On Springs, SuperThings and the Sphinx

(a Quest for Dimensions)

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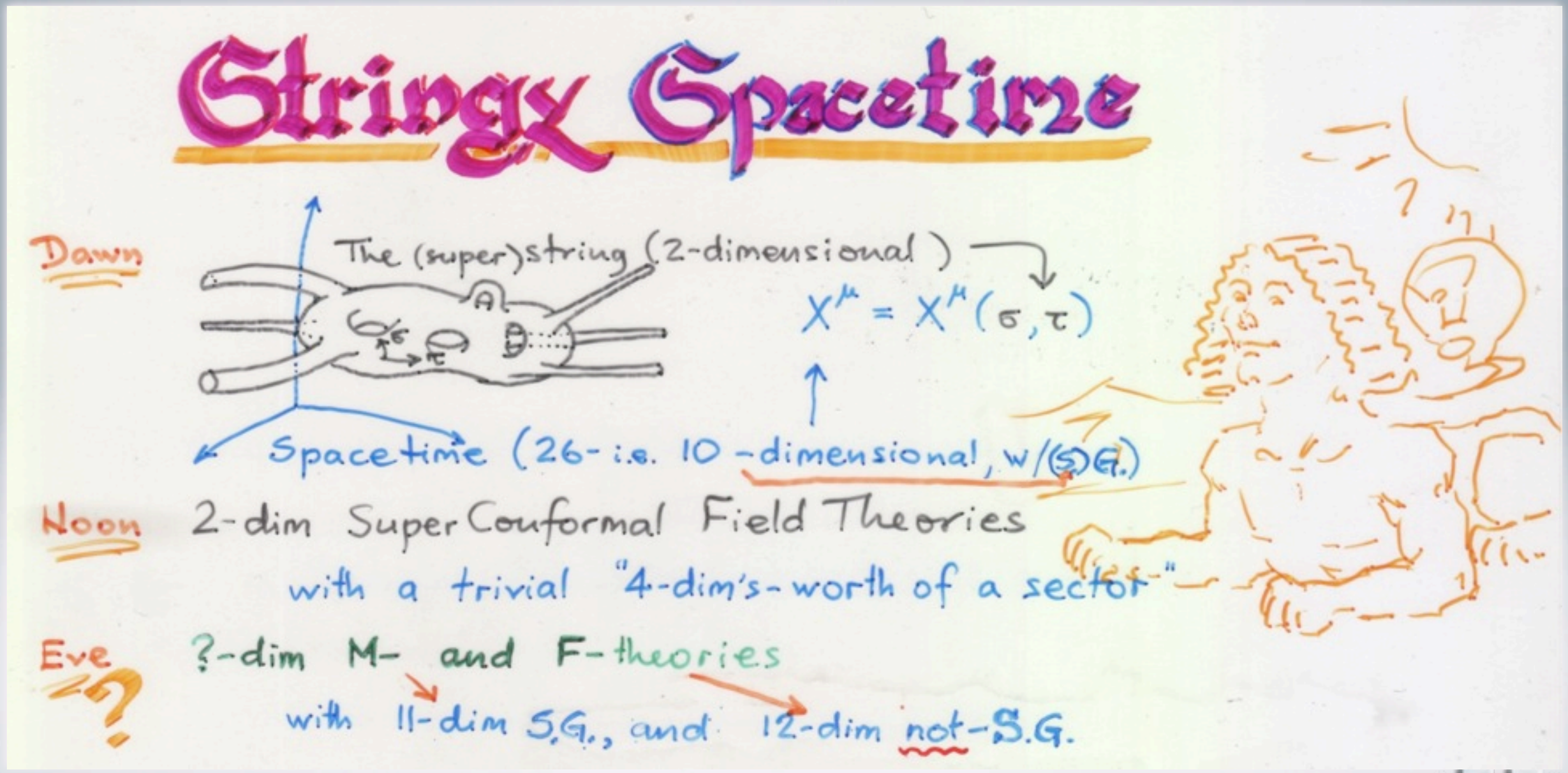
But, Before We Begin…

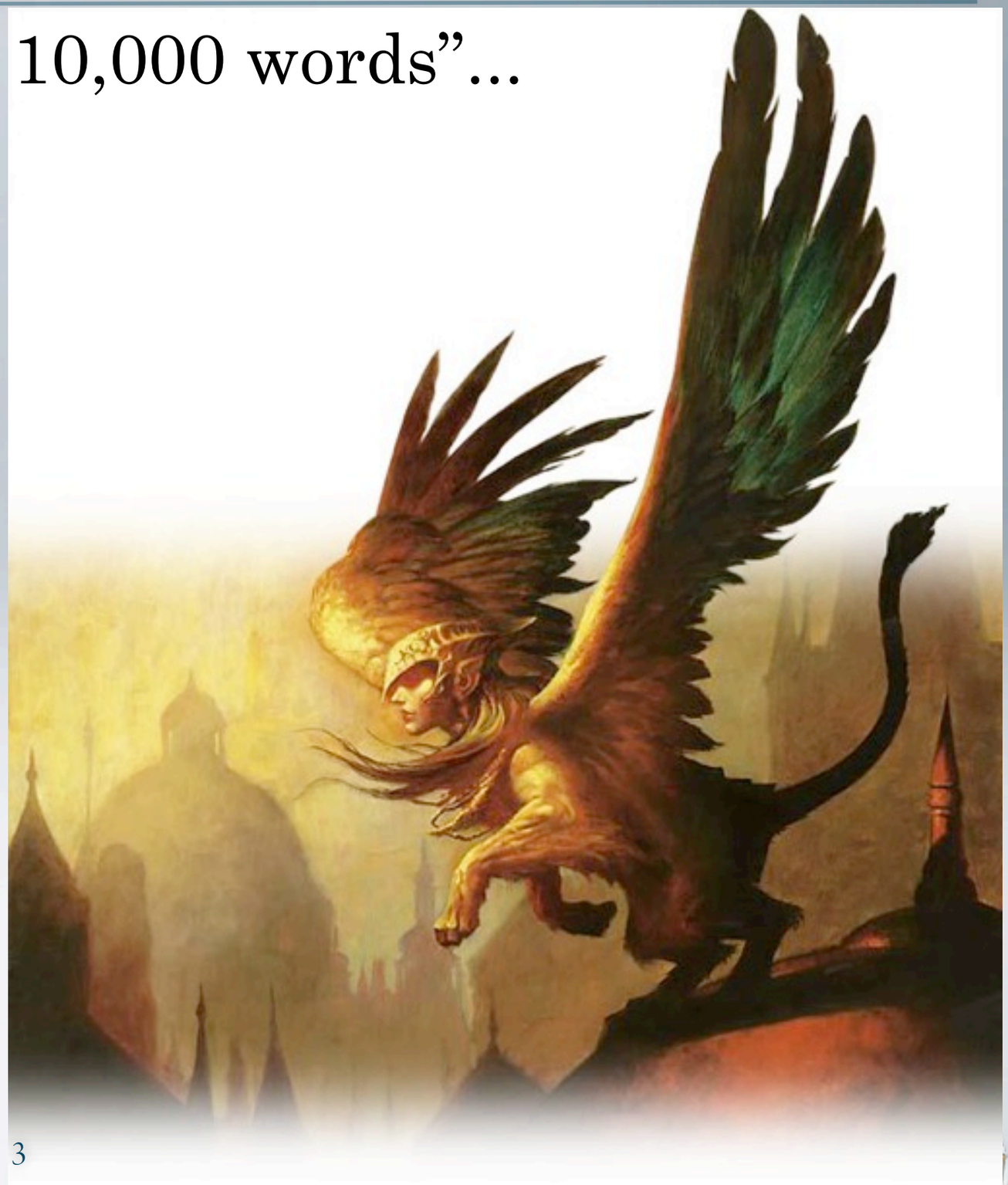
- Some of you may recall October 1992, TKH.103…



