Through the Looking-Glass, & What Physics is Found There

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http://homepage.mac.com/thubsch/

Playbill

"If you don't know where you're going,
it doesn't matter which road you take."

- Through a Looking Glass:
  What is mirror duality
- First Few Fragment's Fruition
  Why it is worthwhile
- Forging and Fanciful Framing
  How a mirror is made
  in collaboration with Per Berglund
- Festivities of a Fine Finish
  *Where the foundations lie… er, I mean, **are***

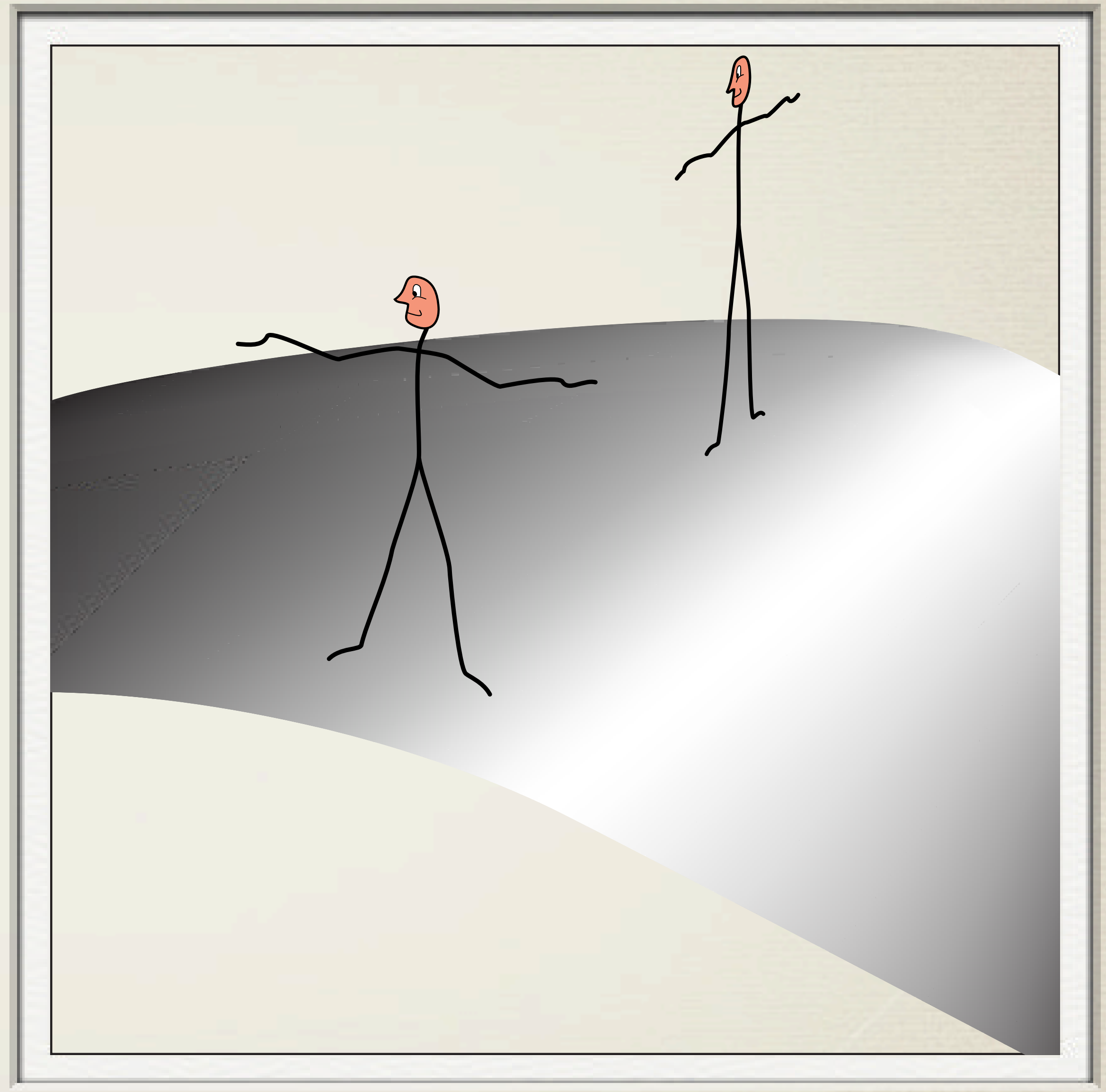
Physics & Astronomy


Mirror Duality in
String Theory

Mirror Duality in String Theory

- Compactifying space-time
  - The surface to the right is…

A very 2-dimensional surface, both indefinitely long and indefinitely wide



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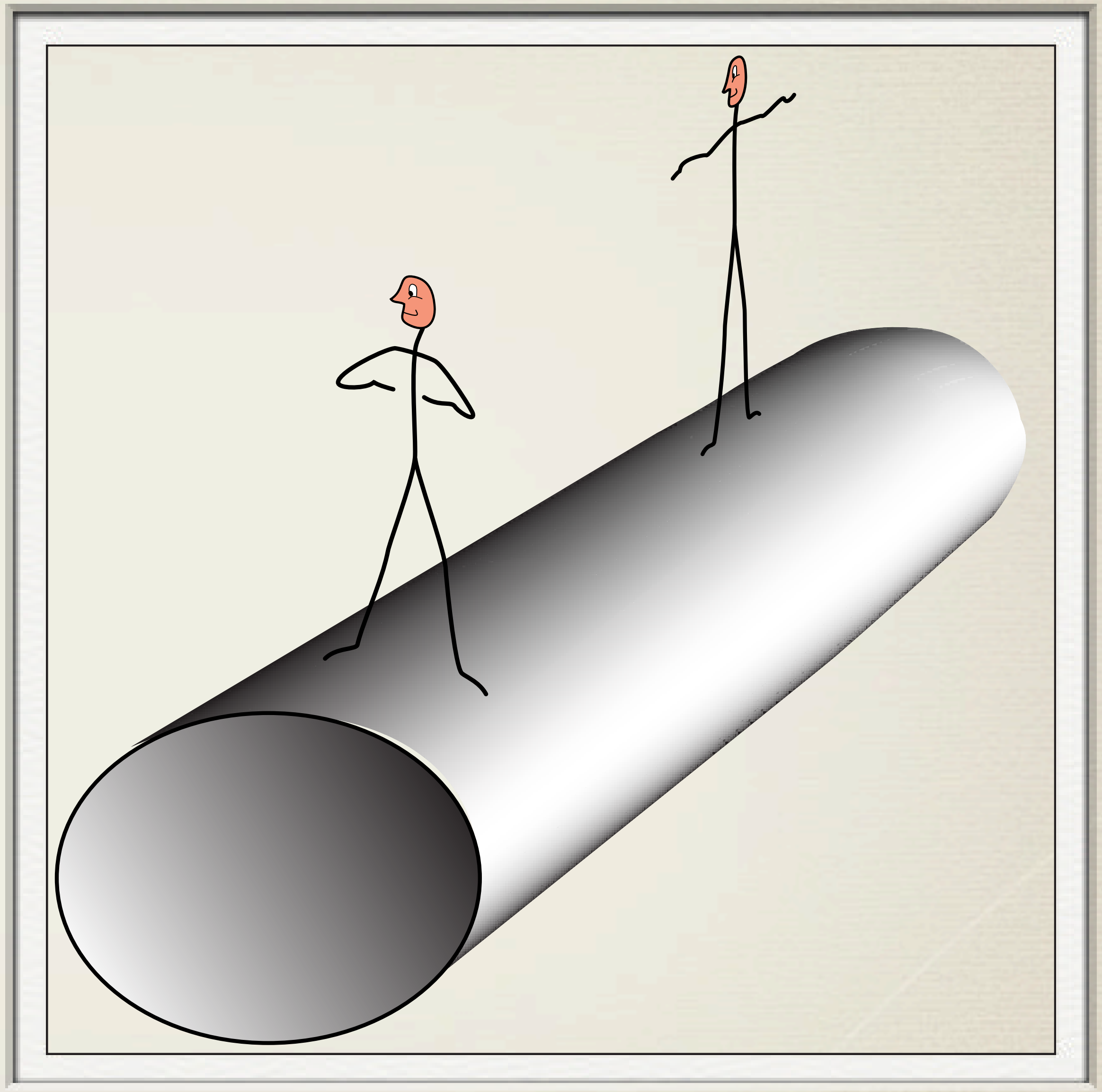
Mirror Duality in String Theory

