Introspections on Quantum Introspection

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Quantum Introspections

Playbill

- Axioms and Assumptions
- Binary Breakdown?
- Critical Coaxing
- Division Drill
- Elusive Extensions
- Fusing Force-Fields
- Grand-Gödelian Guide
- …


BRAND CAMP
by Tom Fishburne
THE FIVE STAGES OF RECESSION
IT WILL BE A QUICK RECOVERY
DENIAL
MY PLANS ARE RUINED
ANGER
HOW THE HECK AM I SUPPOSED TO DO MORE WITH LESS?
BARGAINING
KEEP CALM AND CARRY ON
DEPRESSION
ACCEPTANCE
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“Whereof we cannot speak clearly, we must shut up.”
—w/apologies to L. Wittgenstein

Axioms and Assumptions

- In Quantum Mechanics:

  (1) Every state of a system ↔ a “state function,” $|\psi\rangle \in \mathcal{S}$

  (2) Every observable ↔ an operator, Q, acting on $|\psi\rangle$’s

  (3) Only eigenvalues of Q are results of any individual measurement

  (4) If $\mathsf{Q}|n\rangle = q_n|n\rangle$, then $\text{Prob}(\mathsf{Q}\mapsto q_n|\psi) = \frac{|\langle n|\psi\rangle|^2}{\langle n|n\rangle\langle\psi|\psi\rangle} =$ “$\cos^2(\theta_{n\psi})$”

  (5) $i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi\rangle = \mathsf{H}|\psi\rangle$ with H the Hamiltonian operator

  (6) $|\psi\rangle \xrightarrow{\mathsf{Q}\mapsto q_n} |n\rangle$

- *One more thing (😇):*

  (7) The Hamiltonian, H, is independent of $|\psi\rangle$

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Axioms and Assumptions

- BTW, "(7) $\mathsf{H} \neq \mathsf{H}(\psi)$" $\Rightarrow$ "(5) $i\hbar \frac{\mathrm{d}}{\mathrm{d}t} |\psi\rangle = \mathsf{H}|\psi\rangle$" a linear ODE
  - $\Rightarrow$ superposition, i.e., $\mathscr{S}$ is a linear vector space
- Implicit: $\langle\psi|\chi\rangle < \infty$, by "(4) $\mathrm{Prob}(\mathsf{Q} \mapsto q_n | \psi) = \frac{|\langle n|\psi\rangle|^2}{\langle n|n\rangle\langle\psi|\psi\rangle}$"
  - So, $\mathscr{S}$ is by (4) & for (7) a Hilbert space (…complete? …rigged? …)
  - …on which Q act as linear operators (…& bounded …)
- (1) lied (🤥): "mixed" states $\not\longleftrightarrow$ $|\psi\rangle$; $\boldsymbol{\rho} := \sum_i r_i |i\rangle\langle i|$
- Most Q are Hermitian; & other, if eigenvalues are real
- (5) $\Rightarrow$ probability conservation; *& particle decay?*
  & particle condensation? & particle creation?

Incomplete

Binary Breakdown?

- Measurement conundrum:
  - How exactly does "(6) $|\psi\rangle \xrightarrow{\mathsf{Q}\mapsto q_n} |n\rangle$" happen?
  - What, who, when, where, why (&how) …*measures*?
    - Collapse? Decoherence? Non-unitary? Non-local?
    - Parallel Universes? (combinatorial → transfinite?) *Mind? Friend? …?*
  - *"Be bothered sleepless, or have rocks in one's head"* [~D. Mermin/TH]

Bohr, Heisenberg, Wigner, ……

- The *transition* $\big(|\psi\rangle \to |n\rangle\big) \in \mathcal{S}$ is discontinuous…
- *Contradicts* $i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi\rangle = \mathsf{H}|\psi\rangle$, i.e., $|\psi(t)\rangle = \exp\{-\frac{1}{\hbar}\int_0^t \mathrm{d}\tau\,\mathsf{H}\}|\psi(0)\rangle$ *!!*

DZA

- Of course, not — $\mathsf{H}=\mathsf{H}(t)$ controls the t-discontinuity

Binary Breakdown?

- OK, $|\psi(t)\rangle = \exp\{-\frac{i}{\hbar}\int_0^t d\tau\, \mathsf{H}(\tau)\}|\psi(0)\rangle$ *can* be discontinuous
  - "quantum jumps" — including the "collapse" in measurements
  - as *modeled* by the *choice* of $\mathsf{H}(t)$ & by the choice of BC
- So, how *do* we choose/model $\mathsf{H}(t)$?
  - Typically, $\mathsf{H}(t) = H_0 + V(t),\ H_0 = KE + V_0$
  - Only " $KE = \frac{1}{2m}\mathsf{P}^2$ " is *intrinsic* to the quantum system at hand
  - Both V_0 & $V(t)$ encode interaction with its *environment*
    - Like "potential well," "Coulomb potential," …
    - ……which is not quantal, but is treated as if classical

Binary Breakdown?

- OK, $|\psi(t)\rangle = \exp\{-\frac{1}{\hbar}\int_0^t d\tau\, \mathsf{H}(\tau)\}|\psi(0)\rangle$ <u>*can*</u> be discontinuous
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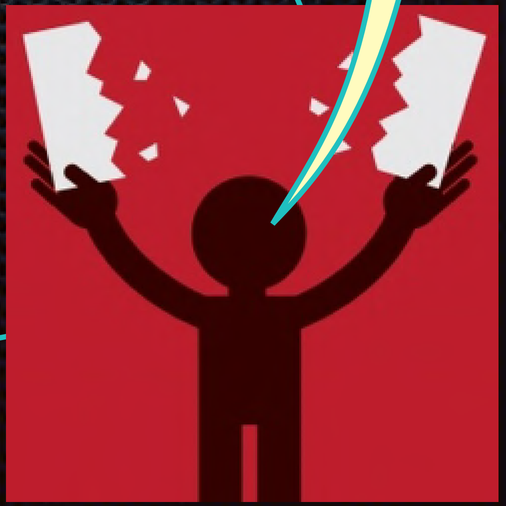
    - ……which is not quantal, but is treated as if classical

I have seen the problem. — It is us.



Binary Breakdown?

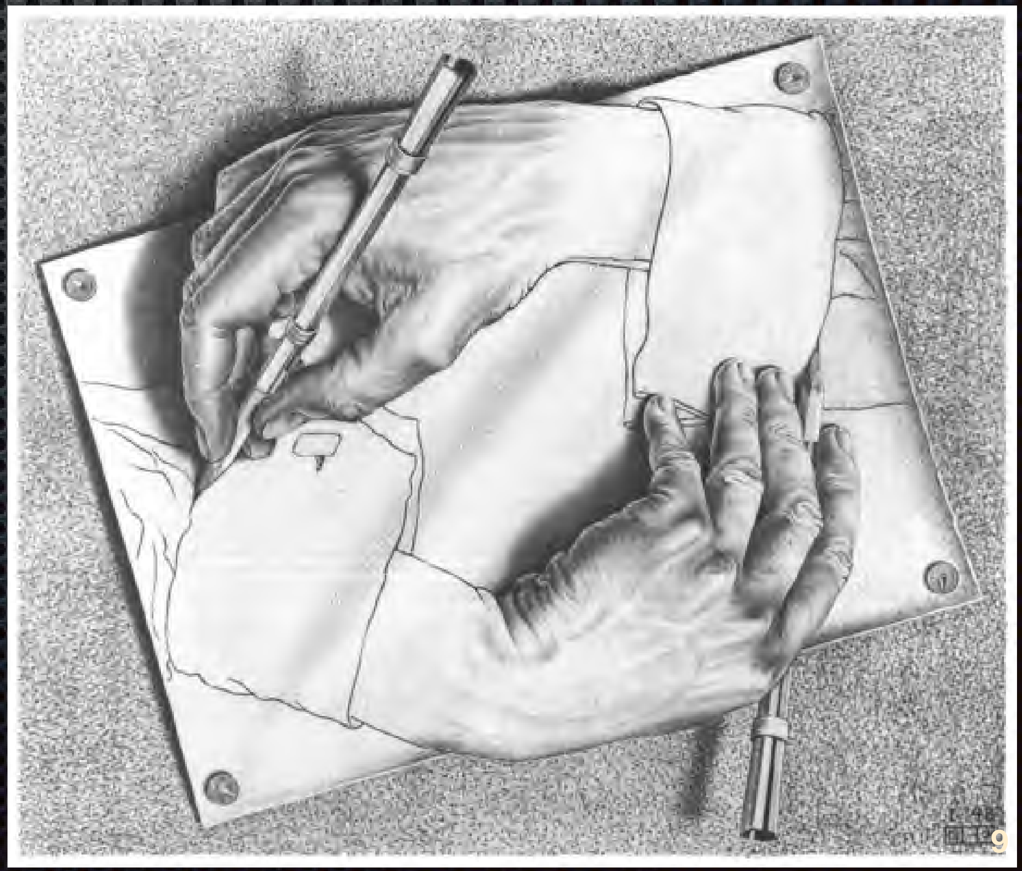
- So, $|\psi(t)\rangle = \exp\{-\frac{i}{\hbar}\int_0^t \mathrm{d}\tau\, \mathsf{H}(\tau)\}|\psi(0)\rangle$ <u>*can*</u> have solutions
  - and <u>*can*</u> model the discontinuous change in $|\psi(t)\rangle$ under the influence…
  - …of a measuring device — represented by the "external" $V(t)$
- How precisely *does* discontinuous change occur?
  - The "quantized" subsystem $\mapsto |\psi(t)\rangle$
  - The "classical" subsystem = environment/device/…
  - but *…just how do "quantum" and "classical" interact?*
    (since they are inherently diametrically different)
    - *Reminds of the unobservable "Führungsfeld" (pilot-wave)*
    - *that encodes all quantum interference & guides observable particles*
    - *by way of a never specified coupling/interaction to it (& which one?)*