Physics & Astronomy

But, Before We Begin…

- …since "a picture is worth 10,000 words"…



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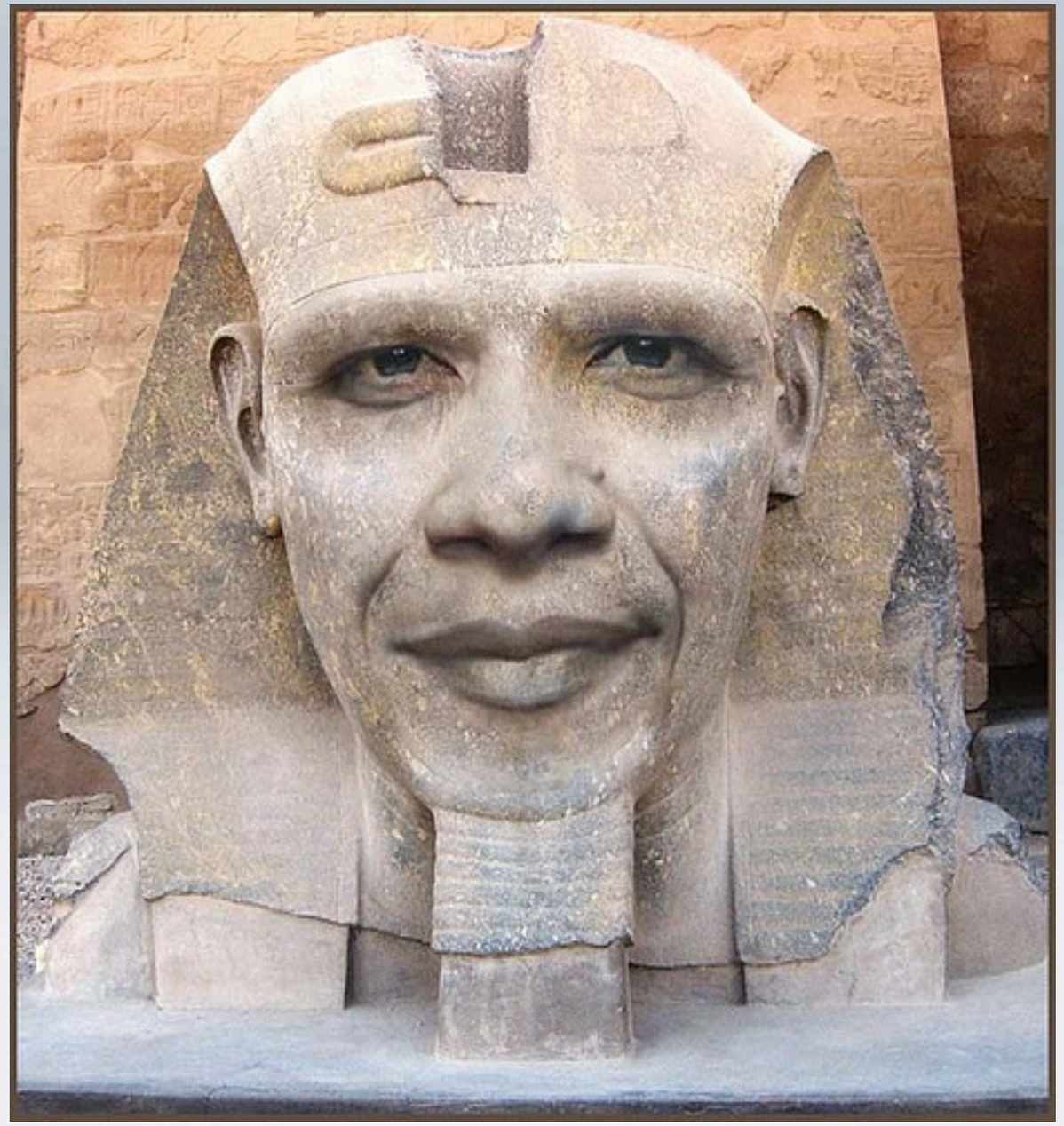


But, Before We Begin…

- …since "a picture is worth 10,000 words"…

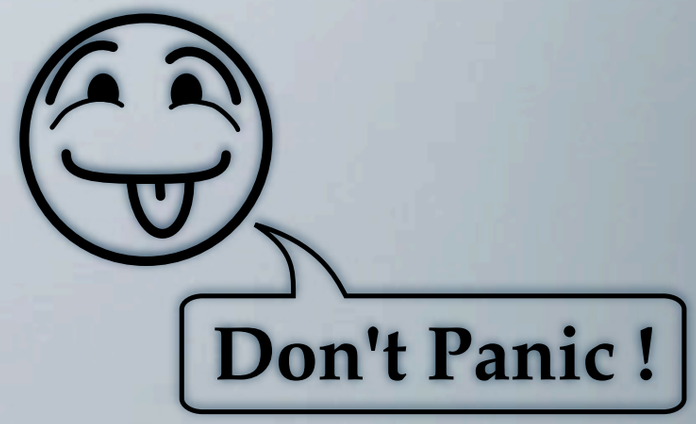
Searching and *re*-searching

the internet does *not* research make

…my ever so subtle hint to our students…

But, Before We Begin… *cont'd*

- Q.: Which creatures move
  - in four (*+six*) dimensions in the morning,
  - in two dimensions at noon,
  - in three dimensions in the evening?



- A.: The creatures formerly known as (super)strings,

…to be called (just today?) *superThings*.

- Well, these *superThings* turn out to be *shapeshifters*:
  - if moving on a cylinder, $R \simeq 1/R$; *(reciprocal lattice: $S^1 = \mathbb{R}^1/\sim$)*
  - if compactified on a CY 3-fold, then $M \simeq W$; *(mirror symm.)*
  - …and many other *dualities*…
  - …including between diverse dimensions and structures!

holography … generalized … quite a bit!

Physics & Astronomy

Playbill

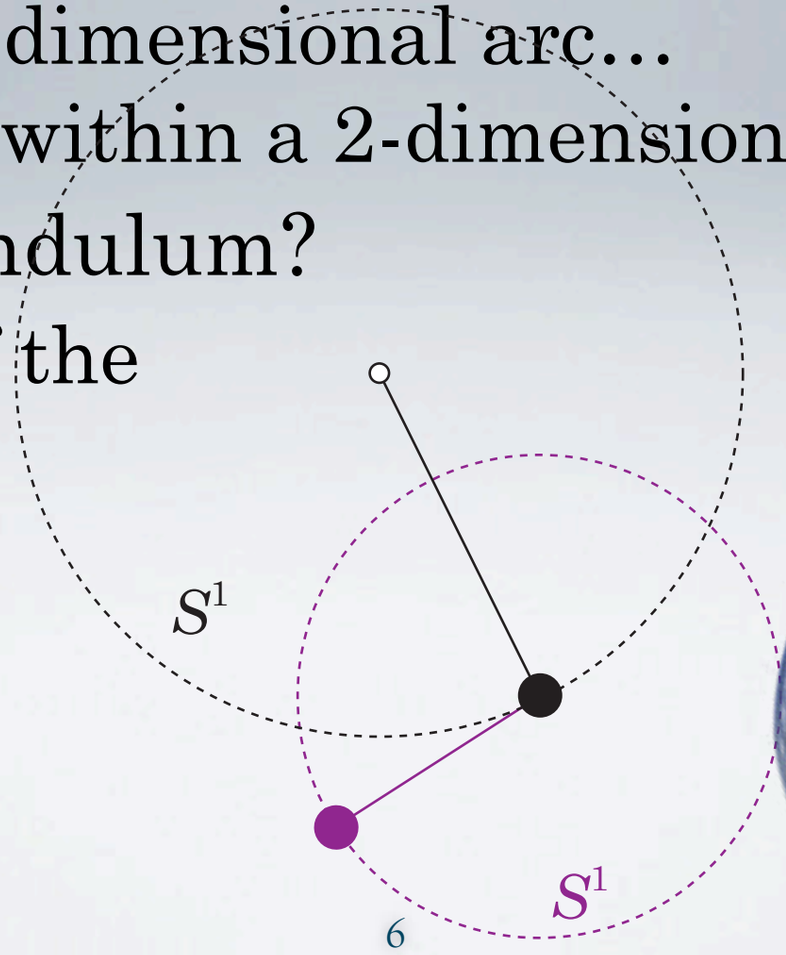
- How many dimensions do we live in?
  - *What* do we *mean* by "dimensions"?
- When 3+1 jus' ain't 'nuff
  - Maxwell-Lorentz-Minkowski, Nordström-Kałuża-Klein:
    - Compactification = Crystallization

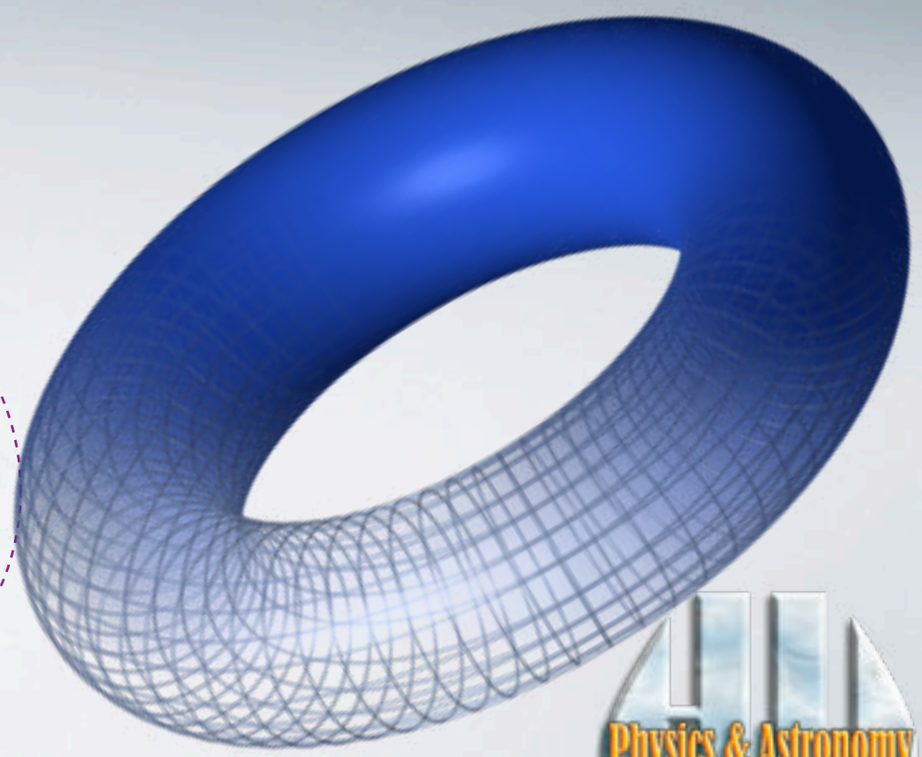
Superstring interlude

  - Randall-Sundrum:
    - Brane-World Cosmology
- A concrete toy-model
  - …and some afterthoughts…

How Many Dimensions?

- D'oh:
  - *Obviously*, it's three: up-down, left-right, front-back. No?
- Well,
  - How many dimensions does a planar pendulum have?
    - It moves on a 1-dimensional arc…
    - …which curves within a 2-dimensional plane…
- And a double pendulum?
  - At each point of the black circle,
  - there's a whole plum circle…
  - $S^1 \times S^1$ = torus!





How Many Dimensions?