- Compactifying space-time
  - The surface to the right is…

Still, a 2-dimensional surface, indefinitely long and somewhat wide



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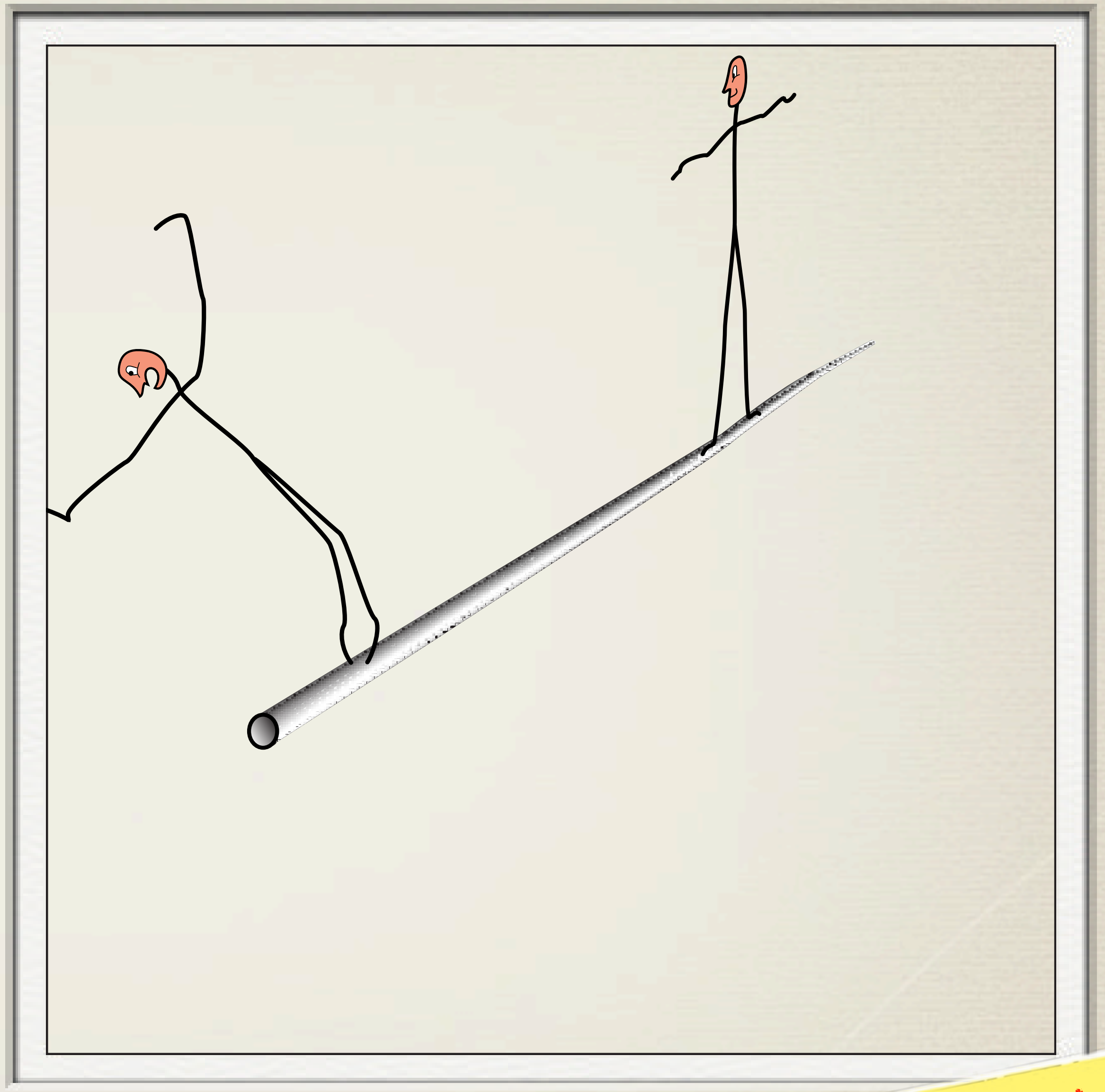
Mirror Duality in String Theory

- Compactifying space-time
  - The surface to the right is…

Only a 1-dimensional
line
indefinitely long and
veeery thin



- Topologically, it remains $\mathbb{R}^1 \times S^1$, even if the circle
  - is too big to be seen as curved, or
  - is too small to be seen

And, strings perceive space differently!

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Mirror Duality in String Theory
cont'd

- But… *why do we need strings?*
  - Because we can't see ***points***, that's why:
  - When a probe of energy $E_\mathrm{p} = \hbar/\lambda_\mathrm{p} = m_\mathrm{p}c^2$, interacts with a target of mass m_t…
  - …during interaction, they form a system with a composite (effective) mass $(m_\mathrm{t} + \hbar/\lambda_\mathrm{p}c^2)$,
  - …from which the escape velocity equals the speed of light at the distance $R_\mathrm{S} = 2G_\mathrm{N}(m_\mathrm{t} + \hbar/\lambda_\mathrm{p}c^2)/c^2$.
  - So, once $\lambda_\mathrm{p} = R_\mathrm{S}(= \ell_\mathrm{P} \sim 10^{-35}\mathrm{m}$ = "Planck length"), the probe stops seeing "inside" the target.
- And strings are the next best thing.

"multipole expansion"

Physics & Astronomy

Mirror Duality in String Theory

cont'd

- Indeed, in the multipole expansion, the next "thing" after a point is a dipole—*i.e.*, a stick.
- Except, relativistic "sticks" are not rigid.
- Strings have a finite and nonzero tension:

$$T_0 = \frac{1}{2\pi\alpha'\hbar c} = \frac{c^4}{2\pi G_N}$$

- …and so also a characteristic size, $\ell_S = \ell_P \sim 10^{-35}$m.
- And strings (modeled on mesons) interact:
  - by breaking in two, and
  - by joining ends.

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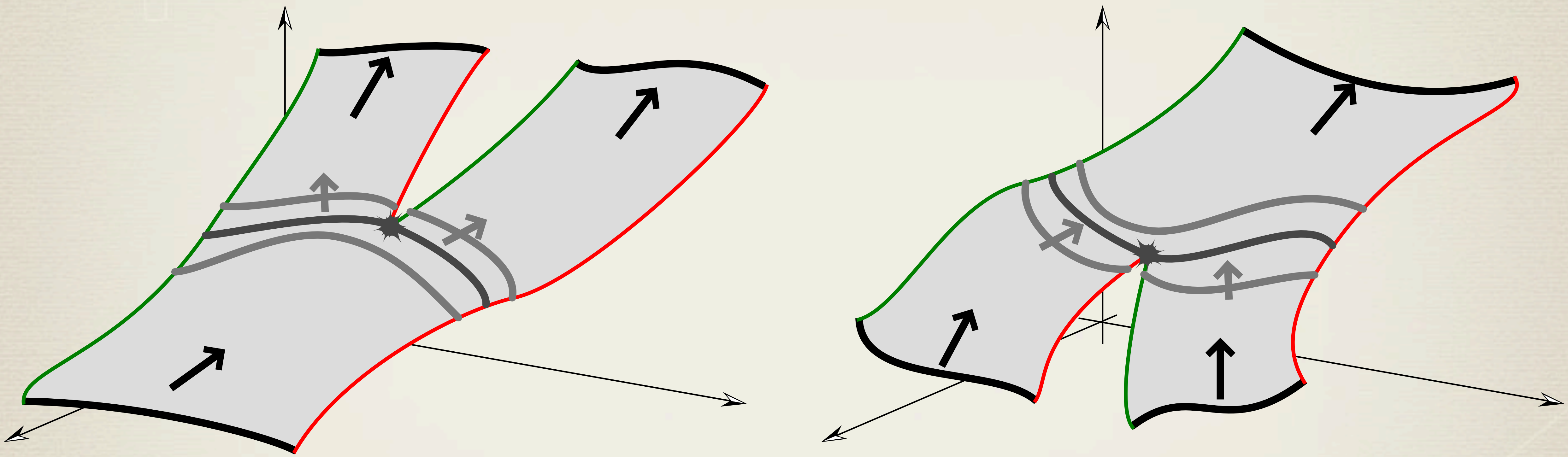
Mirror Duality in String Theory

cont'd

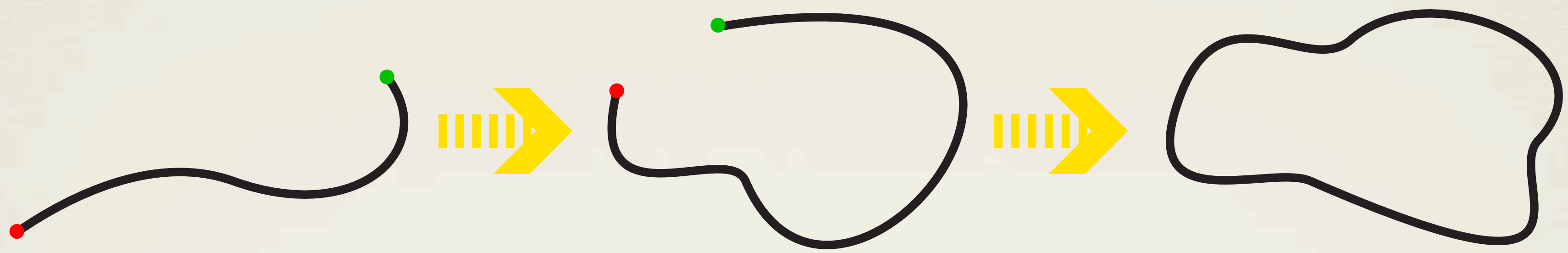
- Something like this:



- It is then consistent to have opposite charges at the endpoints. (*In fact, only SO(32) charges will do, but that's technical.*)
- Also: there are always closed (O-like) strings.