…I'll be back!

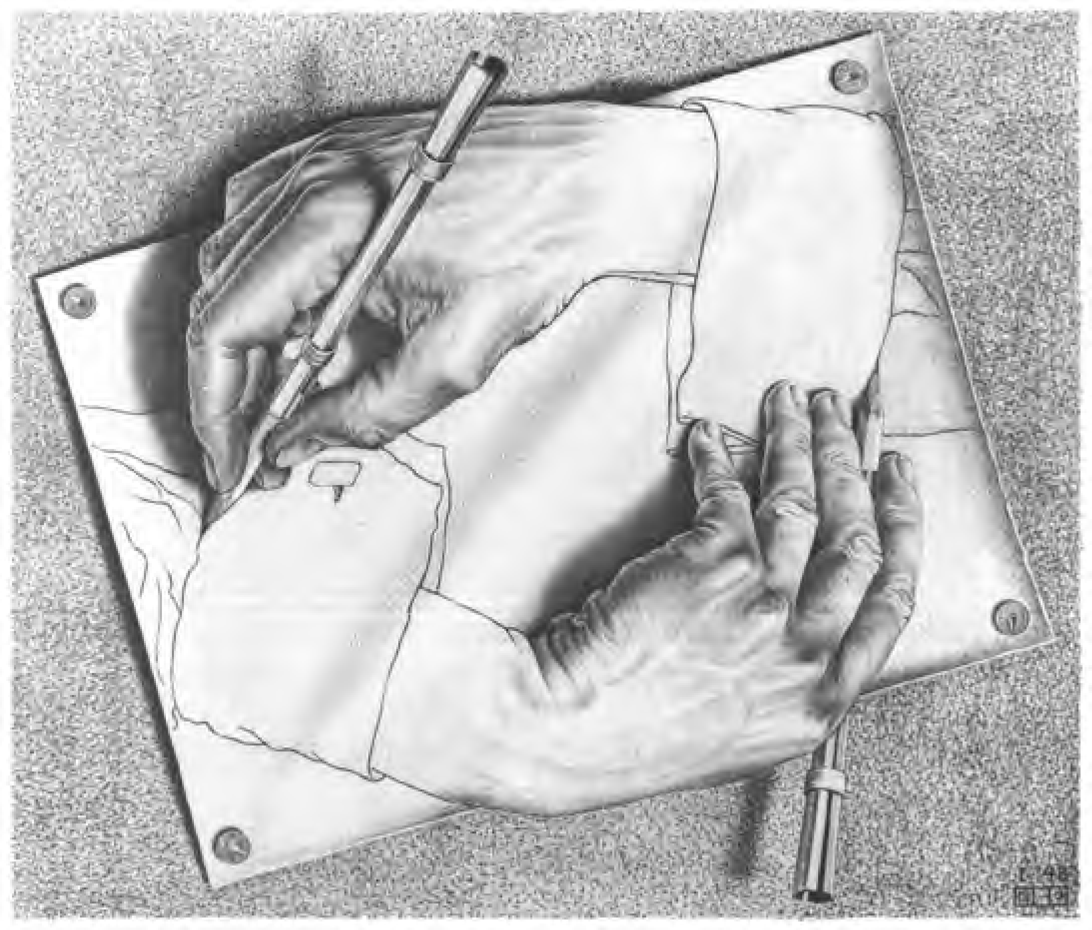
…getting our hands dirty…

Critical Coaxing

- "Let's treat the environment *also* quantum-ly!"
  - If the "object" $\mapsto |\psi(t)\rangle$, its "environment" $\mapsto |\phi(t)\rangle$
  - Toy model: $i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi\rangle = V_M|\psi\rangle$, V_M = (classical) measuring device
  - …and *vice versa*: $i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\phi\rangle = \Lambda_M|\phi\rangle$, $|\phi\rangle$ = state of the device
  - …but then $V_M = V_M(\phi)$ and $\Lambda_M = \Lambda_M(\psi)$
  - Coarsely: $|\psi\rangle \mapsto \psi(\ldots, t)$
  - so: $$i\hbar\frac{\mathrm{d}\psi}{\mathrm{d}t} = (a_0 + a_1\phi + \ldots)\,\psi$$ $$i\hbar\frac{\mathrm{d}\phi}{\mathrm{d}t} = (b_0 + b_1\psi + \ldots)\,\phi$$



Critical Coaxing



- This is a coupled system of t-ODEs

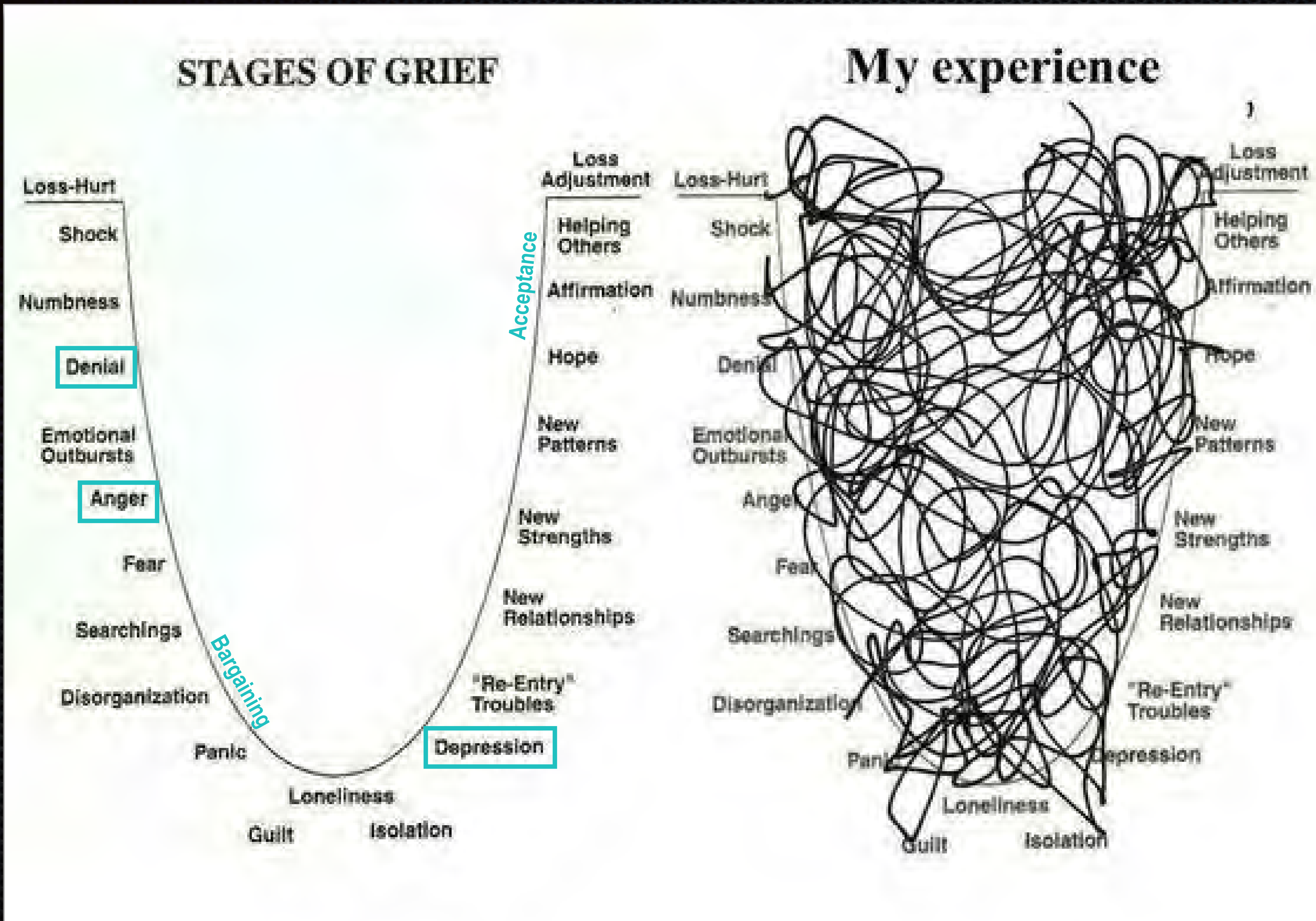
$$i\hbar\frac{\mathrm{d}\psi}{\mathrm{d}t} = \underbrace{(a_0 + a_1\phi + \ldots)}_{V_M(\phi)}\psi$$