- To settle some language:
  - 'Real' physical systems are described by:

model:	input	output	relation
function:	argument	value	func. expr.
mapping:	domain	target	structure
Class. Mech.	time, $\mathbb{R}^1$	space, $\mathbb{R}^3$	continuous
Relat. Mech.	proper time $\mathbb{R}^1$	spacetime $\mathbb{R}^{1,3}$	continuous
Class. Fields	spacetime $\mathbb{R}^{1,n-1}$	field space $\mathbb{R}^N$, $\mathbb{C}^N$...	continuous
Quant. Mech.	time, $\mathbb{R}^1$	Hilb.sp., $\mathcal{H}$	continuous*

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How Many Dimensions?

- Configuration space:
  - Coincides with target space, for 1 point-particle
  - Is an n-multiple of the target space, for n point-particles
  - Is infinite-dimensional, for any field
  - Is infinite-dimensional, for quantum mechanics of even just a single, non-relativistic point-particle
  - …even bigger for quantum fields
  - **= space of maps from the domain to target, typically (very) infinite-dimensional**
- Target space: chosen "by hand" & kept so thereafter
  - Ex.: $\mathbb{R}^3$ for a point-particle, $(\mathbb{R}^{1,3}/\mathbb{R}^{1,1})$ for the EM field
- Domain: chosen by the scientific *system* employed

Assigning a dimension may be *very* tricky

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When 3+1 jus' ain't 'nuff:

- Newton's theory of gravity *unifies* celestial and terrestrial matter in the eyes of gravity
- Maxwell's synthesis of all E&M laws into a single, self-consistent *system* of PDE's (laws) w/BC (applic.)
  - *unifies* electric and magnetic fields
  - *unifies* electric and magnetic charges

The $E \leftrightarrow B$ duality

$$\nabla \cdot (\epsilon_0 \boldsymbol{E}) = \rho_E, \qquad \nabla \cdot (\boldsymbol{B}/\mu_0) = \rho_M,$$

$$-\nabla \times (\epsilon_0 \boldsymbol{E}) = \frac{1}{c^2}\frac{\partial (\boldsymbol{B}/\mu_0)}{\partial t} + \boldsymbol{j}_M, \qquad \nabla \times (\boldsymbol{B}/\mu_0) = \frac{\partial (\epsilon_0 \boldsymbol{E})}{\partial t} + \boldsymbol{j}_E$$

- …also written as $\partial^\mu F_{\mu\nu} - j_\nu = 0 = \frac{1}{2}\varepsilon^{\mu\nu\varrho\sigma}\partial_\nu F_{\varrho\sigma} - j^\mu$.
- Also *unifies* space and time into spacetime!

'cos of Lorentz transformations, that's how!

Physics & Astronomy

When 3+1 jus' ain't 'nuff:

- Einstein's theory of special relativity
  - generalizes Newton's mechanics so as to…
  - *couple* it with Maxwell's electromagnetic theory.
- Einstein's theory of general relativity
  - curves spacetime in the presence of matter, thus
  - *coupling* spacetime and matter.
- But, fields and particles, spacetime and matter…
- …remain separate entities. Equal, but separate.
- And: $g_{\mu\nu} = + g_{\nu\mu}$ & $F_{\mu\nu} = - F_{\nu\mu}$, although complementary, just won't combine *natur*ally (couplings don't jibe).

$$G_N \cdot m_1 m_2 / r^2 \quad vs. \quad k_C \cdot q_1 q_2 / r^2$$

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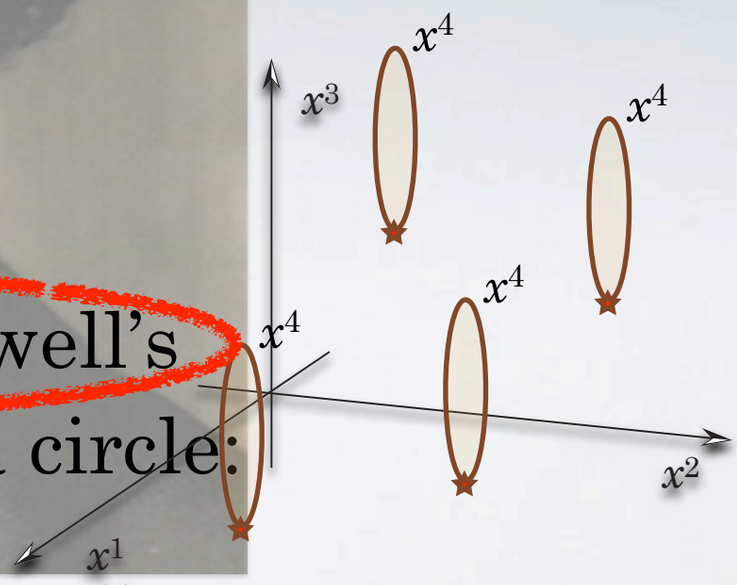
When 3+1 jus' ain't 'nuff:

- 1914: Gunnar Nordström,
  (early competitor of Einstein's)
- 1919: Theodor F.E. Kałuża
- 1926: Oscar Klein
- Our spacetime is a *sub-spacetime* of a bigger (*5-dimensional!*) one:
- The metric tensor
  $ds^2 = g_{ij}\, dx^i dx^j$, where $i, j = 0,—,4$
  - decomposes: ($g_{\mu\nu}$, $g_{\mu 4}$, g_{44})
  - g_{04} plays the role of Φ,
  - g_{i4} play the role of A_i,
  - …of ***electromagnetism***!
- Einstein's eq's ***beget*** Maxwell's
- …provided x^4 curls up in a circle:



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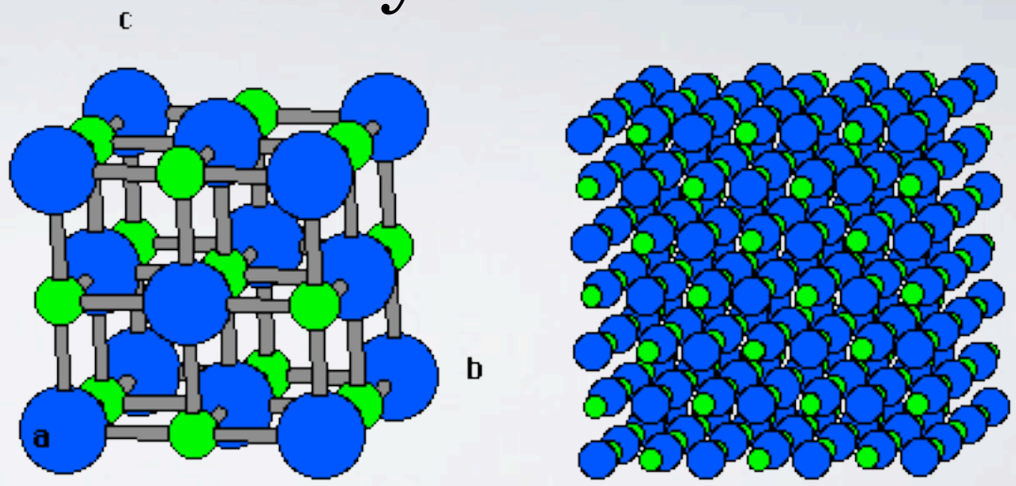
When 3+1 jus' ain't 'nuff

- (Nordström)-Kałuża-Klein compactification:
  - extends spacetime from 3+1 to a total of 4+1 dimensions
  - the "extra" 5^{th} dim is curled up into a circle…
  - …the circumference of which must be small enough not to be noticed directly
- 90 year later, this idea still raises eye-brows…
  - …but should not be unfamiliar; not at all:
  - The crystal of NaCl is…
  - …just as periodic:

"Classical physicists do not come to terms with new physics. It outlives them." – M. Planck



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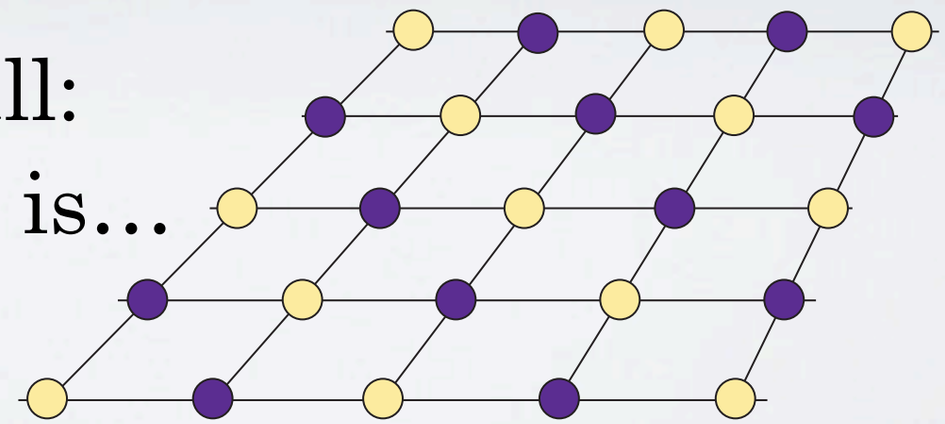

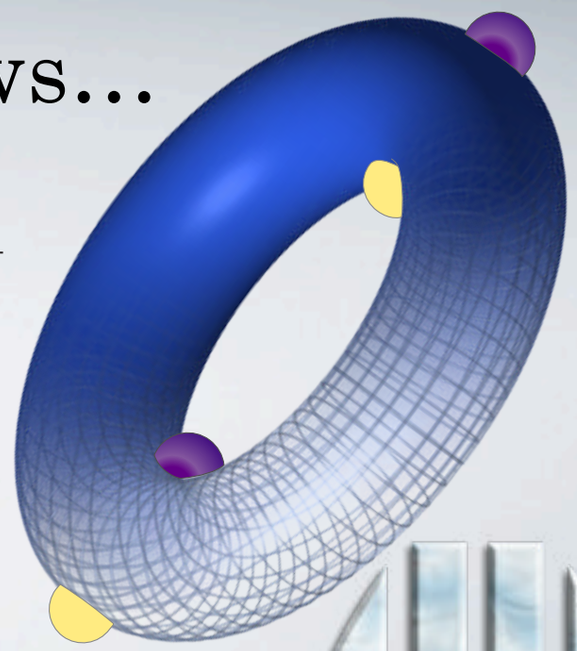

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When 3+1 jus' ain't 'nuff

- Another equivalent model:
  - A collection of vertical, very long line-like objects,
  - free to move horizontally, while staying vertical.
  - Both (Newtonian) gravitational and electrostatic force obey Gauss's law:
    - …which implies a $1/r^1$ power law for the force, not $1/r^2$.
    - Thus, both Newton's and Coulomb's force can "see" only the 2 dimensions (perpendicular to the objects).
- Gauss's law
  - = equation of continuity for the static case,
  - = gauge invariance of the model considered.
- Variety of *things* adds merely *technical* challenge…