Mirror Duality in String Theory
cont'd

- Self-interaction…



- …guarantees: open strings are inconsistent without closed ones, but closed strings are self-consistent.
- Strings vibrate:

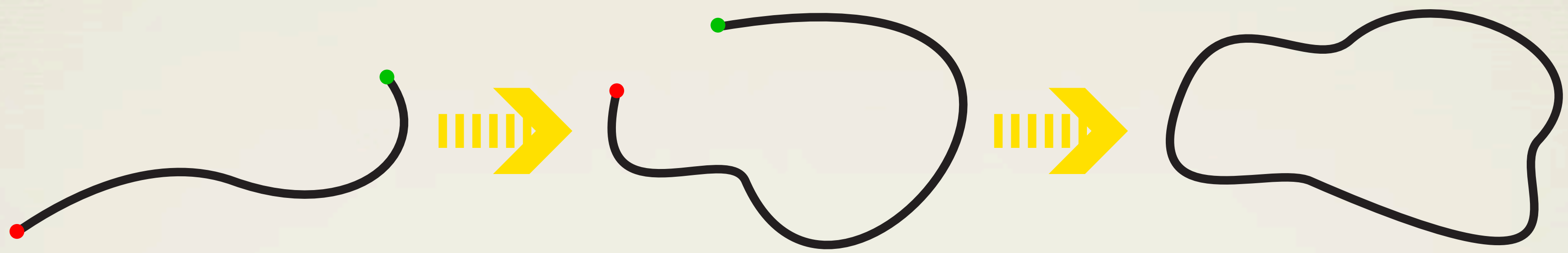
$$X^\mu(\tau,\sigma) = \boxed{x^\mu + \frac{p_\mu}{p_-} c\tau} + i\sqrt{\frac{\alpha'}{2}} \sum_{\substack{n=-\infty \\ n\neq 0}}^{+\infty} \left[\frac{a^\mu_{n,R}}{n} e^{-2\pi i n\varsigma^+} + \frac{a^\mu_{n,L}}{n} e^{2\pi i n\varsigma^-}\right],$$

where *on-shell, c.m. motion*

$$\varsigma^\pm := (\sigma \pm c\tau)/\ell_s, \qquad \text{i} \qquad a^\mu_{-n,R} = (a^\mu_{n,R})^\dagger, \quad a^\mu_{-n,L} = (a^\mu_{n,L})^\dagger,$$

Mirror Duality in String Theory *cont'd*

- Self-interaction…



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oscillations

where

$$\varsigma^\pm := (\sigma \pm c\tau)/\ell_s, \qquad \text{i} \qquad a^\mu_{-n,R} = (a^\mu_{n,R})^\dagger, \quad a^\mu_{-n,L} = (a^\mu_{n,L})^\dagger,$$

Mirror Duality in String Theory

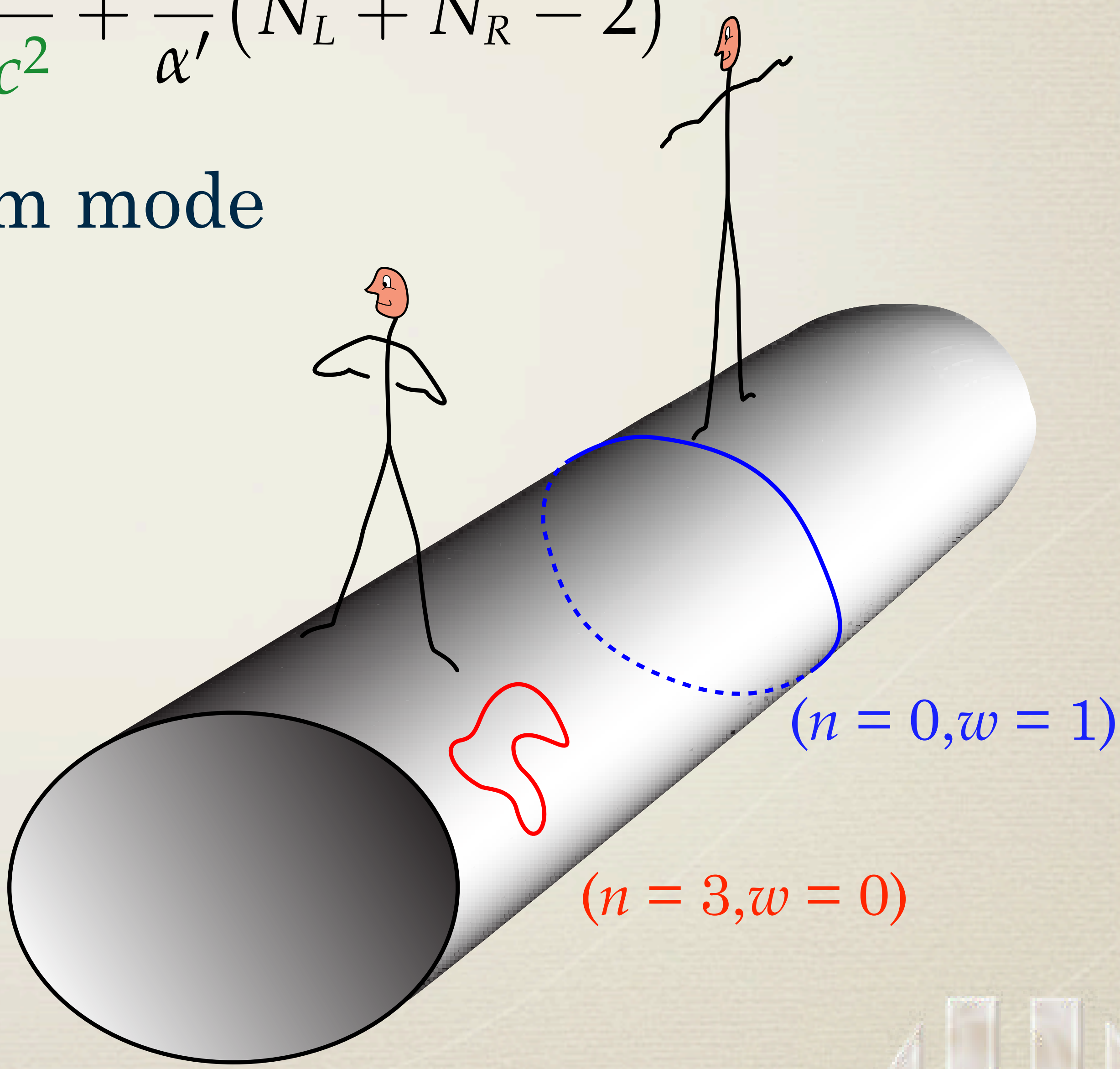
cont'd

- The energy/mass of a mode of string motion then is:

$$(mc^2)^2 = \frac{n^2\hbar^2c^2}{R^2} + \frac{w^2R^2}{\alpha'^2\hbar^2c^2} + \frac{2}{\alpha'}(N_L + N_R - 2)$$

- n = (periodic space) momentum mode
- w = string winding mode
- N_L, N_R = total number of oscillations
- Small-Large transform.:

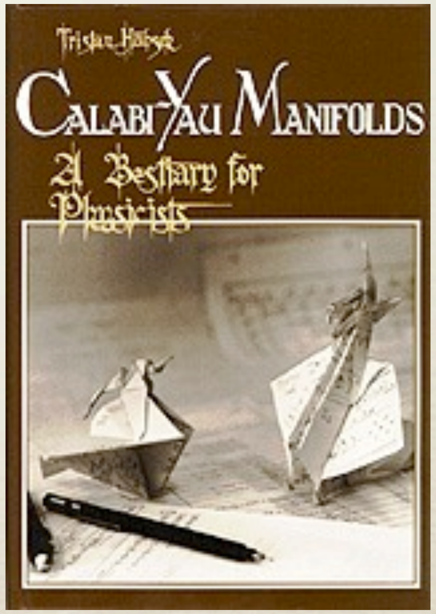
$$n \leftrightarrow w \quad \text{and} \quad R \leftrightarrow \frac{\alpha'\hbar^2c^2}{R} = \frac{\ell_s{}^2}{R}$$



Mirror Duality in String Theory

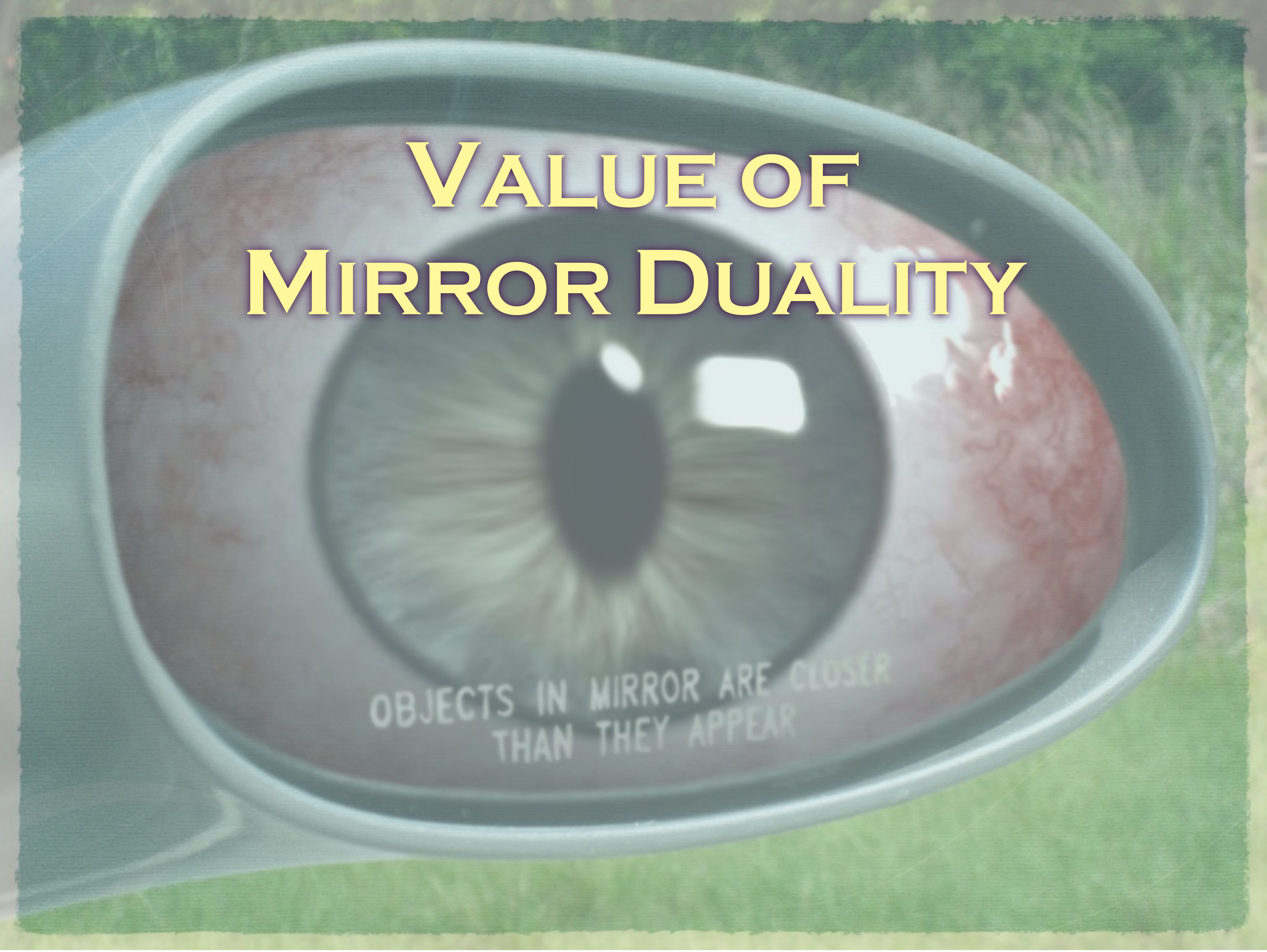
cont'd

- When a dimension is curled up critically, $R \sim \ell_S$,
- …then $R \to \ell_S^2 / R$ is a symmetry.
- Otherwise, it's a small-large ***T-duality***.
- **Clearly**, when more than one dimension is curled up, there are many more ways to ***dualize***.
- When curling up 6 dimensions & preserving 1 susy, one must use a Calabi-Yau manifold.



