hübsch, *Int. J. Mod. Phys. A*9 (1994) 3203–3228  
(stringy cosmic strings, generalized later to stringy cosmic branes)
- T. Hübsch, *Nucl. Phys. (Proc. Suppl.)* 52A (1997) 347–351  
(the “generic/typical” stringy spacetime, jump-gates, warp-drives, etc.)
- P. Berglund, D. Minic & T. Hübsch, *JHEP* 09 (2000) 015
- ———, *JHEP* 01 (2001) 045
- ———, *JHEP* 02 (2001) 010
- ———, *Phys. Lett. B*512 (2001) 155–160
- ———, *Phys. Lett. B*534 (2002) 147–154
- ———, *Phys. Rev. D*67 (2003) 041901
- ———, “work in progress” (challenges)
- A.F. Roane, *Some Brane-World Cosmological Models*, PhD dissertation physics, Howard University: a gedanken-prototype of hyperspace jump-gate)

~~The End~~

*...to be continued...*