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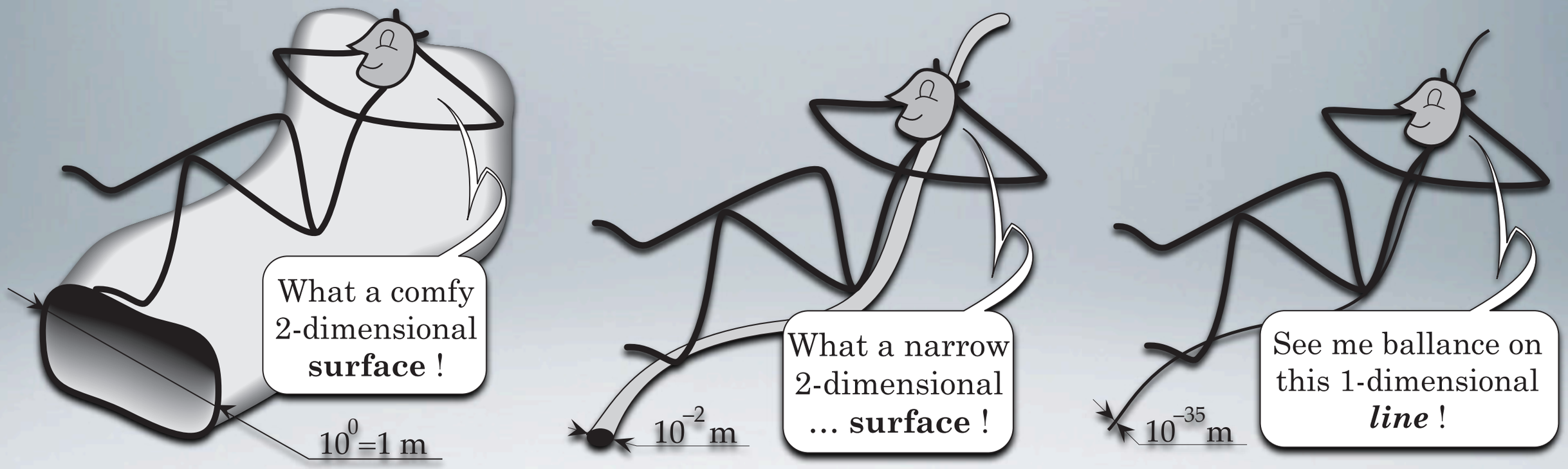
When 3+1 jus' ain't 'nuff

- Depending on the resolution and type of experiment,
- The physical concept of dimension is relative:



- …so, a 4+1-dimensional space, with the 4th spatial dimension curled up too small, will appear as if 3+1-dimensional.

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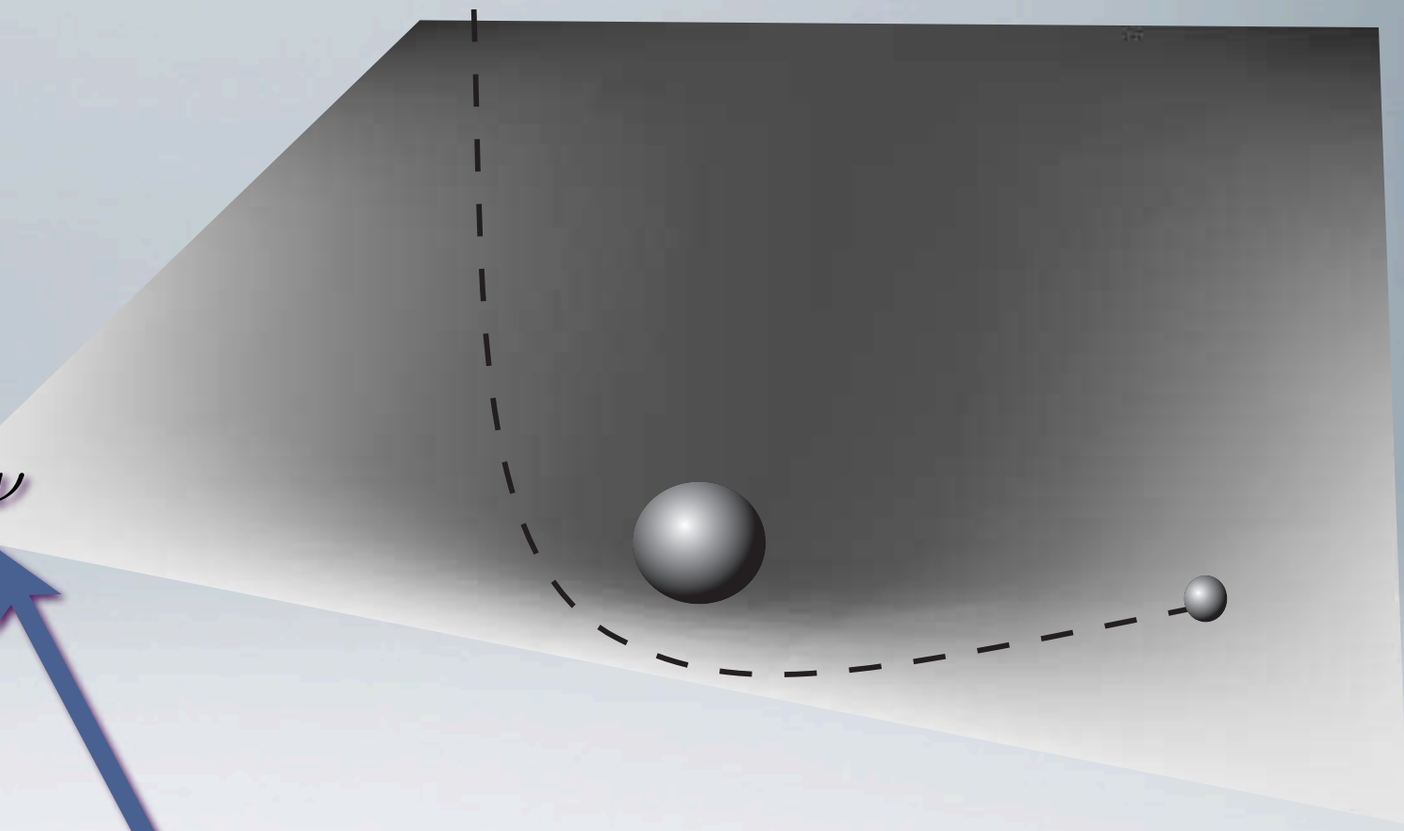
An Interlude

- More seriously:
- Gravity:
  - Newton ➠ Einstein



$$\underbrace{R_{\mu\nu} - \tfrac{1}{2}\, g_{\mu\nu}\, R}_{} = T_{\mu\nu}$$

Spacetime curvature
2nd order (nonlinear)
differential expression
for the metric

Matter
(energy-momentum)

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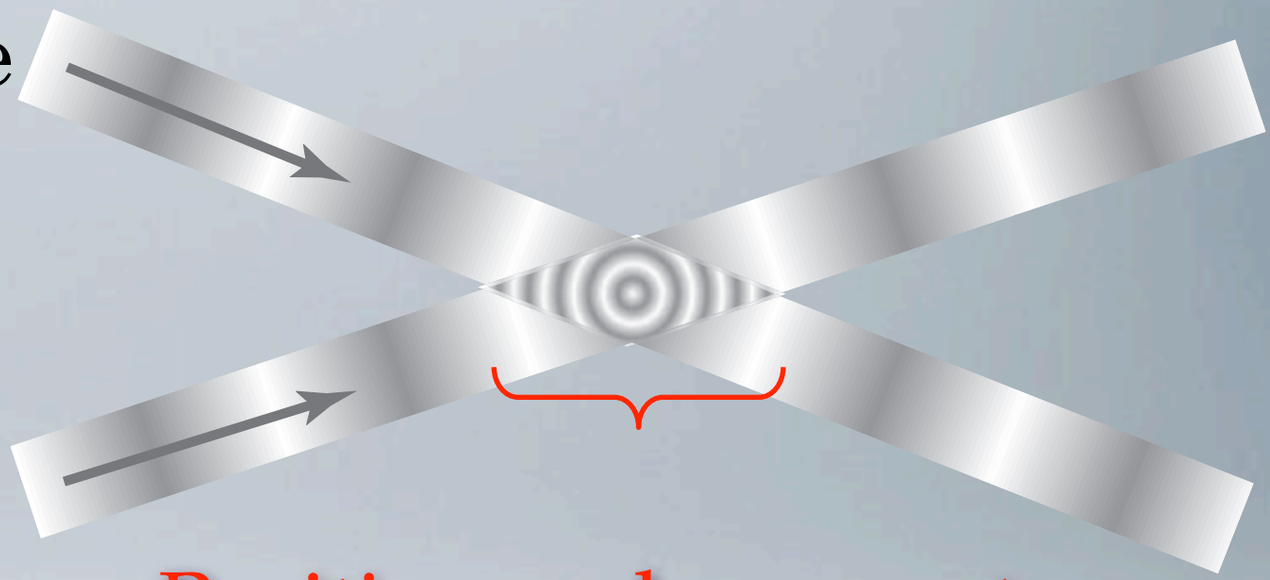
An Interlude