- Strominger-Yau-Zaslow (1996):
  "Mirror duality of Calabi-Yau
  manifolds is a T-duality"

Value of Mirror Duality



Values of Mirror Duality

- First off, ***What*** does mirror duality ***do***?
- When space is curled up on a CY 3-fold $\mathscr{Y}$, the string model in the remaining 3+1-dimensional spacetime has
  - $h^{2,1}$ "generations" of Standard Model matter,
  - and $h^{1,1}$ "anti-generations",
  - plus some completely charge-less "junk".
- Mirror duality swaps that:
  - $M(h^{2,1}, h^{1,1}) \leftrightarrow W(h^{1,1}, h^{2,1})$,
  - discovered (Greene & Plesser, 1990)

...and we thought 'em to be **bonkers!**

Values of Mirror Duality

cont'd

- So Candelas, de la Ossa, Parkes and Green worked out a detailed example (1991).

Note the palindromic year!

- One CY 3-fold, *M*, is:

$$z_1^5 + z_2^5 + z_3^5 + z_4^5 + z_5^5 - 5\psi\, z_1 z_2 z_3 z_4 z_5 = 0,$$

$$(z_1, z_2, z_3, z_4, z_5) \simeq (\lambda z_1, \lambda z_2, \lambda z_3, \lambda z_4, \lambda z_5)$$

- where $\lambda \neq 0$. This is $\mathbb{C}\mathrm{P}^4[5]$.

- Symmetries: $(z_1, z_2, z_3, z_4, z_5) \cong (\omega^a z_1, \omega^b z_2, \omega^c z_3, \omega^d z_4, \omega^e z_5)$, where $\omega^5 = 1$, and so (a,b,c,d,e) are taken (mod 5).

- Turns out, $W = M / (\mathbb{Z}_5 \times \mathbb{Z}_5 \times \mathbb{Z}_5)$; ($h^{1,1}=101$, $h^{2,1}=1$: ψ)

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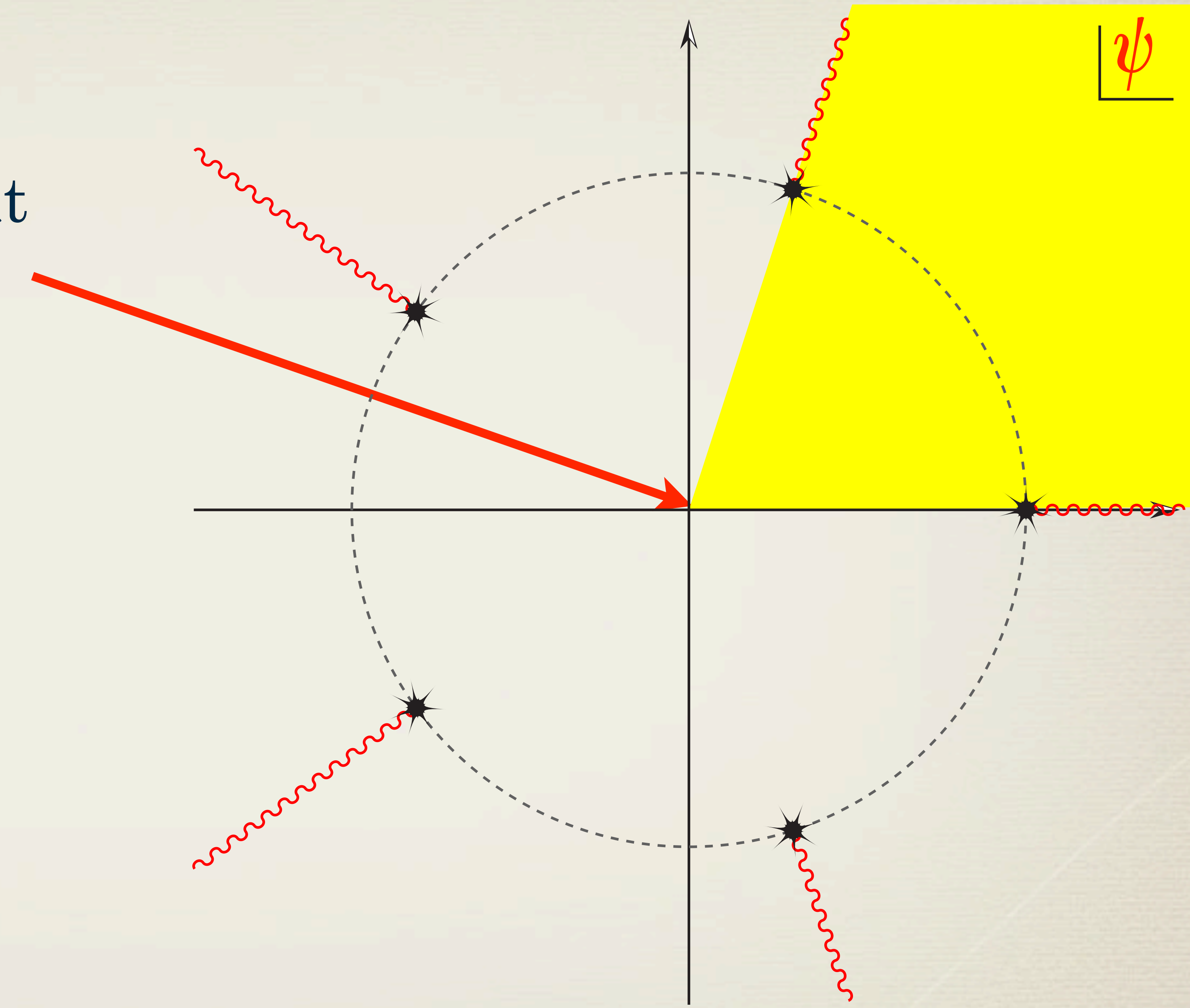
Values of Mirror Duality

cont'd

- So, in the ψ-space:
  - $\psi = 0$ is the "Fermat quintic," $\Sigma_i z_i^5 = 0$;