$$i\hbar\frac{\mathrm{d}\phi}{\mathrm{d}t} = \underbrace{(b_0 + b_1\psi + \ldots)}_{\Lambda_M(\psi)}\phi$$

- Toy model (coarse): *nonlinearly* coupled object-measurer interaction
- So, $\ddot{\psi} = a_0\left(\frac{b_0}{\hbar^2} + \frac{\dot{a}_0}{i\hbar\, a_0} - \frac{\dot{a}_1}{i\hbar a_1}\right)\psi + \left(\frac{\dot{a}_1}{a_1} + \frac{b_0}{i\hbar}\right)\dot{\psi}$

$$+\frac{a_0 b_1}{\hbar^2}\psi^2 + \frac{b_1}{i\hbar}\psi\dot{\psi} + \frac{\dot{\psi}^2}{\psi} + \ldots$$

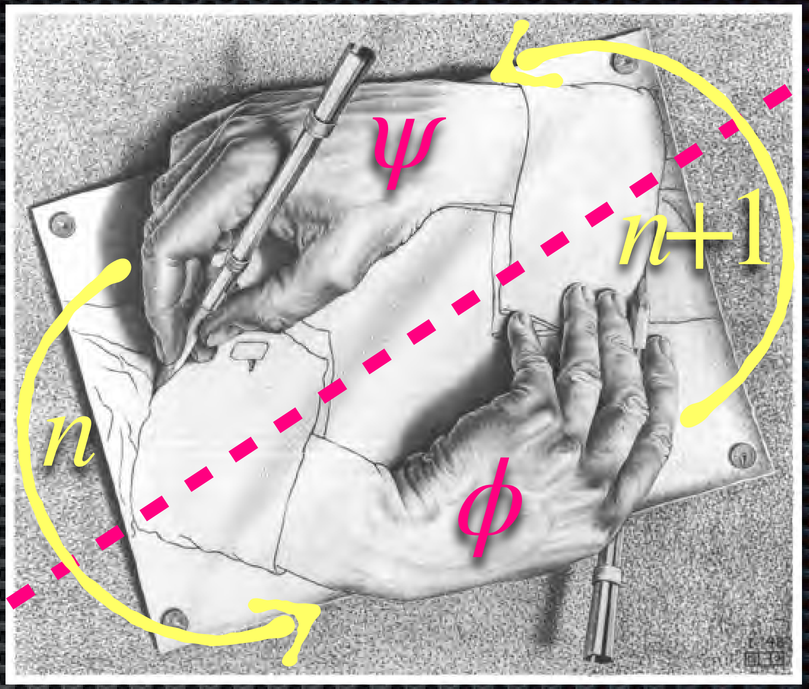
- "QM is Either Non-Linear or Non-Introspective" [quant-ph:9712047]



- “QM is either Non-Linear or Non-Introspective” [quant-ph:9712047]

→ Mod. Phys. Lett. A13 (1998) 2503–2512

Division Drill



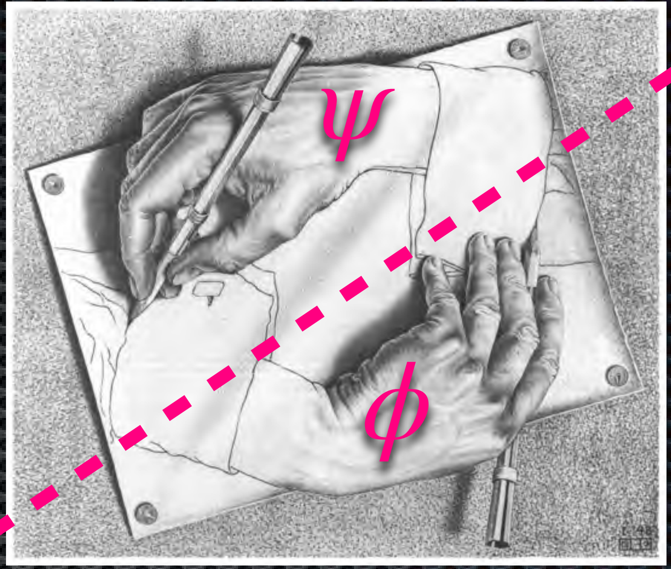
- In fact, <u>*we do "this" all the time!*</u>
  - H_2^+: $2p^+$ (@ $\vec{0}$ & $\vec{R}$) + $1e^-$ (at $\vec{r}$)
  - Fix the p^+'s: $\left[-\frac{\hbar^2}{2m_e}\vec{\nabla}^2 - \frac{e^2}{4\pi\epsilon_0}\left(\frac{1}{r}+\frac{1}{\sqrt{r^2+R^2-2rR\cos\theta}}\right)\right]\psi(\vec{r}) = E_e\,\psi(\vec{r})$
  - solve for $\psi(\vec{r})$ with a ~~"tentative" R.~~ replace
  - Compute the e^- charge density: $-e\,|\psi(\vec{r})|^2 \mapsto V_e(\vec{r}) = \frac{-e^2}{4\pi\epsilon_0}\int d^2\vec{r}\,'\frac{|\psi(\vec{r}\,')|^2}{|\vec{r}-\vec{r}\,'|}$
    - This charge density yields a potential in which the $2p^+$ float
  - That is, now fix the e^-: $\left[-\frac{\hbar^2}{2m_p}\vec{\nabla}^2_{\vec{R}} + \frac{2e^2}{4\pi\epsilon_0}\frac{1}{|\vec{R}|} + V_e(\vec{R})\right]\phi(\vec{R}) = E_p\,\phi(\vec{R})$
  - solve for $|\phi\rangle$. Compute $R := |\langle\phi|\vec{R}|\phi\rangle|$

Go to 1

- Iteratively solving mutually interactive ("observing") system

…getting our hands really dirty…

Division Drill



- So, either
  - A nonlinearly coupled system of two equations

$$\left[-\frac{\hbar^2}{2m_e}\vec{\nabla}^2 - \frac{e^2}{4\pi\epsilon_0}\left(\frac{1}{r}+\frac{1}{\sqrt{r^2+R^2-2rR\cos\theta}}\right)\right]\psi(\vec{r})=E_e\,\psi(\vec{r})$$

$$\left[-\frac{\hbar^2}{2m_p}\vec{\nabla}^2_{\vec{R}} + \frac{2e^2}{4\pi\epsilon_0}\frac{1}{|\vec{R}|}+V_e(\vec{R})\right]\phi(\vec{R})=E_p\,\phi(\vec{R})$$

n $\quad$ $n+1$

  - solved iteratively
- Or, a “unified” description:

(Escher’s view)

$$-\left[\frac{\hbar^2}{2m_e}\vec{\nabla}_e^2+\frac{\hbar^2}{2m_p}\left(\vec{\nabla}_a^2+\vec{\nabla}_b^2\right) + \frac{e^2}{4\pi\epsilon_0}\left(\frac{1}{|\vec{r}_e-\vec{r}_a|}+\frac{1}{|\vec{r}_e-\vec{r}_b|}-\frac{1}{|\vec{r}_a-\vec{r}_b|}\right)\right]\Psi(\vec{r}_e,\vec{r}_a,\vec{r}_b)=E_M\Psi(\vec{r}_e,\vec{r}_a,\vec{r}_b)$$

“Shut up and calculate.”
—D. Mermin

Insider View

“…nobody really understands quantum mechanics.”
—R.P. Feynman

- *Who, what, when, where …is doing the measuring?*
- *How & why does superposition …break?*

Elusive Extensions

Bohr, Heisenberg, Wigner, Everett, DeWitt, …… 👻

- Observation irreversibly collapses (*dis*-superposes) the state
  - The observometer? The human viewer? Her friend? Twice removed?
- Objective-collapse by …nonlinearities and/or mesoscale dynamics
  - Gravity? Spontaneous localization? Origin of the random noise?
- Environmental decoherence by *statistically many* scattering events
  - Quantum Prob → Classical Prob, but still linear QM & superposition?
- Incessant splitting of (increasingly) many worlds
  - Coexistent with decoherence, but indistinguishable from Copenhagen?

Angelo Bassi et al.: "*Models of Wave-function Collapse, Underlying Theories, and Experimental Tests*" *Rev. Mod. Phys.* **85** (2013) 471–527, arXiv:1204.4325

- *Who, what, when, where …is doing the measuring?*
- *How & why does superposition …break?*

Elusive Extensions

Bohr, Heisenberg, Wigner, Everett, DeWitt, …… 👻

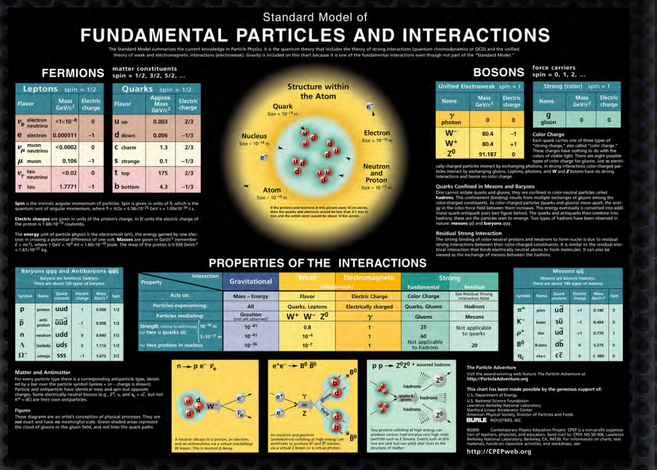
- Modifications of QM …often include nonlinearities
  - Tend to be superluminal? Stochastic prob. distribution *ad hoc*?
- Hidden variables… Trace Dynamics (Themodynamical QM)…
  - Nonlocality? Superluminal? Grassmann matrices? (→ M-Theory?)
- Spacetime: emergent? smeared? non-commutative?
  - Heisenberg? (incl. "Loop Quantum Gravity"?)
- Unseen Führungsfeld (pilot-wave) *guides* observable particles
  - By what interaction? What mediates that? Is that particle-specific?
    (How do the *various Führungfeldern* interact? …& avoid other particles?)

"Quantum Field Theory already is
what Führungsfeldtheorie wants to become when it grows up."

(Pilot-wave theory)

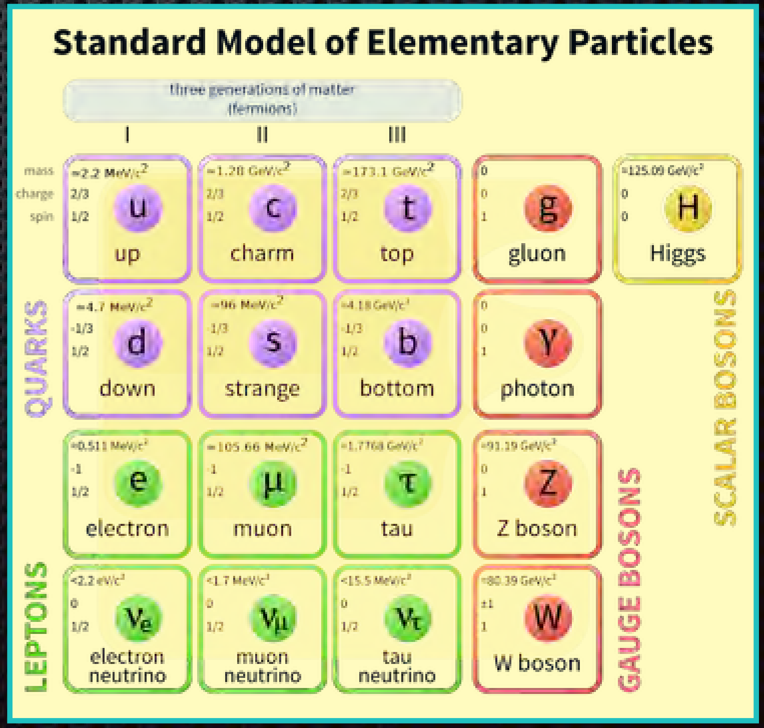
originally so-named by Max Born in 1926

Fusing Force-Fields



- 1972 S. Freedman & J. Clauser's 1st experiment violating Bell('64) Clauser-Horne-Shimony-Holt ('69) inequalities; A. Aspect('81)…
- 1974: The Standard Model
  - *quantum consistency*
  - 1969 Adler-Bell-Jackiw & 1970 Glashow-Iliopoulos-Maiani → charm
  - 1974, Ting & Richter: $J/\psi = [c\bar{c}]$; "November revolution"
  - A concrete QFT model involving
  - *Renormalizable* non-abelian quantum gauge theory *fundamental forces*
  - …with spontaneous symmetry breaking
  - …experimentally detected 2011–13 (Higgs boson)

`…I'll be back!`



Standard Model of

FUNDAMENTAL PARTICLES AND INTERACTIONS

The Standard Model summarizes the current knowledge in Particle Physics. It is the quantum theory that includes the theory of strong interactions (quantum chromodynamics or QCD) and the unified theory of weak and electromagnetic interactions (electroweak). Gravity is included on this chart because it is one of the fundamental interactions even though not part of the "Standard Model."

FERMIONS

matter constituents
spin = 1/2, 3/2, 5/2, ...

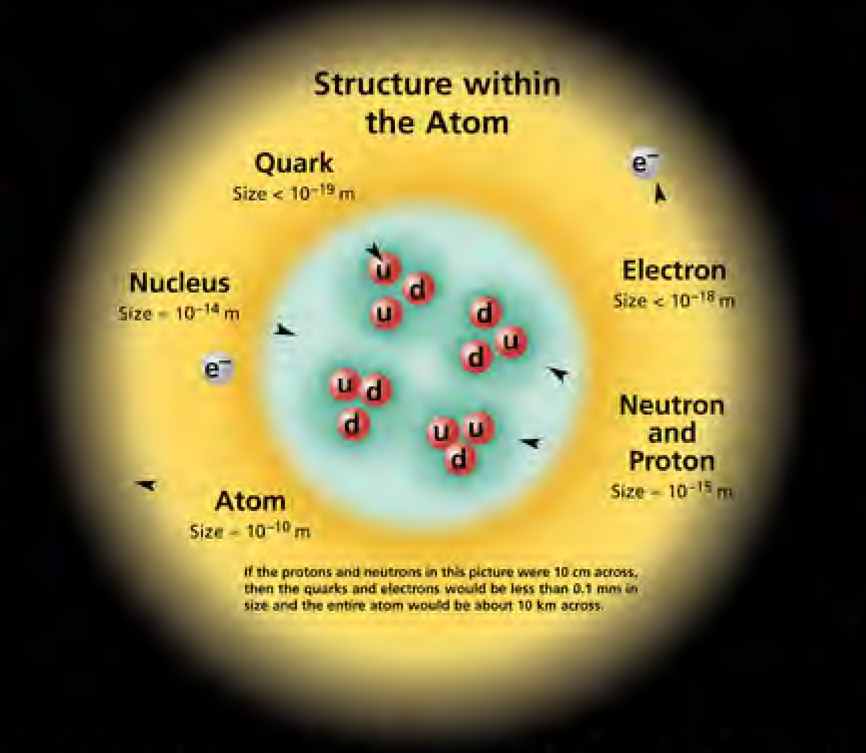
Leptons spin = 1/2		
Flavor	Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	–1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	–1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	–1

Quarks spin = 1/2		
Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.003	2/3
d down	0.006	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	175	2/3
b bottom	4.3	–1/3

Spin is the intrinsic angular momentum of particles. Spin is given in units of $\hbar$, which is the quantum unit of angular momentum, where $\hbar = h/2\pi = 6.58\times10^{-25}$ GeV s $= 1.05\times10^{-34}$ J s.