- More seriously:
- Quantum nature of Nature
  - Some observations are mutually exclusive



Position and momentum (wavelength) cannot be determined precisely

Conjugate variables

$$\triangle_\Psi x \cdot \triangle_\Psi p_x \geq \frac{1}{2}\hbar$$

State (Ψ)-dependent indeterminacy

Planck constant

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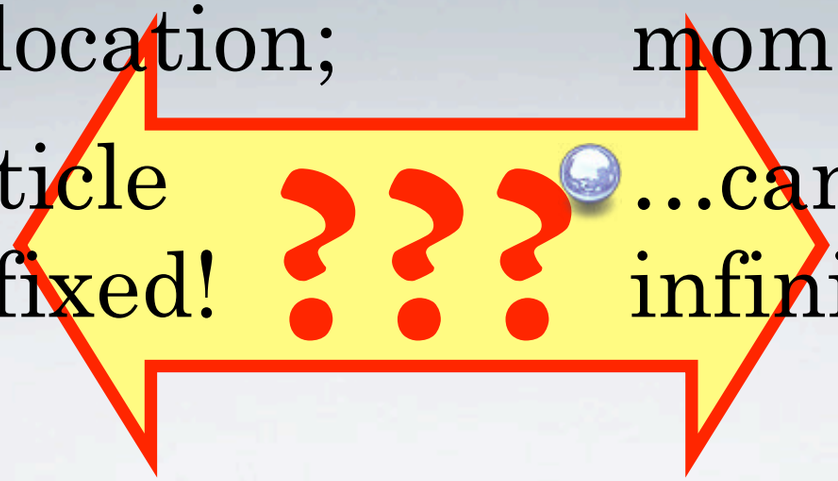
An Interlude

- Really, really, really seriously:

- Gravitation
- Presence of matter curves spacetime;
- In curved spacetime, one knows the location;
- The origin (particle location) stays fixed!

- Quantum physics
- Heisenberg's relations of indeterminacy hold;
- Both location & momentum of object…
- …cannot be known infinitely precisely!



But, we know full well that Nature *both* gravitates *and* is quantum!?

An Interlude

- Classical Mechanics:
  3-body problem = chaos
  - 3-body problem cannot be solved in general
- Quantum Mechanic & Classical Field Theory:
  most potentials are unsolvable *back-reaction ⇛ pre-acceleration*
  - 2-body problem cannot be solved in general
- Quantum Field Theory:
  self-interaction: divergent in general
  - 1-body problem cannot be solved in general
- Quantum Gravity:
  empty spacetime: divergent in general
  - 0-body problem cannot be solved in general
- A dawning conclusion:
  - Particles are a bad idea.
  - ...do strings instead!

THE NIGHTMARE BEFORE STRING-MASS

Superstrings

- In string theory, the building blocks of Nature,
- strings, well… oscillate:

- To the center of mass, with location and momentum known perfectly, Heisenberg's principle does not apply.
- Oscillatory modes (a.k.a particles), to which Heisenberg's principle does apply, are smeared, near but away from at the center of mass.

➠ *no contradiction!*

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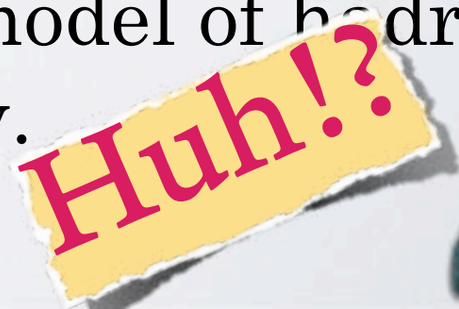
Superstrings

- Actually, that's not at all what happened in the 1960's:
  - Hadrons (particles that interact via strong *and* weak interactions) are not point-like, but spring/string-like
  - Use rubber strings to *model* hadrons (1969, Susskind),
    - characteristic energy ~ $\Lambda_{QCD} \approx 200$ MeV (string tension)
  - Didn't pan out:
    - Predicts spin-2 massless particle (Pomeranchuk), and…
    - Quark model & Quantum Chromodynamics won out
      - Hadrons consist of *point-like* quarks (and anti-quarks)…
      - …bound by gluons (= $SU(3)$ gauge interaction quanta)
  - And strings?
    - Effective, phenomenological & approx. model of hadrons
    - …or (1974, Sherk & Schwarz), of gravity.



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Superstrings RECYCLED

- Well,…there must exist a spin-2 massless particle (graviton),
  - …if gravity is ever to become a quantum theory;
- So, *if* Nature's building blocks were stringy (springy),
  - …they'd *force* the inclusion of gravitons and gravity.
- Summer 1984 (Green, Schwarz): no known anomaly
  - *i.e.*, quantum effects do not break classical symmetries
- November 1984 (Gross, Harvey, Martinec, Rohm)
  - supersymmetry, $E_8 \times E_8$ gauge interaction & 9+1 dimensions
- December 1984 (Candelas, Strominger, Horowitz & Witten)
  - Nordström-Kałuża-Klein compactification on Calabi-Yau complex 3-folds ➠ simple supersymmetry in 3+1 dimensions, & Standard Model parameters = CY geometry!

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Superstrings

- "Standard Model from CY geometry?"
  - $SU_c(3) \times SU_w(2) \times U_y(1)$ gauge group
  - 3 generations of 15 spin-½ matter particles (chirality eigenstates)
  - all their (different!) masses
  - the CKM mixing matrix
  - Baryon & (flavored) lepton conservation
  - θ-vacua, instantons, PCAC,
  - Higgs particle: MIA
  - The Grand Desert (10^2 — 10^{17} GeV)
  - 3+1-dimensional de Sitter spacetime, w/a teeeeeeeeeeeensy Λ
  - …and 10^{-5} cosmic background fluctuation…

15 orders of magnitude of boooo-ring…

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Can you say "continuum"?

15 orders of magnitude of boooo-ring…

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Superstrings

- Yet, strings (& particles) without supersymmetry
  - have no (stable) ground state (vacuum), most likely.
  - So, just as quantum-ness of Nature stabilizes the atom, supersymmetry stabilizes the vacuum.
- Sooo:
  - Stable
  - Quantum
  - Gravity

⇒ Superstring theoretical system w/gravity, matter and gauge interactions

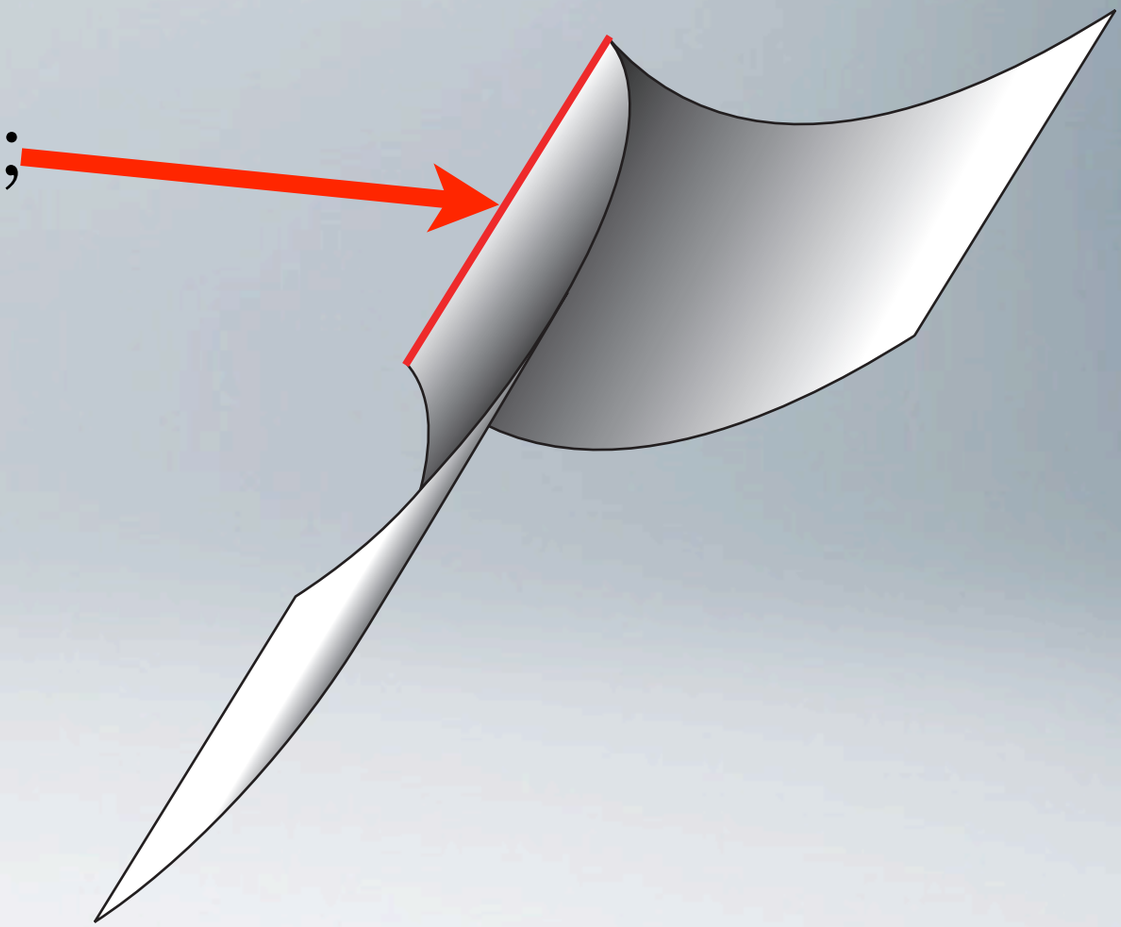
concrete choices are still "up for grabs"

a multi-dimensional continuum, organized into The Web* (of Worlds)

* T. Hübsch, *Calabi-Yau Manifolds—A Bestiary for Physicists* (World Scientific, 2nd ed., 1994).