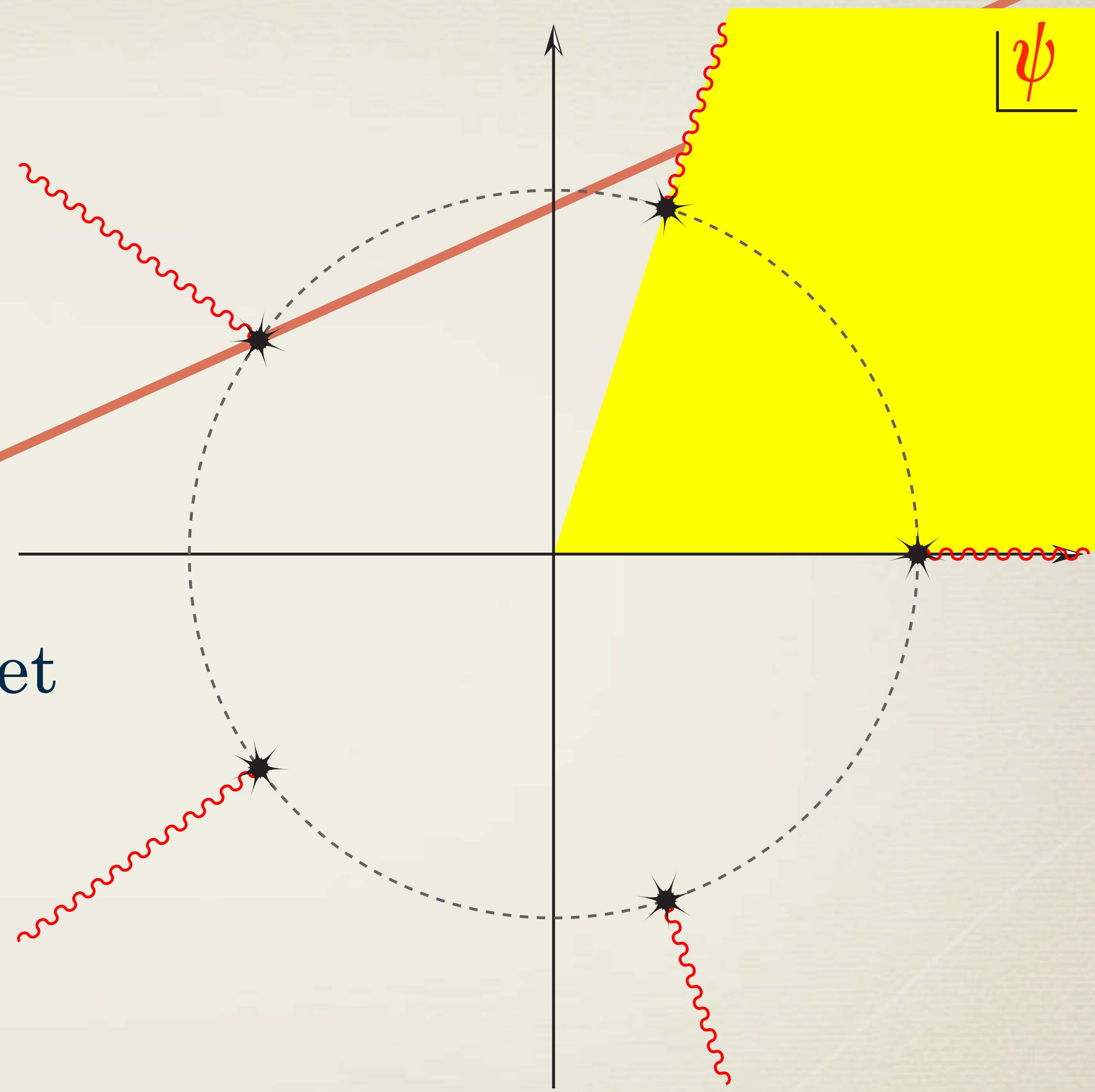


Friday, November 19, 2010

Values of Mirror Duality

cont'd

- So, in the ψ-space:
  - $\psi = 0$ is the "Fermat quintic," $\Sigma_i z_i^5 = 0$;
  - $\psi = \infty$ is the "*quint-ogicon*", $\prod_i z_i = 0$:
  - (5 $\mathbb{C}^3$'s meet in 10 $\mathbb{C}^2$'s meet in 10 $\mathbb{C}^1$'s meet in 5 $\mathbb{C}^0$'s);



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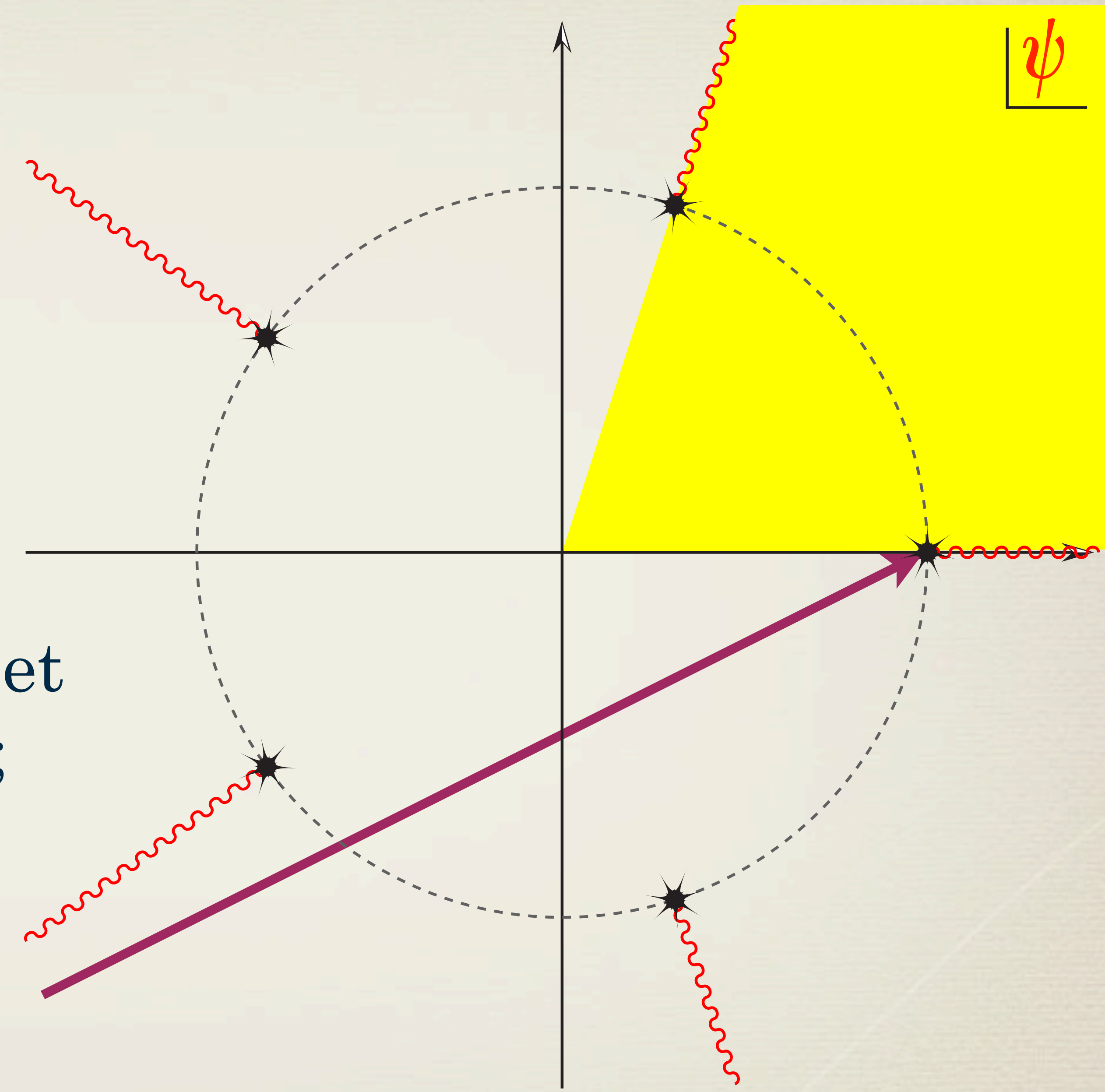
Values of Mirror Duality

cont'd

- So, in the ψ-space:
  - $\psi = 0$ is the "Fermat quintic," $\sum_i z_i^5 = 0$;
  - $\psi = \infty$ is the "*quint-ogicon*", $\prod_i z_i = 0$:
  - (5 $\mathbb{C}^3$'s meet in 10 $\mathbb{C}^2$'s meet in 10 $\mathbb{C}^1$'s meet in 5 $\mathbb{C}^0$'s);
  - $\psi = 1$ is the "quintic *conifold*": $f(z) = 0 = \mathrm{d}f(z)$…
  - …which is *singular**. [http://en.wikipedia.org/wiki/Conifold]



* $\mathrm{d}f = \sum_i (\partial_i f)\, \mathrm{d}z^i$, so that $\{\mathrm{d}f = 0\} \Leftrightarrow \{(\partial_1 f, \partial_2 f, \ldots, \partial_5 f) = 0\}$.

Values of Mirror Duality

cont'd

- Long story short, physicist *compute*. And, have found [COGP] that the Yukawa coupling on W may be expressed as a power series

RECOGNIZE THIS?

$$5+\sum_{k=1}^{\infty}\frac{n_k k^3 e^{2\pi i k t}}{1-e^{2\pi i k t}} = 5 + 2875\, e^{2\pi i t} + \ldots$$

$$t := -\frac{5}{2\pi i}\left\{\log(5\psi) - \frac{1}{\varpi_0(\psi)}\sum_{m=1}^{\infty}\frac{(5m)!}{(m!)^5(5\psi)^{5m}}\left(\psi^{(0)}(1+5m)-\psi^{(0)}(1+m)\right)\right\}$$

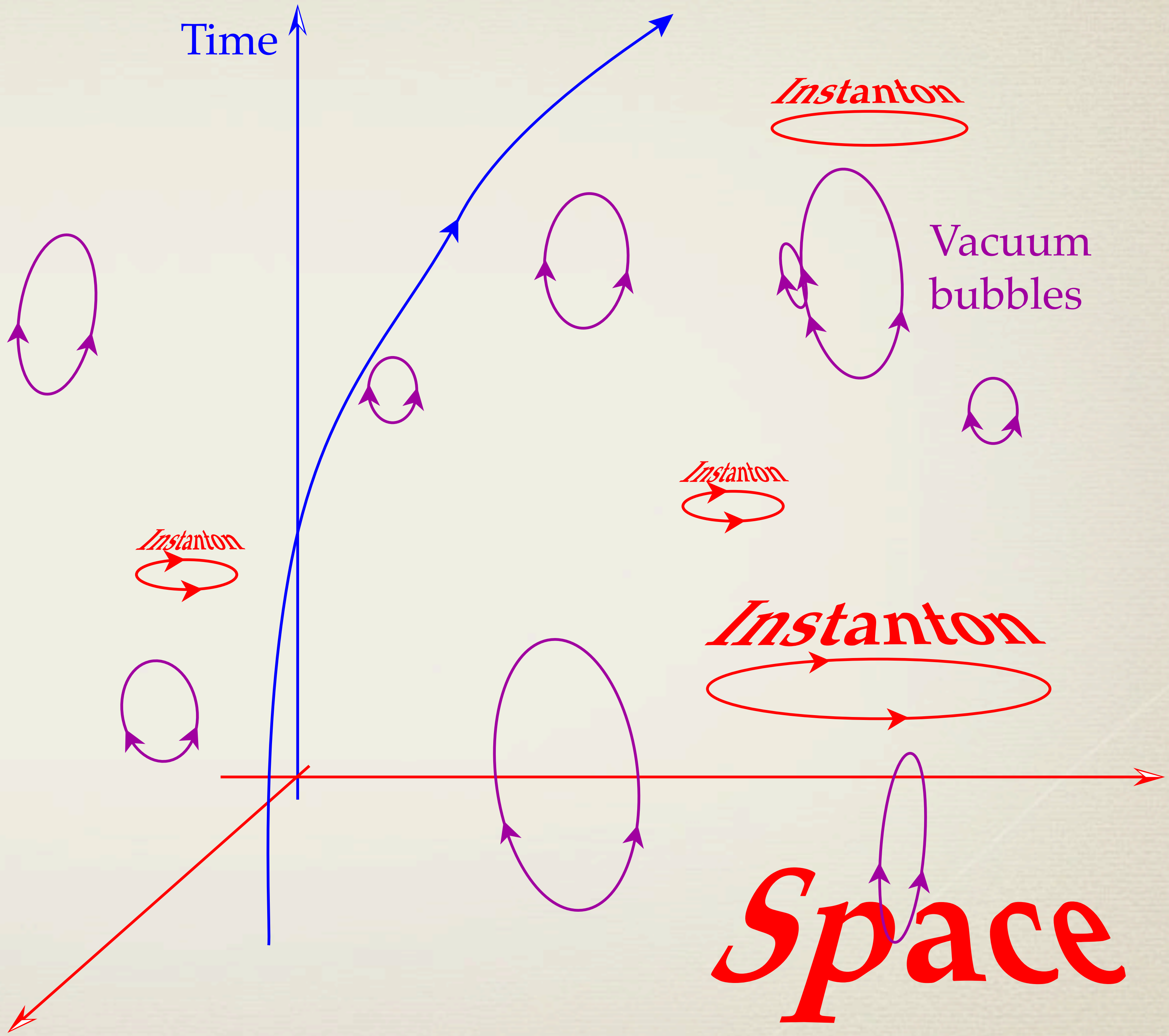
- This tells the interaction strength for *every* model in this 1-parameter family.