Electric charges are given in units of the proton's charge. In SI units the electric charge of the proton is 1.60×10^{-19} coulombs.

The **energy** unit of particle physics is the electronvolt (eV), the energy gained by one electron in crossing a potential difference of one volt. **Masses** are given in GeV/c^2 (remember $E = mc^2$), where 1 GeV = 10^9 eV = 1.60×10^{-10} joule. The mass of the proton is 0.938 GeV/c^2 = 1.67×10^{-27} kg.



If the protons and neutrons in this picture were 10 cm across, then the quarks and electrons would be less than 0.1 mm in size and the entire atom would be about 10 km across.

BOSONS

force carriers
spin = 0, 1, 2, ...

Unified Electroweak spin = 1		
Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.4	–1
W^+	80.4	+1
Z^0	91.187	0

Strong (color) spin = 1		
Name	Mass GeV/c^2	Electric charge
g gluon	0	0

Color Charge

Each quark carries one of three types of "strong charge," also called "color charge." These charges have nothing to do with the colors of visible light. There are eight possible types of color charge for gluons. Just as electrically-charged particles interact by exchanging photons, in strong interactions color-charged particles interact by exchanging gluons. Leptons, photons, and ***W*** and ***Z*** bosons have no strong interactions and hence no color charge.

Quarks Confined in Mesons and Baryons

One cannot isolate quarks and gluons; they are confined in color-neutral particles called **hadrons**. This confinement (binding) results from multiple exchanges of gluons among the color-charged constituents. As color-charged particles (quarks and gluons) move apart, the energy in the color-force field between them increases. This energy eventually is converted into additional quark-antiquark pairs (see figure below). The quarks and antiquarks then combine into hadrons; these are the particles seen to emerge. Two types of hadrons have been observed in nature: **mesons** $q\bar{q}$ and **baryons** qqq.

Residual Strong Interaction

The strong binding of color-neutral protons and neutrons to form nuclei is due to residual strong interactions between their color-charged constituents. It is similar to the residual electrical interaction that binds electrically neutral atoms to form molecules. It can also be viewed as the exchange of mesons between the hadrons.

PROPERTIES OF THE INTERACTIONS

Property \ Interaction	Gravitational	Weak (Electroweak)	Electromagnetic (Electroweak)	Strong: Fundamental	Strong: Residual
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge	See Residual Strong Interaction Note
Particles experiencing:	All	Quarks, Leptons	Electrically charged	Quarks, Gluons	Hadrons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons	Mesons
Strength relative to electromag for two u quarks at: 10^{-18} m	10^{-41}	0.8	1	25	Not applicable to quarks
Strength relative to electromag for two u quarks at: 3×10^{-17} m	10^{-41}	10^{-4}	1	60	
for two protons in nucleus	10^{-36}	10^{-7}	1	Not applicable to hadrons	20

Baryons qqq and Antibaryons $\bar{q}\bar{q}\bar{q}$ Baryons are fermionic hadrons. There are about 120 types of baryons.					
Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
p	proton	uud	1	0.938	1/2
$\bar{p}$	anti-proton	$\bar{u}\bar{u}\bar{d}$	–1	0.938	1/2
n	neutron	udd	0	0.940	1/2
Λ	lambda	uds	0	1.116	1/2
Ω^-	omega	sss	–1	1.672	3/2

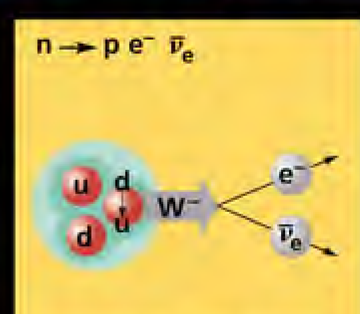
Mesons $q\bar{q}$ Mesons are bosonic hadrons. There are about 140 types of mesons.					
Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
π^+	pion	$u\bar{d}$	+1	0.140	0
K^-	kaon	$s\bar{u}$	–1	0.494	0
ρ^+	rho	$u\bar{d}$	+1	0.770	1
B^0	B-zero	$d\bar{b}$	0	5.279	0
η_c	eta-c	$c\bar{c}$	0	2 .980	0

Matter and Antimatter

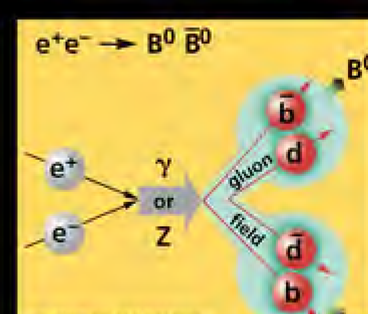
For every particle type there is a corresponding antiparticle type, denoted by a bar over the particle symbol (unless + or – charge is shown). Particle and antiparticle have identical mass and spin but opposite charges. Some electrically neutral bosons (e.g., Z^0, γ, and $\eta_c = c\bar{c}$, but not $K^0 = d\bar{s}$) are their own antiparticles.

Figures

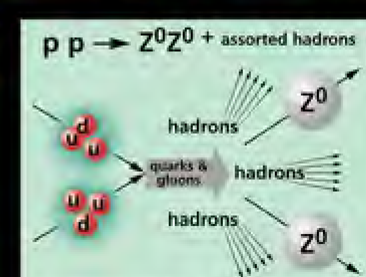
These diagrams are an artist's conception of physical processes. They are ***not*** exact and have ***no*** meaningful scale. Green shaded areas represent the cloud of gluons or the gluon field, and red lines the quark paths.



A neutron decays to a proton, an electron, and an antineutrino via a virtual (mediating) W boson. This is neutron β decay.



An electron and positron (antielectron) colliding at high energy can annihilate to produce B^0 and $\bar{B}^0$ mesons via a virtual Z boson or a virtual photon.



Two protons colliding at high energy can produce various hadrons plus very high mass particles such as Z bosons. Events such as this one are rare but can yield vital clues to the structure of matter.

The Particle Adventure

Visit the award-winning web feature *The Particle Adventure* at
http://ParticleAdventure.org

This chart has been made possible by the generous support of:
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Lawrence Berkeley National Laboratory
Stanford Linear Accelerator Center
American Physical Society, Division of Particles and Fields
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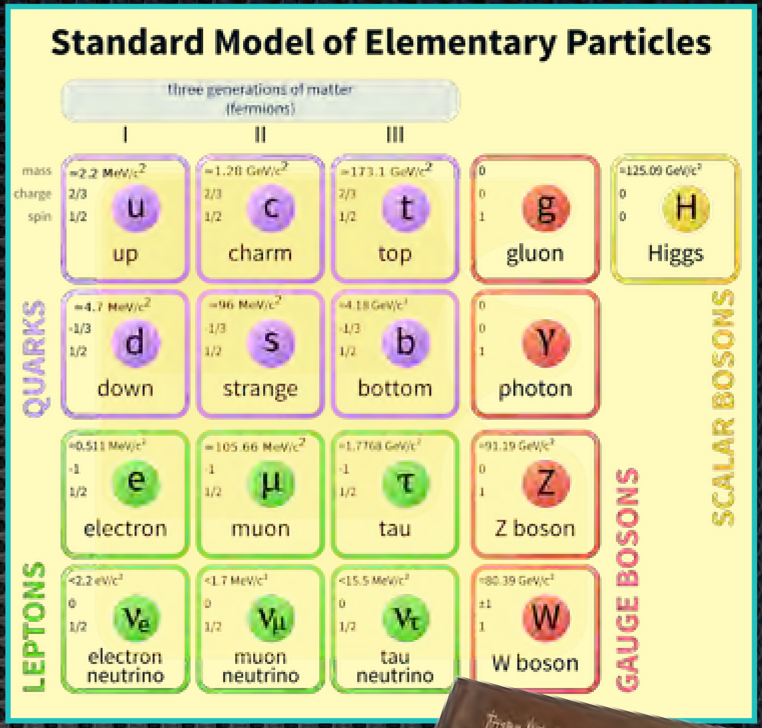
"Quantum Field Theory already is what Führungsfeldtheorie wants to become when it grows up."