When 3+1 jus' ain't 'nuff

- "Generic/typical" stringy spacetimes feature*:
- "isolated" 4D subspace(time)s;



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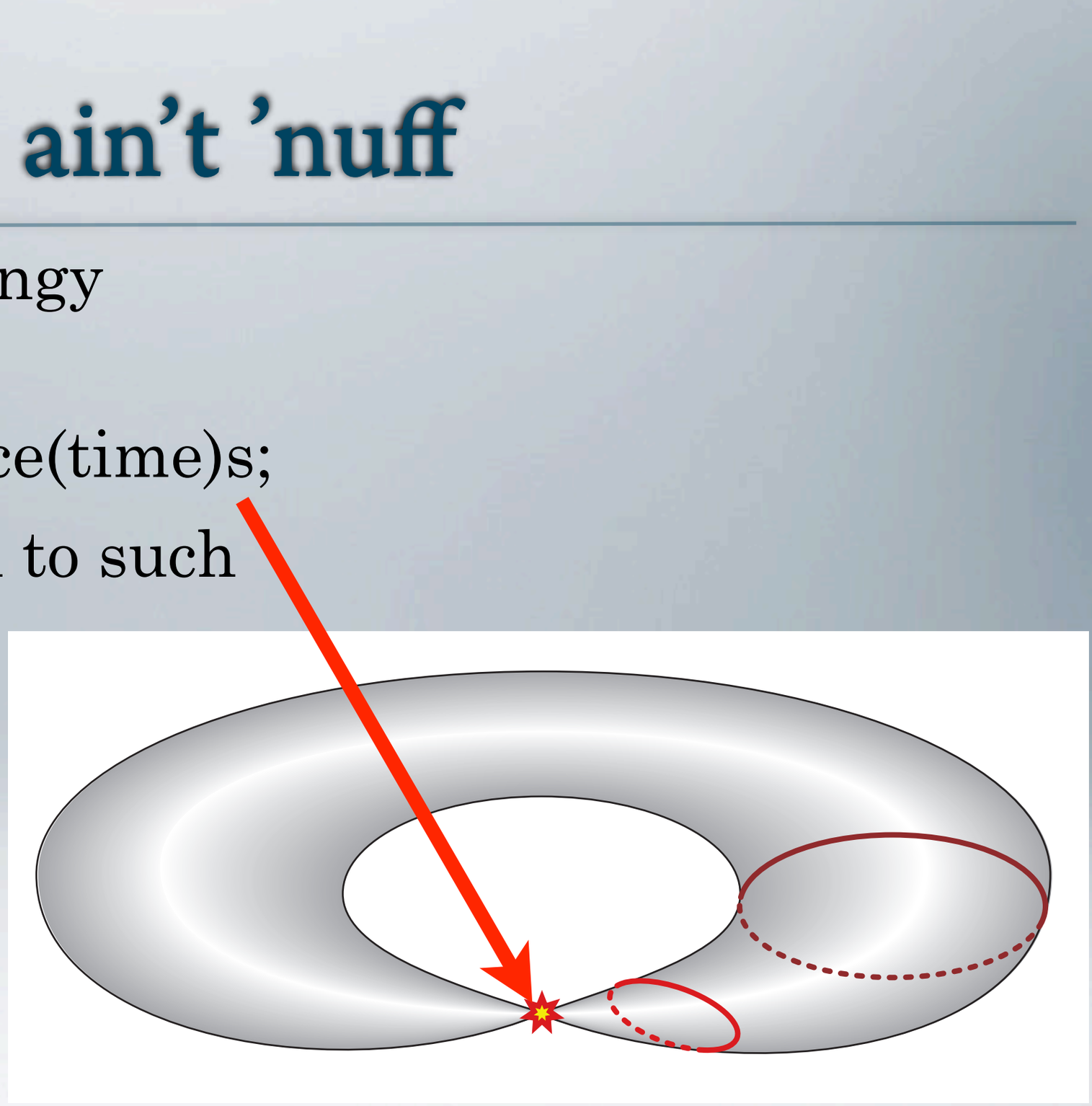

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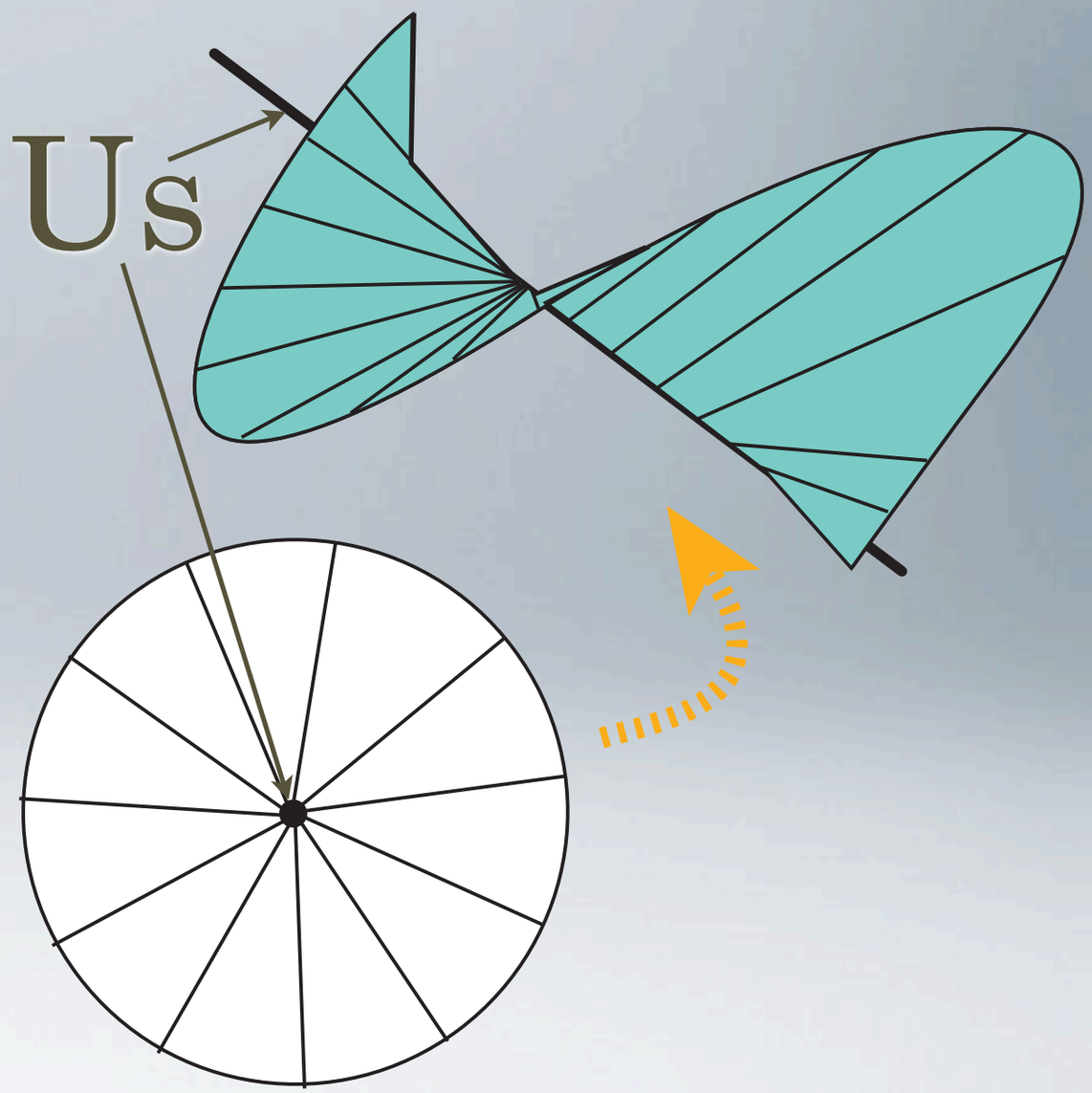
When 3+1 jus' ain't 'nuff

- "Generic/typical" stringy spacetimes feature*:
- "isolated" 4D subspace(time)s;
- with matter localized to such subspace(time)s;
- the first *gedanken-prototype** of jump-gates and warp-drive: detouring into the "trans-versal" directions (hyper-space), then return ☺
- …and gravity?! ☹



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Superstrings ➠ SuperThings

- D-branes: ends of open strings must satisfy Dirichlet boundary conditions …located *somewhere*: D-branes
- Superstrings include higher rank-r gauge potentials
  - $E = \mathrm{d}A$, $B = {*}E$ (e.g., $F_{\mu 0}$ *vs.* $\frac{1}{2}\varepsilon_{\mu 0}{}^{\varrho\sigma}F_{\varrho\sigma} = {*}F_{\mu 0}$)
  - $A_{(r)}$-form ➠ $E_{(r+1)}$ ➠ $B_{(n-r-1)}$-form
- Gauss's law:
  - $\Phi_E = \Phi_E(S^{n-r-1})$ ➠ $n-(n-r-1)-2 = (r-1)$-dim. e-source
  - $\Phi_B = \Phi_B(S^{r+1})$ ➠ $n-(r+1)-2 = (n-r-3)$-dim. m-source
    - E.g.: $n = 4$, $r = 1$ ➠ (0,0)-dim'l (e,m)-sources
    - E.g.: $n = 4$, $r = 2$ ➠ (1,–1)-dim'l (e,m)-sources
    - E.g.: $n = 10$, $r = 2$ ➠ (1,5)-dim'l (e,m)-sources

  –1 (*radial*) –1 (*temporal*)

  p-branes
- A large selection of *superThings*.