- The n_k give the number of degree-k embeddings of $\mathbb{C}P^1$'s (S^2's – *string instantons*) in W.

Values of Mirror Duality

digression

- Instantons?
- Vacuum "bubbles" = virtual particles
  - pop out of the vacuum,
  - propagate,
  - vanish into the vacuum.
- Espec. interesting,
  - when they are noncontractible!



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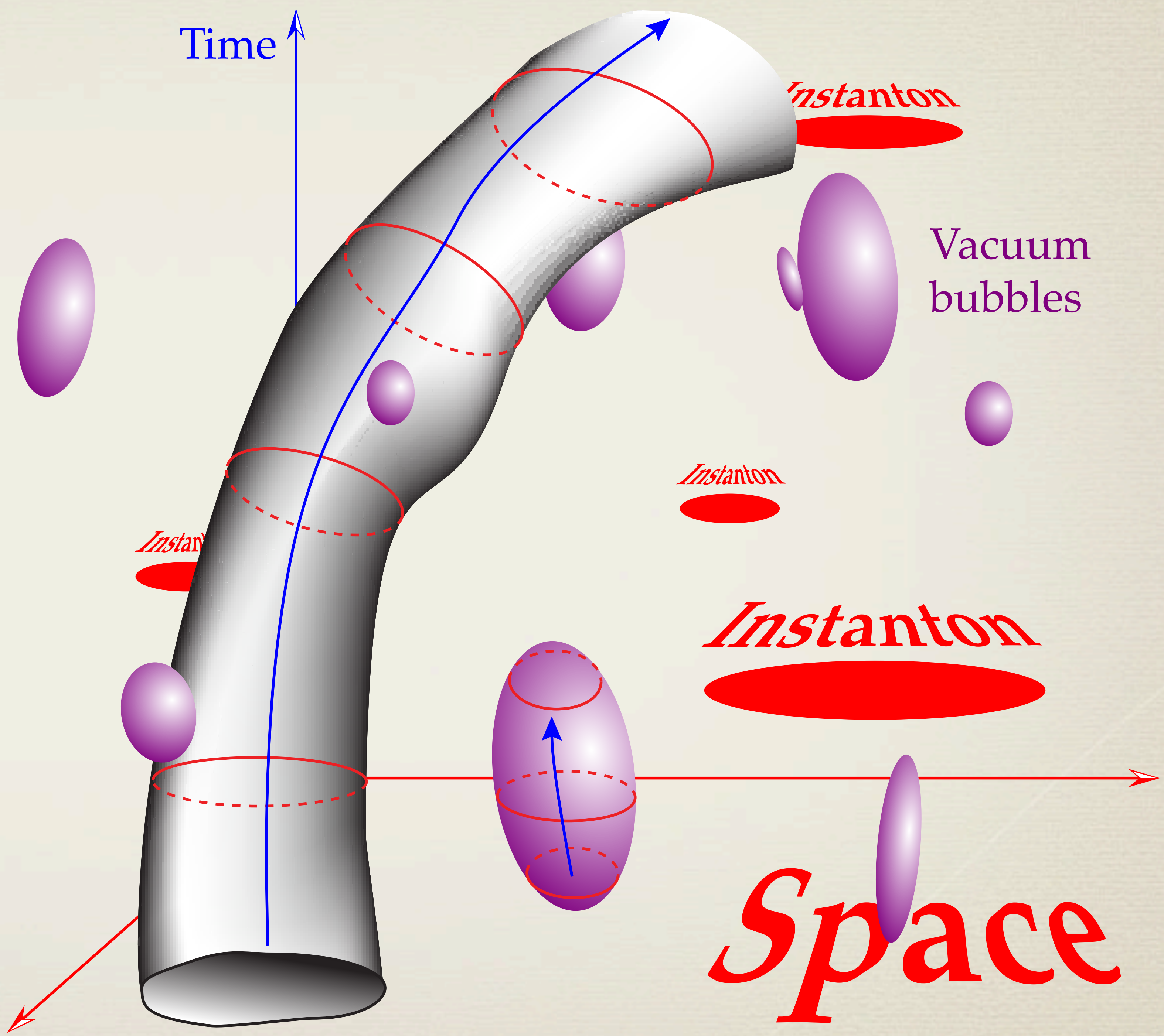
Values of Mirror Duality

digression cont'd

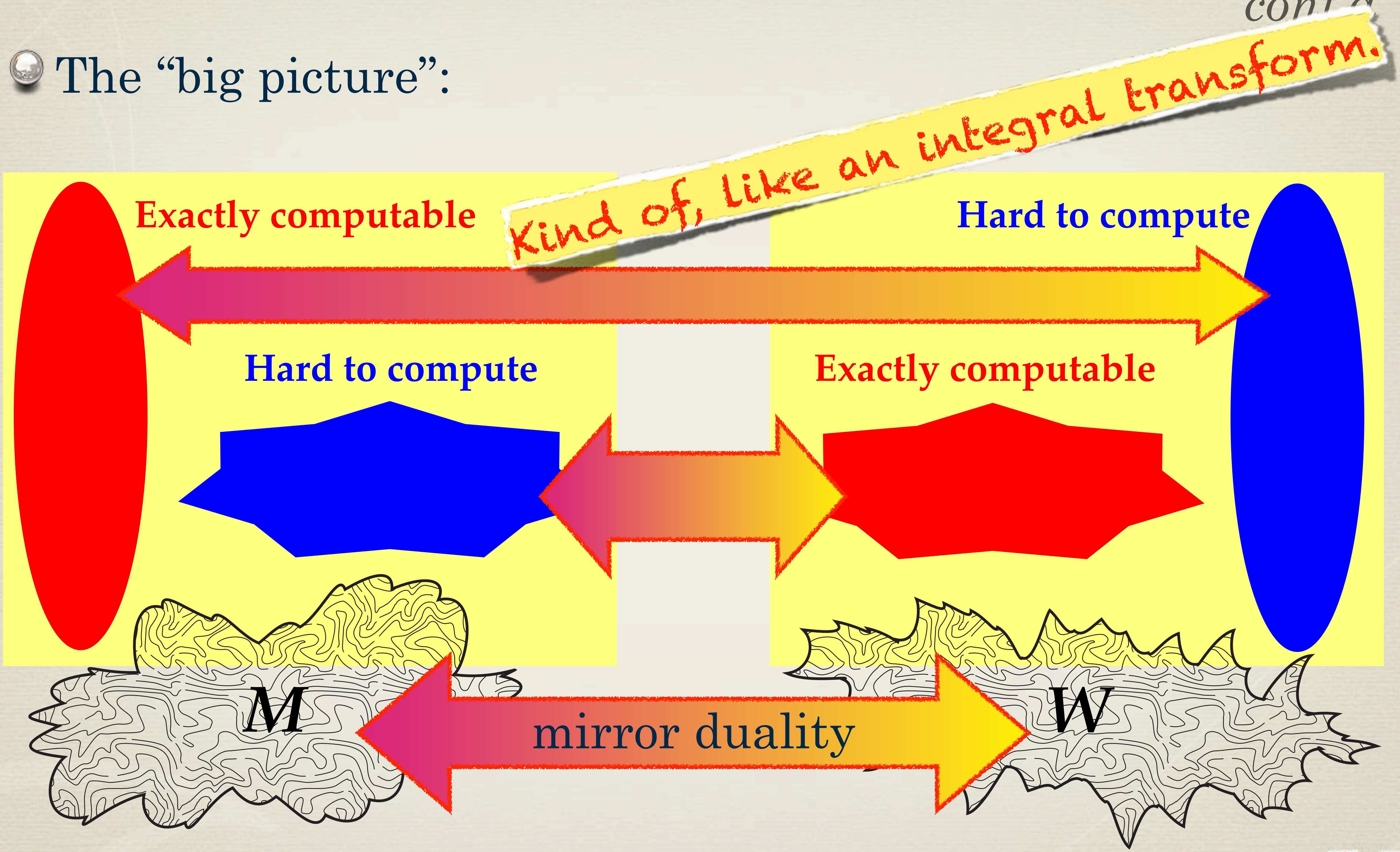
- String instantons?
- Vacuum bubbles = virtual strings
  - pop out of the vacuum,
  - propagate,
  - vanish into the vacuum.
  - Must be bosons!
- Bose-Einstein distribution!
- May well be noncontractible