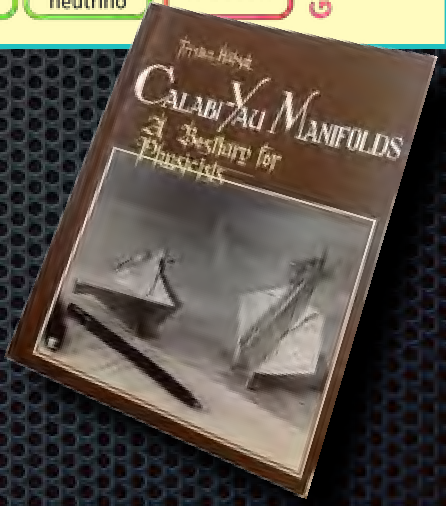
Fusing Force-Fields

- Paraphrasing "The pilot-wave interpretation of quantum mechanics is wishful thinking that aims to achieve what quantum field theory has been for the past half a century." [Quora answers: 1 & 2]
- Quantum fields exist continuously throughout spacetime
- Change in the field is quantized → [particle, …packet,… wave]
  - *Fermions*: Pauli-exclude from condensing → "background Fermi field" = ∅
  - *Bosons*: can condense (in any particular one state) → "background Bose field"
- Very large ensemble ("sea") of quanta comprises the continuous field
- A field and its quanta do not *interact* with each other…
  - …they are part and parcel of *the same entity*

(no need for velcro)

Fusing Force-Fields



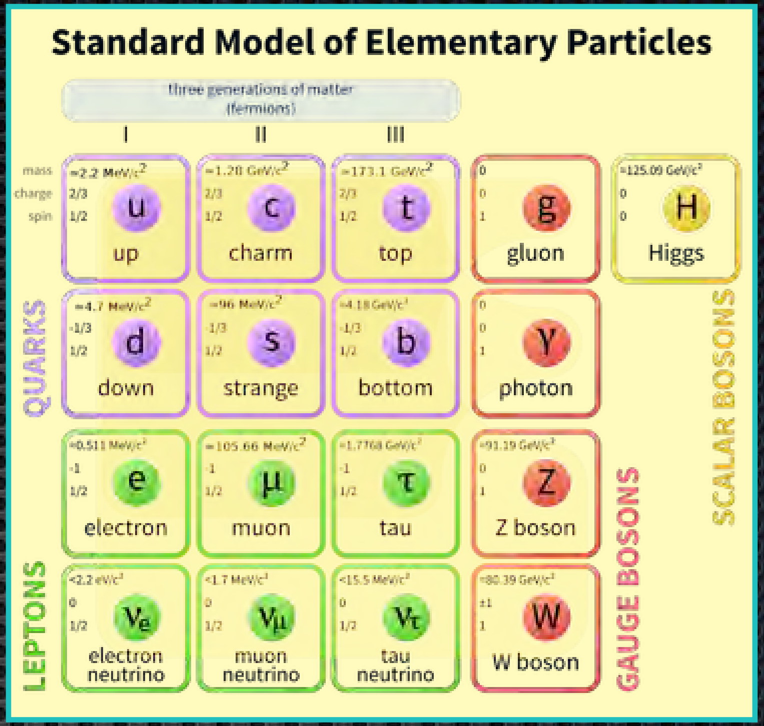


- 1974: (Super)string theory = theory of gravity
  - A finite inherently quantum theory
  - with gauge fields and fermions and (quantum!) gravity
  - 1979 (D. Friedan): *quantum stability* ⇒ Einstein equations
  - 1984 (Alvarez-Gaumé &Witten): *gravitational anomalies* & cancellation
  - 1984 (Green-Schwarz): one more *anomaly cancellation*

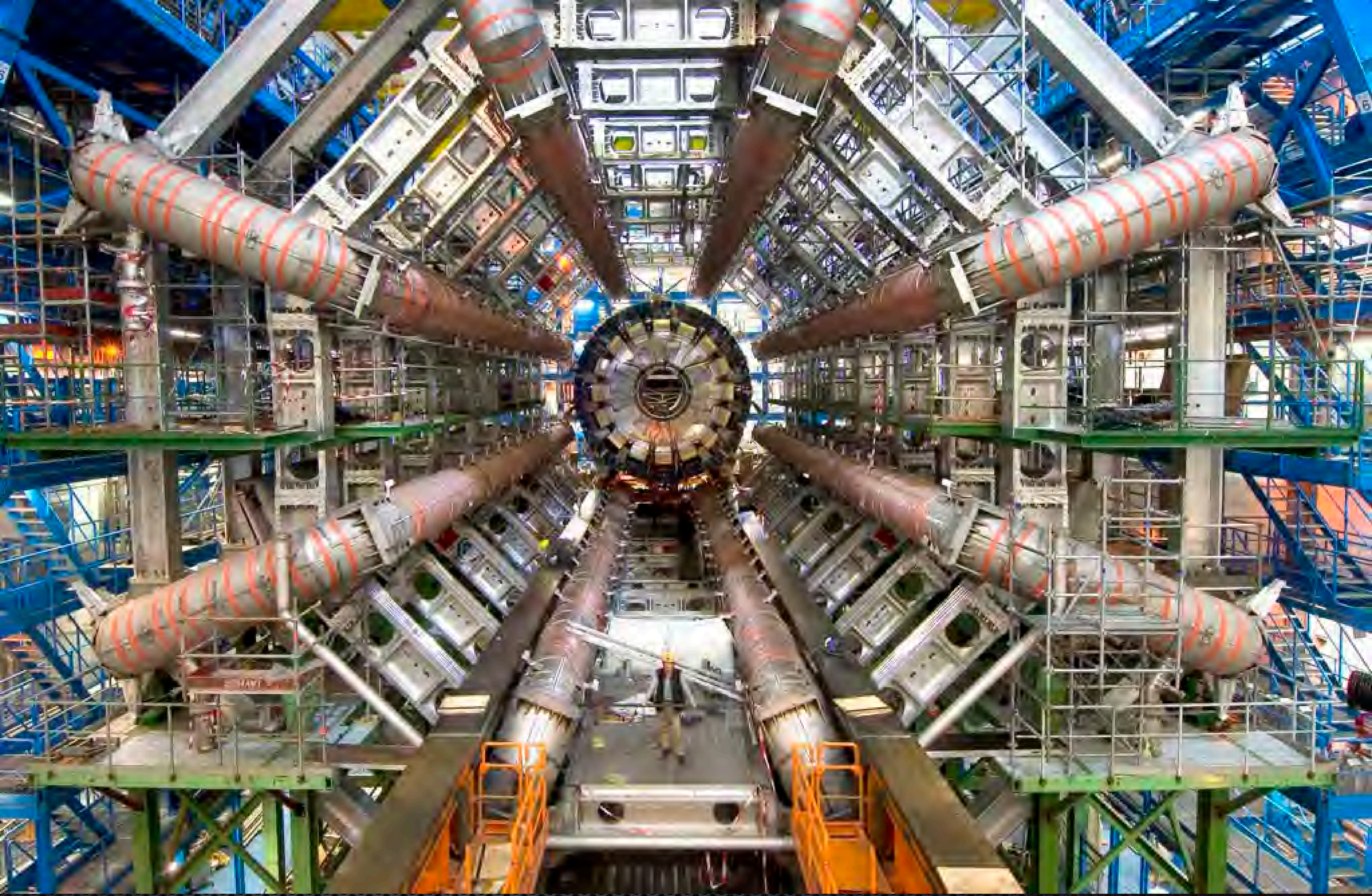
    quantum consistency
  - 1984 (GHMR): heterotic string model with $E_8 \times E_8$ gauge fields
  - 1984 (CHSW): supersymmetric stability ⇒ Calabi-Yau compactification
  - 1984 me@UMD 1987: $O(10^3)$ models; → UT: ’88 connected web