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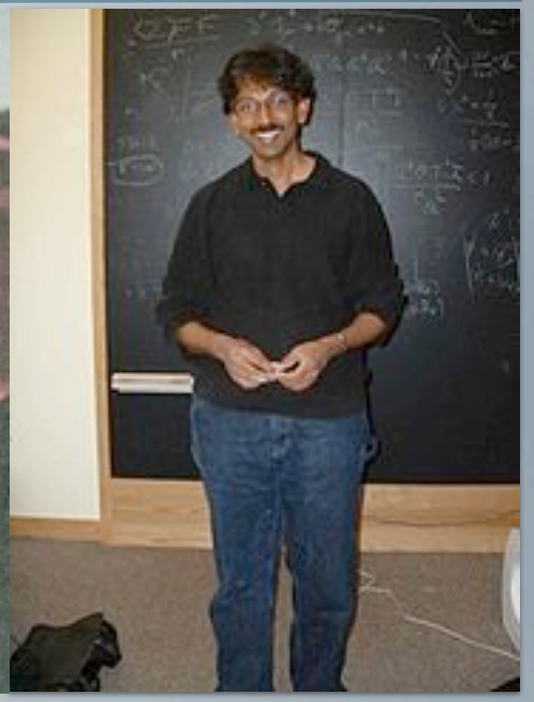
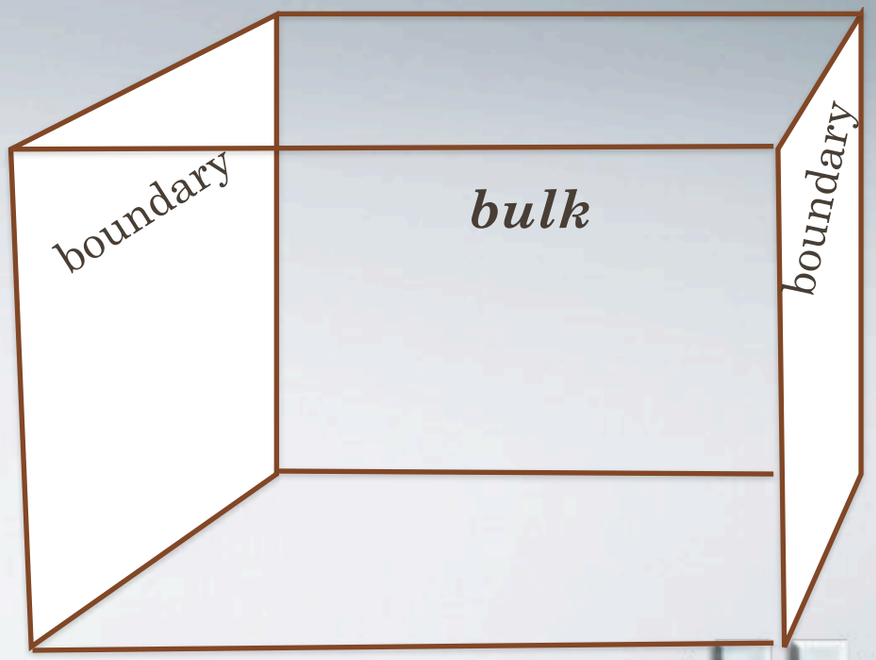
When 3+1 jus' ain't 'nuff:

- 1999: Lisa Randal
- & Raman Sundrum

Fourscore years after Kaluża



- Our spacetime is a sub-spacetime in a bigger (5-dimensional) one,
- …however, not compactified!
- The "heterotic M-theory" of Hořava and Witten:
- two 10-dimensional boundaries in an 11-dimensional spacetime
- each with an E8 gauge group
- …room enough for us, right? ☺



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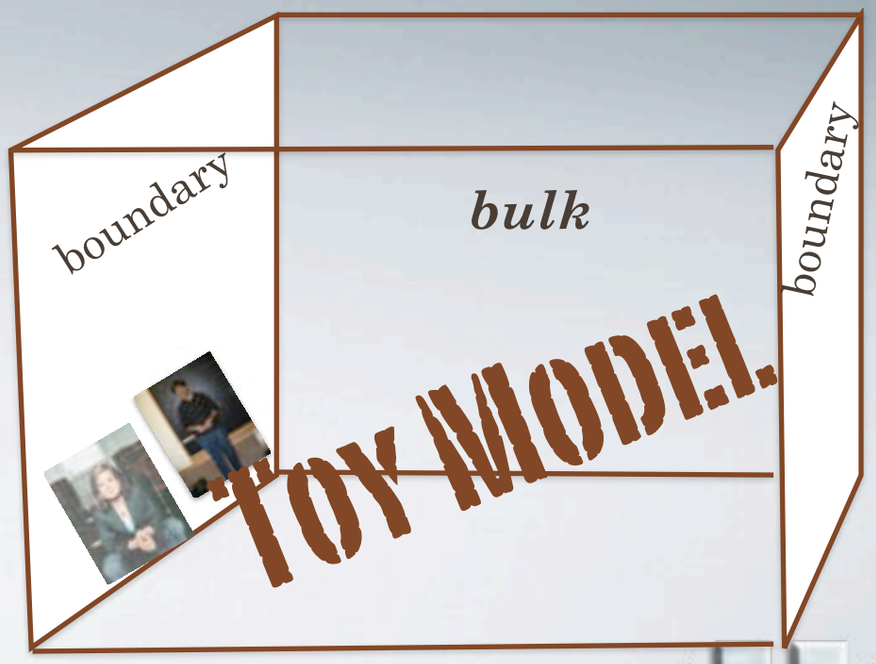

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When 3+1 jus' ain't 'nuff

- (Party like it's) 1999:
  the two most-cited papers in the past decade
- RS-1: exponential hierarchy
  - $M_P \sim 10^{19}$ GeV (gravity becomes confiningly strong)
  - $M_W \sim 10^{2}$ GeV (masses of $W^{\pm}$,Z^0)
  - …exponentially related through x^4 *curvature* ☺
- RS-2: localized gravity
  - bulk gravity ~ $1/r^8$ force
  - "boundary" gravity ~ $1/r^2$ force ☺
- But, not in one and the same end-of-the-world World! ☹

…and, we'd like 'em both, and in the same World! (Ours!)

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A SuperThing Toy Model:

- The SL(2, $\mathbb{Z}$) monodromy τ-driven brane-Worlds*:
  - have exponential $M_P : M_W$ hierarchy
  - localized gravity … *because* $g_{\mu\nu} = g_{\mu\nu}(|x^4|)$.
  - an induced de Sitter metric in the brane-World
  - an acceptable value of the cosmological constant…
  - …related to supersymmetry breaking mass-scale…
  - …which is geometry-induced.
  - The energy-loss in bulk-roaming fields
    - either traps them into the brane-World,
    - or bans them from it…
  - …and allows faster-than light travel through hyperspace
    (A.F. Roane's Howard University PhD dissertation)

* P. Berglund, T. Hübsch & D. Minić: see homepage.mac.com/thubsch/publiTH.html

Physics & Astronomy

Afterthoughts

- Superstring theory is in its 2nd reincarnation
- It is a theoretical system (like classical mechanics)
  - shifting the basic underlying paradigm of point-like, structure-less "basic building blocks" of Nature
  - to stringy basic building blocks.

- Its continuum of choices is organized into a universal (moduli) space, the geometry of which we are learning
- 3 layers:

worldsheet field theory	2D underlying field theory
spacetime field theory	4⁺D effective field theory — Us (among many)
moduli space field theory	∞D meta-field theory — The Web of Worlds

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MATRIX

Physics & Astronomy

Afterthoughts

Magnetic monopoles in spin ice

C. Castelnovo[1], R. Moessner[1,2] & S. L. Sondhi[3]

Electrically charged particles, such as the electron, are ubiquitous. In contrast, no elementary particles with a net magnetic charge have ever been observed, despite intensive and prolonged searches (see ref. 1 for example). We pursue an alternative strategy, namely that of realizing them not as elementary but rather as emergent particles—that is, as manifestations of the correlations present in a strongly interacting many-body system. The most prominent

Nature, Letters

Vol 451|3 January 2008|doi:10.1038/nature06433

78 years after P.A.M. Dirac predicted that electric monopoles could be mirrored by magnetic monopoles, they may have been found… sort of.

articles in Science. If we can expect benefits in the areas of spintronics and magnetic memories, it seems difficult to imagine applications in high energy physics. But we can not be sure. After all, the boson Higgs itself derives from work on superconductivity.