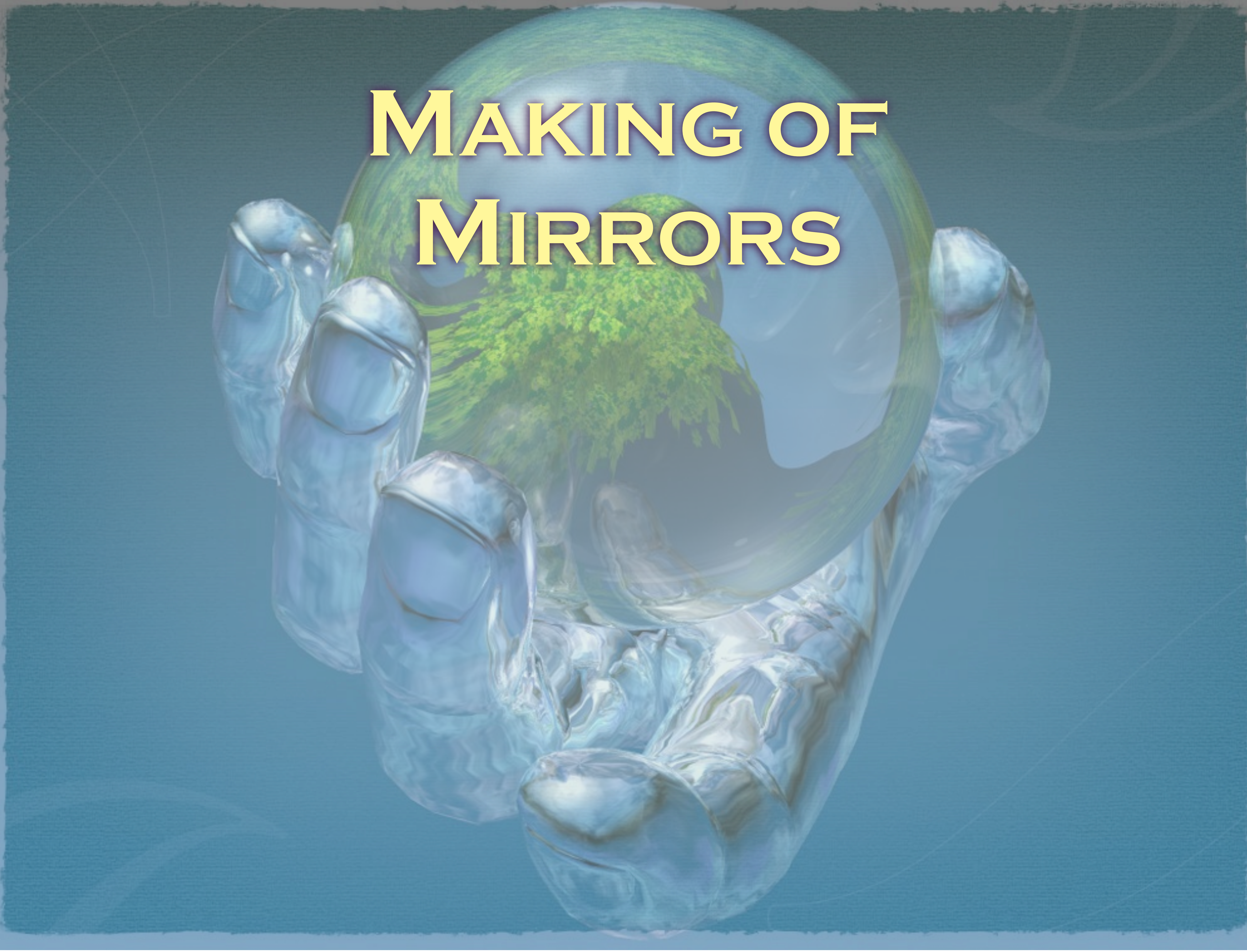


Values of Mirror Duality

cont'd

- The "big picture":

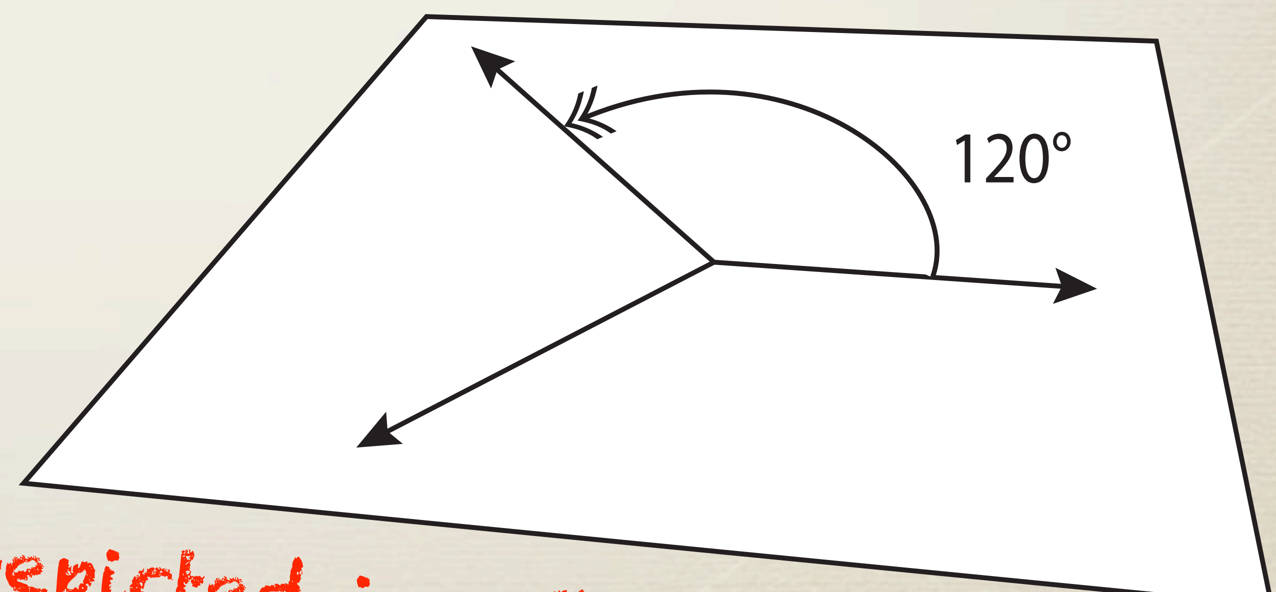



Making of
Mirrors

How Mirrors are Made

- So, how does one ~~find~~ make the mirror dual?
- In the mirror pair of quintics, if $M := \{z \in \mathbb{CP}^4, f(z) = 0\}$, then $W := M/(\mathbb{Z}_5 \times \mathbb{Z}_5 \times \mathbb{Z}_5)$.
- But, this is not true in general.
- So we take $f(z) = z_1^5 + z_2^5 + z_3^5 + z_4^5 + z_5^5$,
  - analyze (*disect*) how does $\mathbb{Z}_5 \times \mathbb{Z}_5 \times \mathbb{Z}_5$ act,
  - where do mirror *modes* come from,
  - how do they interact?

quotient ("quantum") symmetry



Depicted is a Z_3 quotient.

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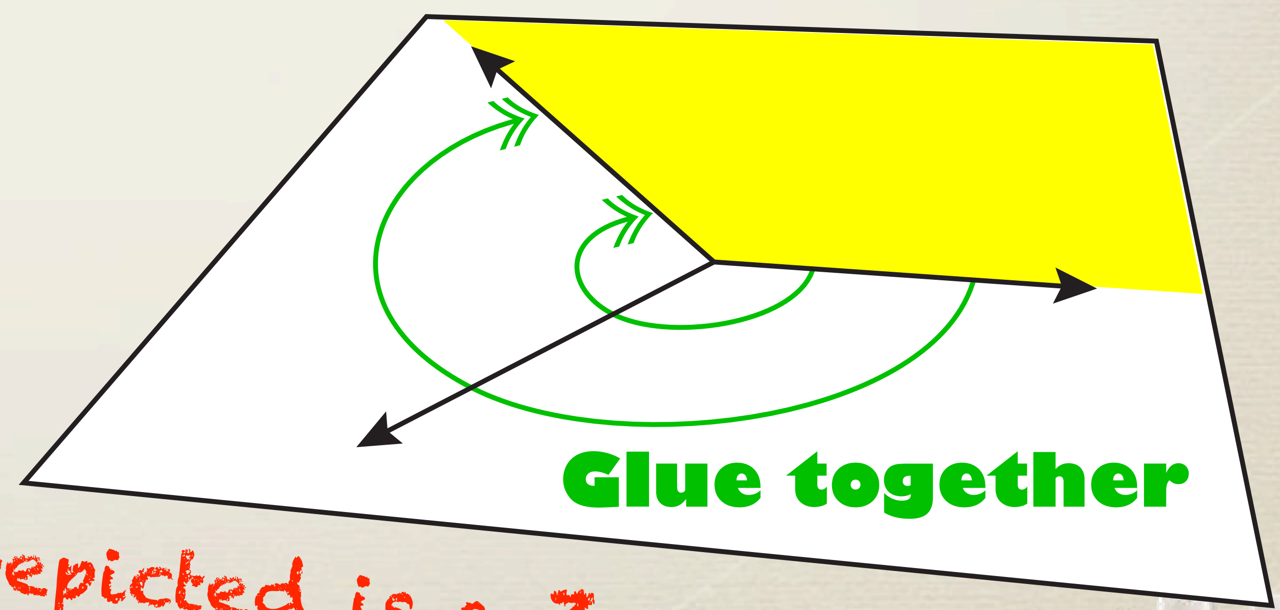

Depicted is a $\mathbb{Z}_3$ quotient.