    connected by phase transitions

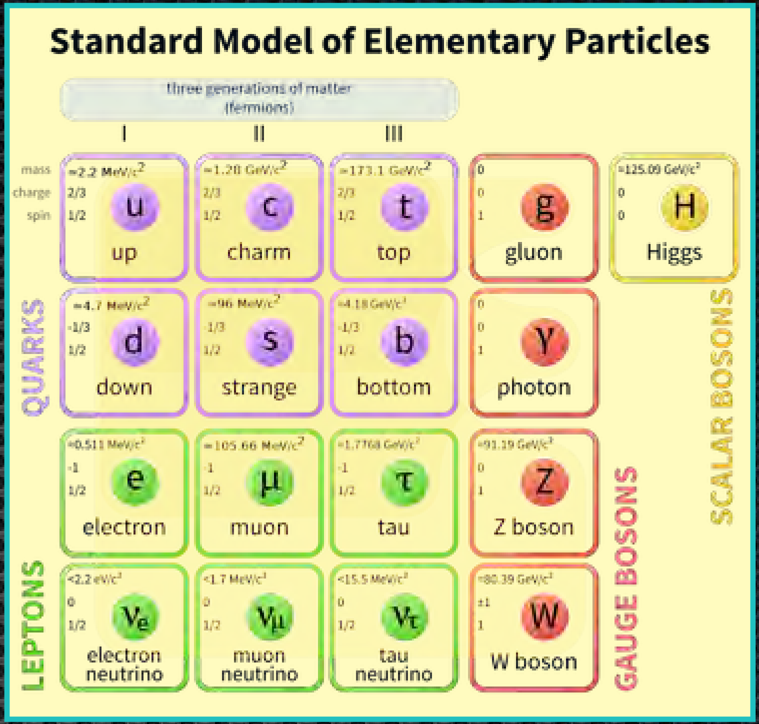
Fusing Force-Fields



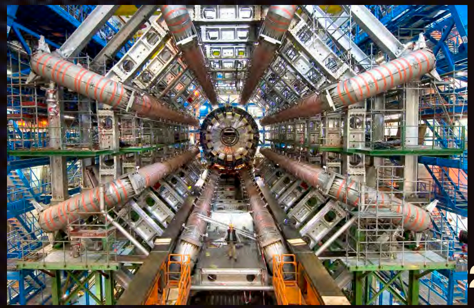
- Quantum Field Theory
  - A *framework* to construct concrete *models*
  - The Standard Model has ~26 continuous parameters & *many* other choices
- What is it that *is being* observed/measured?
  - Such as in the 2011–13 discovery of the Higgs boson?

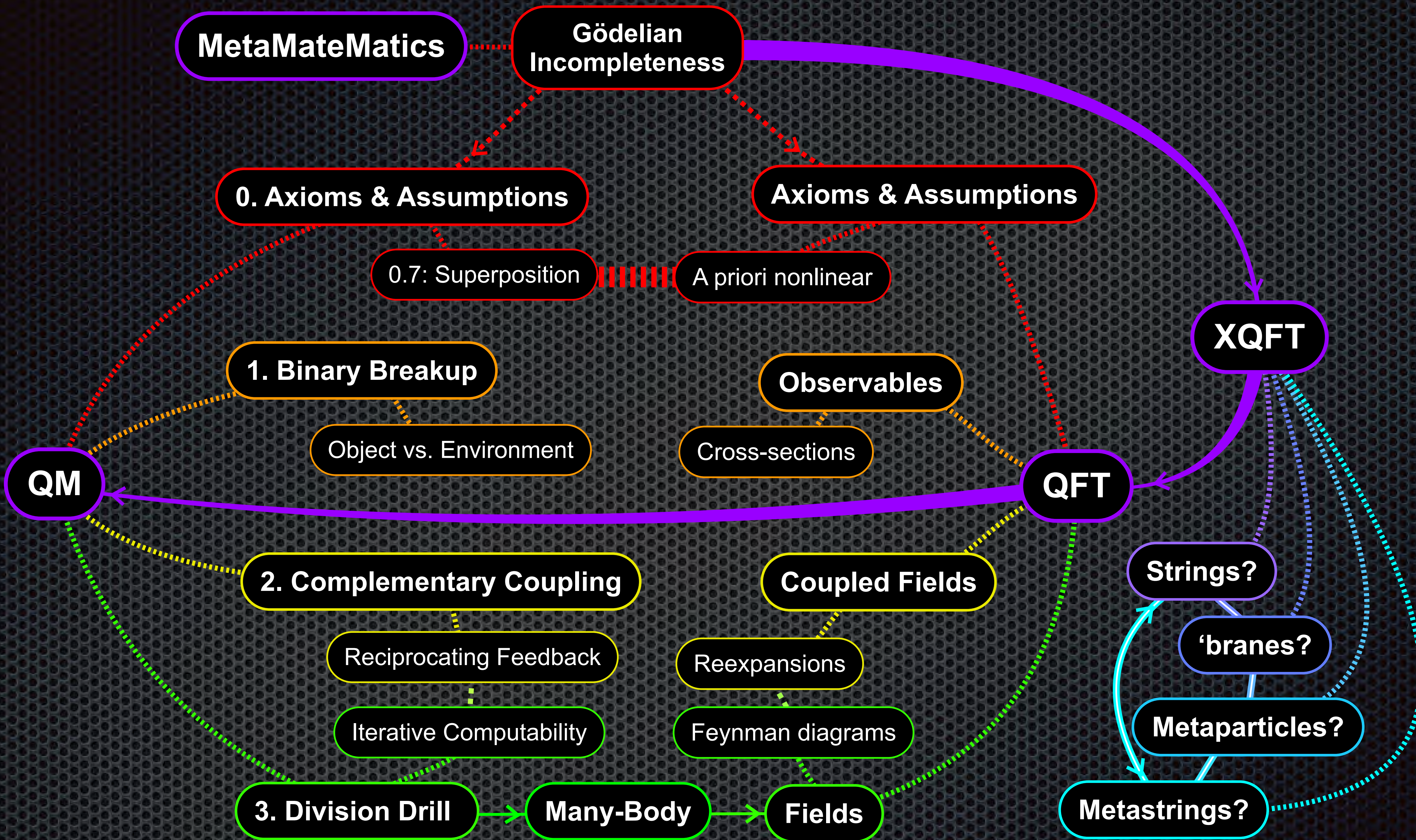


Fusing Force-Fields



- Quantum Field Theory
  - A *framework* to construct concrete *models*
  - The Standard Model has ~26 continuous parameters & *many* other choices
  - The QFT framework is axiomatic (Wightman; Osterwalder-Schrader; Haag-Kastler; …)
- What is it that *is being* observed/measured?
  - Such as in the 2011–13 discovery of the Higgs boson?
  - Terabytes upon terabytes of data from hierarchically triggered and computer-controlled layers of tons of detectors …stored until data-mined
  - Relative process amplitudes: common factors cancel (~Wigner-Eckart) and are largely …irrelevant





The Big Picture

Grand-Gödelian Guide

- QM: 6 (oft-quoted) axioms & (7) $\mathsf{H} \neq \mathsf{H}(\psi)$ in $i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi\rangle = \mathsf{H}|\psi\rangle$
  - (7) ⇒ linear superposition ⇒ quantum entanglement/non-factorizability
  - But the division drill ⇒ nonlinearity ⇒ no superposition
  - which is then, logically, the "axiom (¬7)"
  - The six axioms thereby imply neither "axiom (7)" nor "axiom (¬7)"

— a Gödelian incompleteness!!