Science Niche, 09/09/09; editorial; scienceniche.com

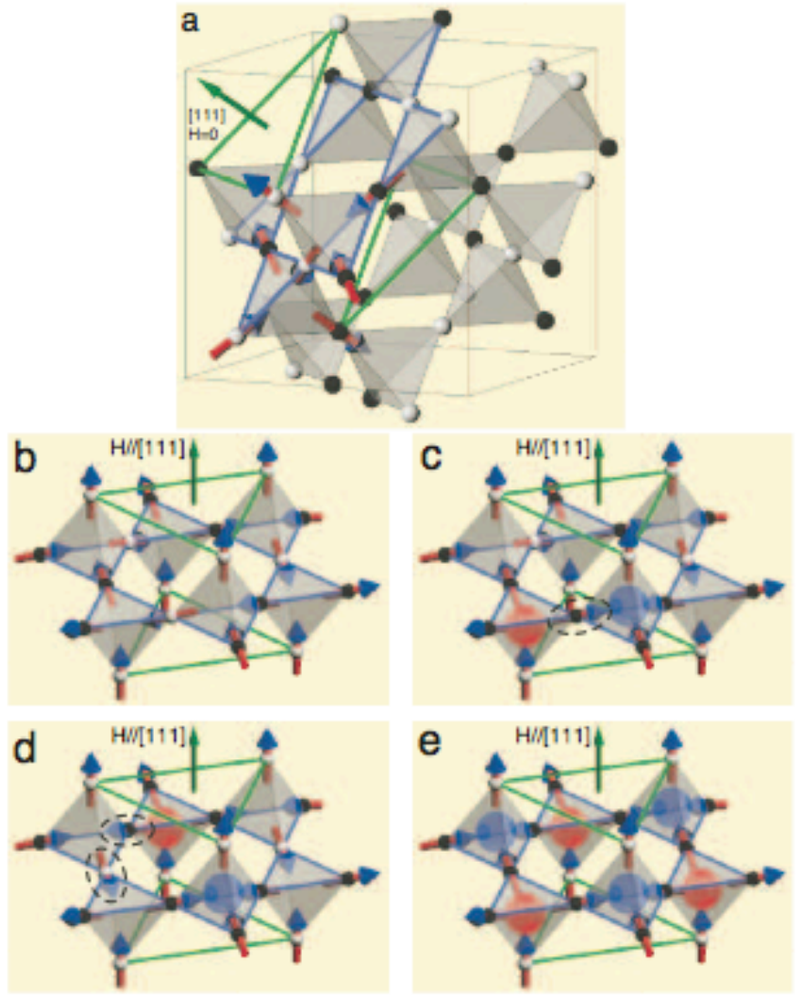

Fig. 1. (Color) Magnetic moments of $Dy_2Ti_2O_7$ reside on the pyrochlore lattice.[8] At low temperatures, four magnetic moments on each tetrahedron obey the ice rule (2-in, 2-out). The resulting spin ice state is shown in (a). The pyrochlore lattice consists of stacked triangular and kagomé lattices, shown by green and blue lines, respectively, along a [111] direction. (b) Under small [111] magnetic fields, spins on the kagomé lattice remain in the disordered kagomé ice state.[11] (c) An excited state is induced by flipping a spin from (b), enclosed by a dashed circle, where neighboring tetrahedra have 3-in, 1-out and 1-in, 3-out configurations. These ice-rule-breaking tetrahedra are represented by magnetic monopoles with opposite charges depicted by spheres. (d) By consecutively flipping two spins from (c), the monopoles are fractionalized. (e) As the magnetic field is increased, $H \gg H_c$, spins realize a fully ordered, staggered arrangement of monopoles.

H. Kadowaki *et al.*, arXiv:0908.3568

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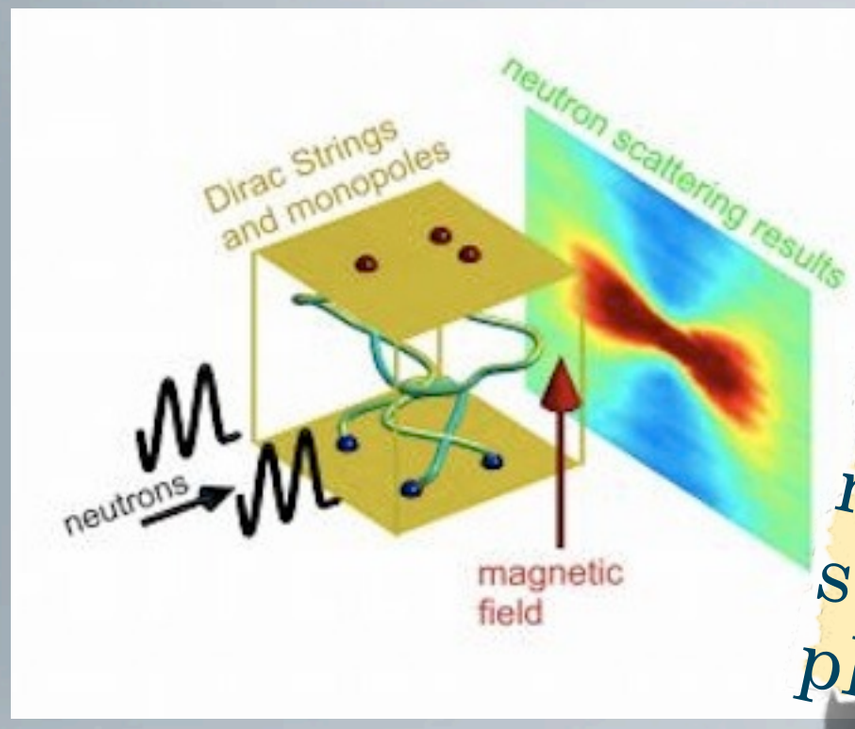
Afterthoughts



The magnetic material with thin strings that end in monopoles stuck at the plates

Two 2-branes with super-Things attached to them

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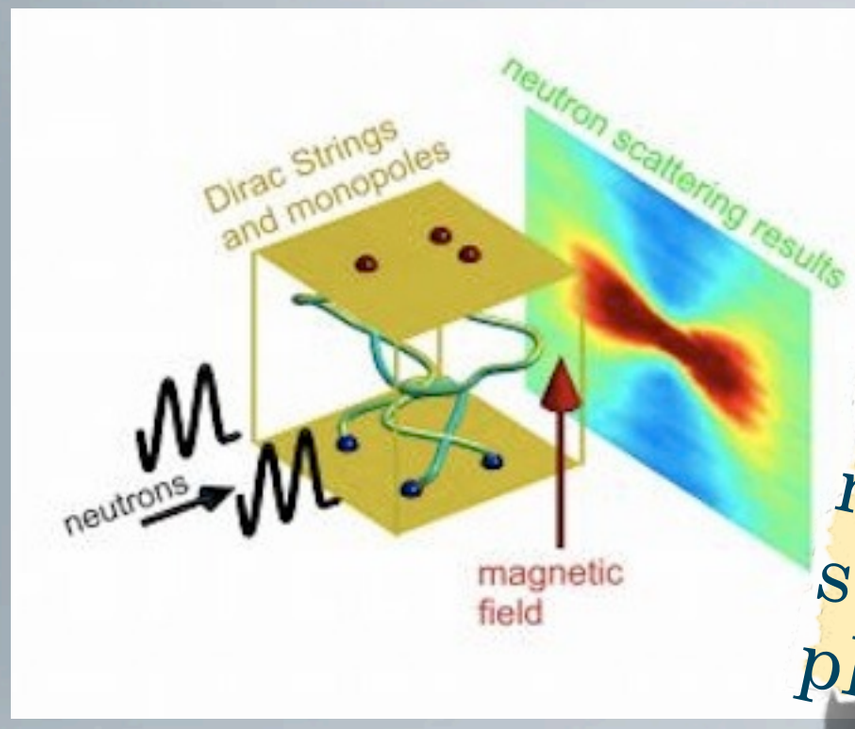
Afterthoughts



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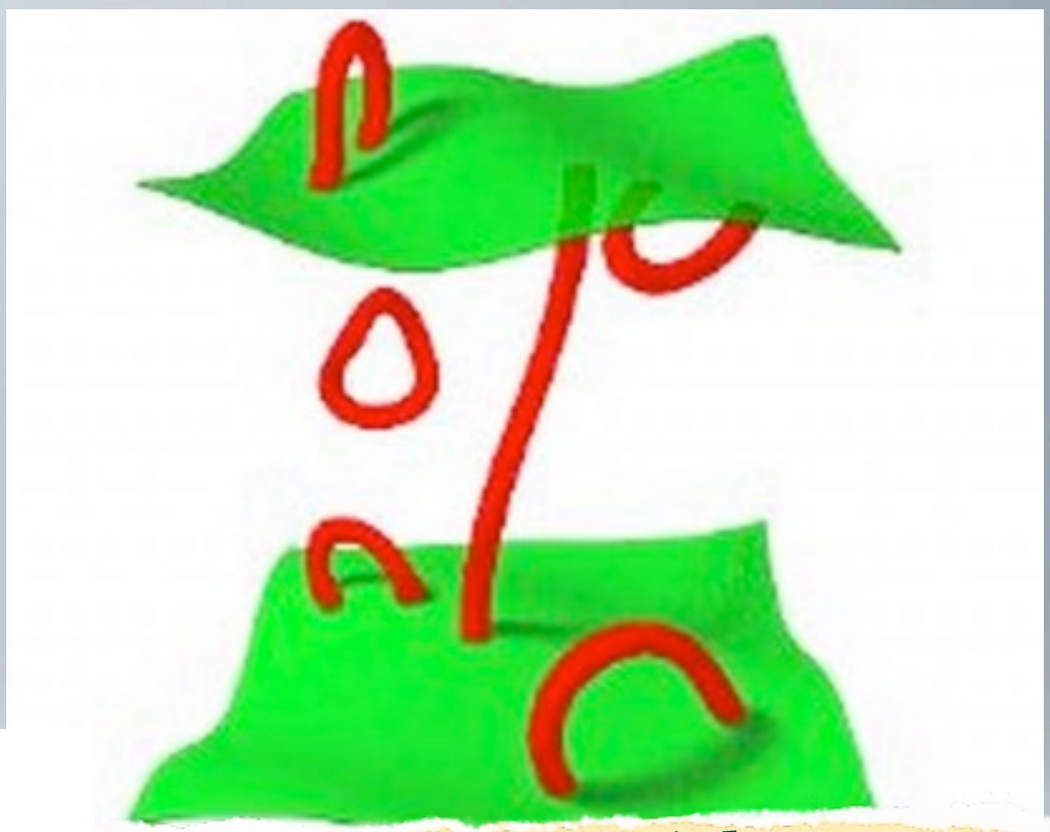

Two 2-branes with super-Things attached to them

Obvious conclusion:
Solid State Physics
has nicer illustrations

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Thank You!

visit me @

http://string.howard.edu/~tristan/
http://physics1.howard.edu/~thubsch/
http://homepage.mac.com/thubsch/
& TKH.213

…to be continued…

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