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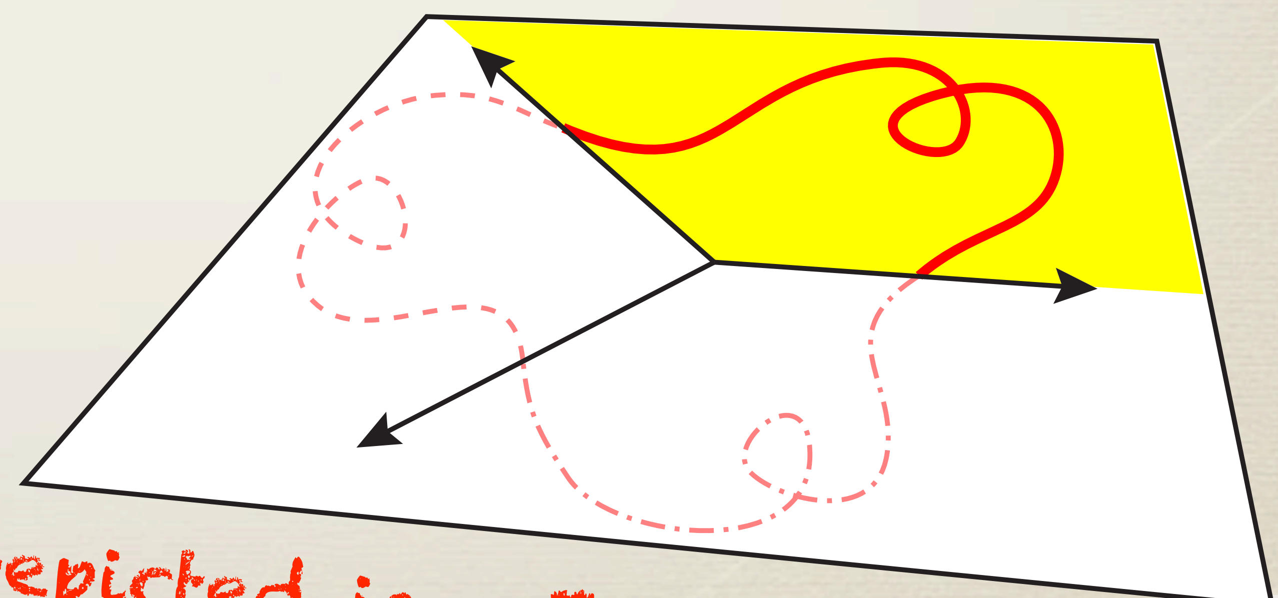

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How Mirrors are Made

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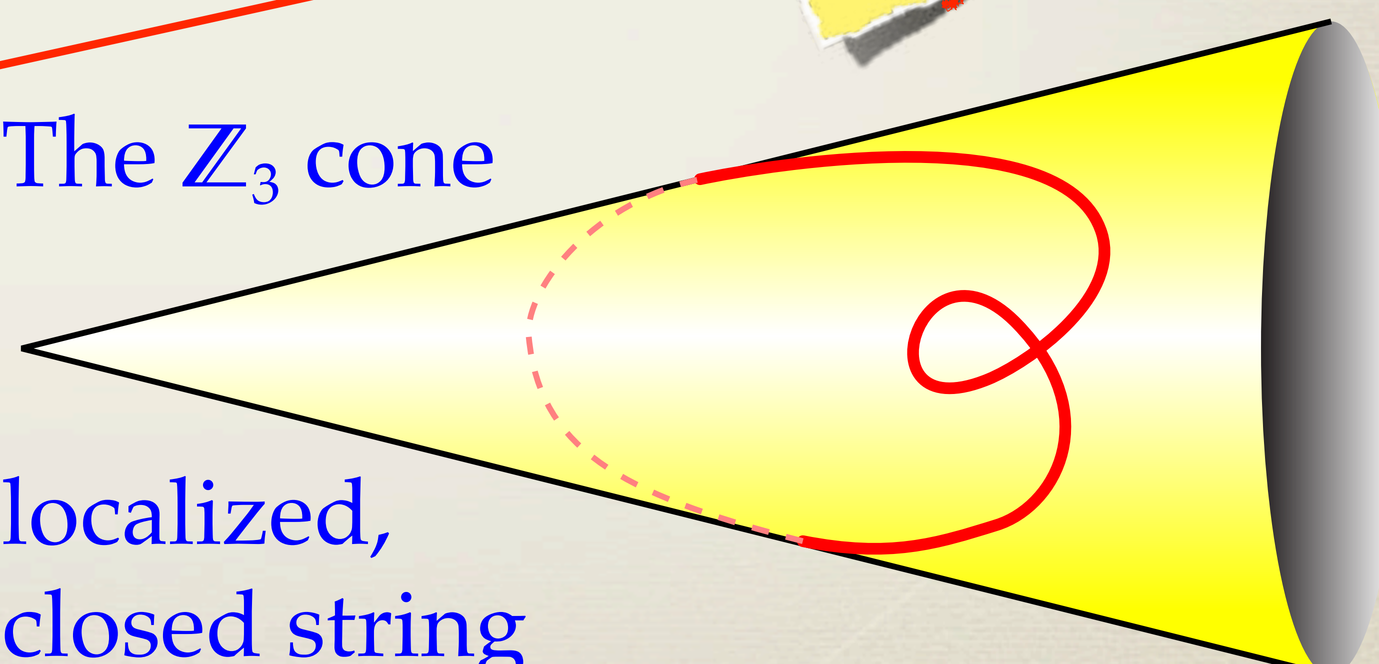
quotient ("quantum") symmetry

The $\mathbb{Z}_3$ cone



localized, closed string

How Mirrors are Made

cont'd

- String modes (that's all matter and all force fields!):
  - from closed strings (as before),
  - from open strings that close via quotient symmetry.
- Find a procedure so that the total Hilbert spaces
  - $\mathscr{H}(M; \boldsymbol{Sym}_M) = \mathscr{H}(W; \boldsymbol{Sym}_W)$, with states
    - symmetric (*identified*) w.r.t. ***Sym***,
    - generated by G, if $W = X/G$.
- Ha!

Find X with $\boldsymbol{Sym}_X = G{\times}H$, so:
$M = X/H$, with $\boldsymbol{Sym}_M = G$, &
$W = X/G$, with $\boldsymbol{Sym}_W = H$.

How Mirrors are Made

cont'd

- So, how to find X with $\boldsymbol{Sym}_X = G{\times}H$?
- By analyzing "building blocks"
  - z^k,
  - $z_1{}^k + z_1 \cdot z_2{}^m + \ldots + z_{p-1} \cdot z_p{}^n$,
- …one finds, one needs to *transpose* a minimal polynomial:
- If $f(z) = \sum_i \prod_j z_i{}^{a(i,j)}$, then $f^{\mathrm{T}}(z) = \sum_i \prod_j z_i{}^{a(j,i)}$.
  - All such $f(z) = 0$ "hypersurfaces" are smooth.
  - Then, $\boldsymbol{Sym}[f(z)] = \boldsymbol{Sym}[f^{\mathrm{T}}(z)] = F{\times}G{\times}H$.
  - Then, $M{:=}\{f(z) = 0\} / F{\times}G$ has $\boldsymbol{Sym}_M = H$,
  - and $W{:=}\{f^{\mathrm{T}}(z) = 0\} / F{\times}H$ has $\boldsymbol{Sym}_W = G$.

} mirror duals

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How Mirrors are Made

cont'd

- This is what one might call an experimental proof.
- That is, a proof in ***experimental mathematics***.
- That is, within a class of constructions:
  - most general, known, at the time,
  - define a large class of cases (16 × *unknown**),
  - and prove the duality relation rigorously.
- (It suffices to show that the states in the Hilbert spaces for candidate mirror duals transform in complementary ways w.r.t. $F \times G \times H$.)

Remember Wigner-Eckardt's theorem from QM?

*Eventually, a computer search (by others) found many thousands…

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How Mirrors are Made

cont'd

- And then…

…silence…


SOMEWHERE ELSE
THIS WAY
THAT WAY
Finding
One's Way

Where the Foundations Are

- And then, out of the blue…

...16 years later: (old enough for a driver's license)

ALESSANDRO CHIODO AND YONGBIN RUAN

http://arxiv.org/abs/0908.0908v2

ABSTRACT. We prove the *classical mirror symmetry conjecture* for the mirror pairs constructed by Berglund, Hübsch, and Krawitz. Our main

MARC KRAWITZ

http://arxiv.org/abs/0906.0796v1

ABSTRACT. In this article, we study the Berglund–Hübsch transpose construction W^T for invertible quasi-

BERGLUND-HÜBSCH MIRROR SYMMETRY VIA VERTEX ALGEBRAS

http://arxiv.org/abs/1007.2633v2

Cool title, dont you think?

Physics & Astronomy

Where the Foundations Are

cont'd

BERGLUND-HÜBSCH MIRROR SYMMETRY VIA VERTEX ALGEBRAS

LEV A. BORISOV

Abstract. We give a vertex algebra proof of the Berglund-Hübsch duality of nondegenerate invertible potentials. We suggest a way to unify it with the Batyrev-Borisov duality of reflexive Gorenstein cones.

http://arxiv.org/abs/1007.2633v2

from Lev A. Borisov's web-site

- The proof by Lev A. Borisov (Math Dept. @ Rutgers U.) uses the algebra of *vertex operators*. These represent the worldsheet localization of the states in (Fock-)Hilbert space of the given string theory model.
- Borisov also pinpoints the key structural differences between the BH-construction and the previous, "standard" construction of the previous 16 years, called the… …the Batyrev-*Borisov* construction.
- This permits a (now studied) unification of these two approaches.

Where the Foundations Are

cont'd

- And the moral to this story?

> If you do not expect the unexpected,
> you will not find it.
>
> — Aristotle

> Only dead fish swim with the stream.
>
> — Thomas Malcolm Muggeridge

> Time is the only judge worth any respect;
> 'xcept, I have no time to wait for it.
>
> — Yours Truly


Thank You!