(1) Every state of a system ↔ a "state function," $|\psi\rangle \in \mathcal{S}$

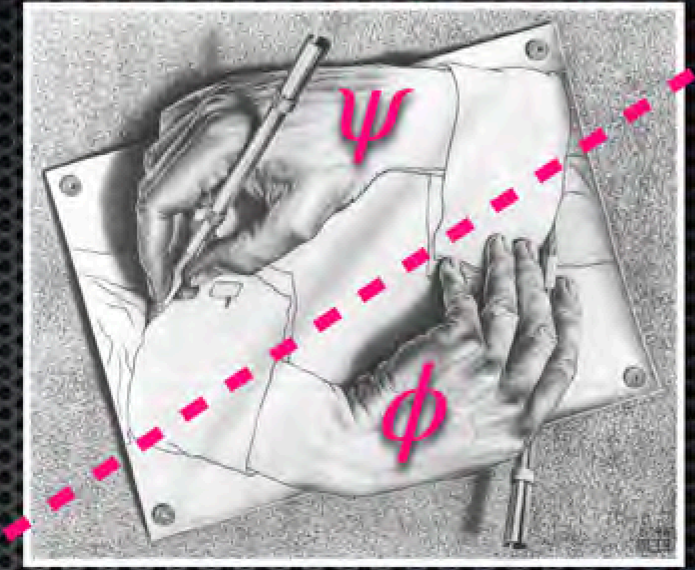
(2) Every observable ↔ an operator, Q, acting on $|\psi\rangle$'s

(3) Only eigenvalues of Q are results of any individual measurement

(4) If $\mathsf{Q}|n\rangle = q_n|n\rangle$, then $\mathrm{Prob}(\mathsf{Q}\mapsto q_n|\psi) = \frac{|\langle n|\psi\rangle|^2}{\langle n|n\rangle\langle\psi|\psi\rangle} =$ "$\cos^2(\theta_{n\psi})$"

(5) $i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi\rangle = \mathsf{H}|\psi\rangle$ with H the Hamiltonian operator

(6) $\Pi(\mathsf{Q}\mapsto q_n):\ |\psi\rangle \to |n\rangle$



Division Drill

- So, either
  - A nonlinearly coupled system of two equations

$$-\left[\frac{\hbar^2}{2m_e}\vec{\nabla}^2 + \frac{e^2}{4\pi\epsilon_0}\left(\frac{1}{r}+\frac{1}{\sqrt{r^2+R^2-2rR\cos\theta}}\right)\right]\psi(\vec{r})=E_e\,\psi(\vec{r})$$

$$-\left[\frac{\hbar^2}{2m_p}\vec{\nabla}^2_{\vec{R}} + \frac{2e^2}{4\pi\epsilon_0}\frac{1}{|\vec{R}|} + V_e(\vec{R})\right]\phi(\vec{R})=E_p\,\phi(\vec{R})$$

n

$n+1$

  - solved iteratively

- Or, a combined description:

(Escher's view)

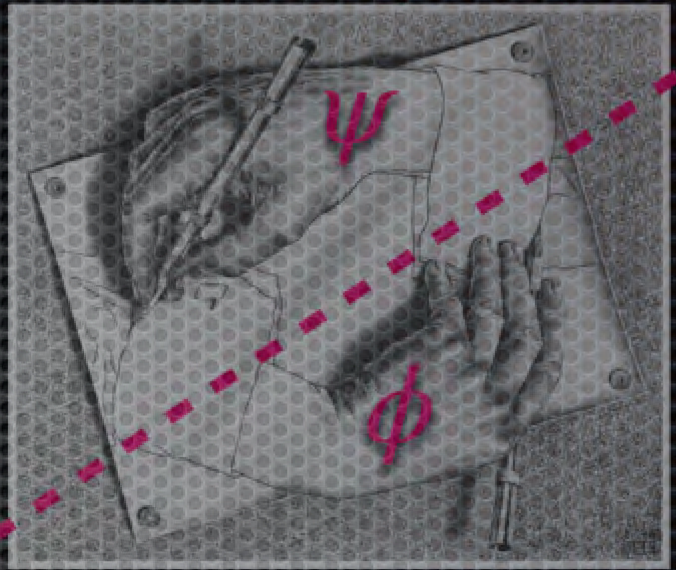
$$-\left[\frac{\hbar^2}{2m_e}\vec{\nabla}_e^2 + \frac{\hbar^2}{2m_p}\left(\vec{\nabla}_a^2+\vec{\nabla}_b^2\right) + \frac{e^2}{4\pi\epsilon_0}\left(\frac{1}{|\vec{r}_e-\vec{r}_a|}+\frac{1}{|\vec{r}_e-\vec{r}_b|}-\frac{1}{|\vec{r}_a-\vec{r}_b|}\right)\right]\Psi(\vec{r}_e,\vec{r}_a,\vec{r}_b) = E_t\Psi(\vec{r}_e,\vec{r}_a,\vec{r}_b)$$

…but quantum ***mechanics*** are not so restricted.

Division Drill

In fact, we use **both** — **combined!**

“duality”



- So, either
  - A nonlinearly coupled system of two equations

$$-\left[\frac{\hbar^2}{2m_e}\vec{\nabla}^2+\frac{e^2}{4\pi\epsilon_0}\left(\frac{1}{r}+\frac{1}{\sqrt{r^2+R^2-2rR\cos\theta}}\right)\right]\psi(\vec{r})=E_e\,\psi(\vec{r})$$

$$-\left[\frac{\hbar^2}{2m_p}\vec{\nabla}^2_{R}+\frac{2e^2}{4\pi\epsilon_0}\frac{1}{|\vec{R}|}+V_e(\vec{R})\right]\phi(\vec{R})=E_p\,\phi(\vec{R})$$

n → $n+1$

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- Or, a combined description:

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Grand-Gödelian Guide

- In fact Gödelian incompleteness = <u>*extensibility*</u> $\in$ Physics
  - System of equations = a particular model in classical fluid mechanics
  - $\to$ Maxwell's equations
- Just as a QFT model with only two families of fermions
  - or a strong $SO(3)_g$-force (Higgs-broken from $SU(3)_c$)

Phys. Rev. **D33** (1986) 1429
erratum: *Phys. Rev.* **D34** (1986) 3536

$e+\frac{df}{dx}+\frac{dg}{dy}+\frac{dh}{dz}=0$	(1)	Gauss' Law
$\mu\alpha=\frac{dH}{dy}-\frac{dG}{dz}$ $\mu\beta=\frac{dF}{dz}-\frac{dH}{dx}$ $\mu\gamma=\frac{dG}{dx}-\frac{dF}{dy}$	(2)	Equivalent to Gauss' Law for magnetism
$P=\mu\left(\gamma\frac{dy}{dt}-\beta\frac{dz}{dt}\right)-\frac{dF}{dt}-\frac{d\Psi}{dz}$ $Q=\mu\left(\alpha\frac{dz}{dt}-\gamma\frac{dx}{dt}\right)-\frac{dG}{dt}-\frac{d\Psi}{dy}$ $R=\mu\left(\beta\frac{dx}{dt}-\alpha\frac{dy}{dt}\right)-\frac{dH}{dt}-\frac{d\Psi}{dz}$	(3)	Faraday's Law (with the Lorentz Force and Poisson's Law)
$\frac{d\gamma}{dy}-\frac{d\beta}{dz}=4\pi p'$ $\quad p'=p+\frac{df}{dt}$ $\frac{d\alpha}{dz}-\frac{d\gamma}{dx}=4\pi q'$ $\quad q'=q+\frac{dg}{dt}$ $\frac{d\beta}{dx}-\frac{d\alpha}{dy}=4\pi r'$ $\quad r'=r+\frac{dh}{dt}$	(4)	Ampère-Maxwell Law
$P=-\xi p \quad Q=-\xi q \quad R=-\xi r$		Ohm's Law
$P=kf \quad Q=kg \quad R=kh$		The electric elasticity equation (**E** = **D**/ε)
$\frac{de}{dt}+\frac{dp}{dx}+\frac{dq}{dy}+\frac{dr}{dz}=0$		Continuity of charge

Grand-Gödelian Guide

- How to construct *Models* in *Theories*?
  - The domain space, $\mathfrak{D}$,
  - The target space, $\mathfrak{T}$, $\leftarrow$ *presumed!* 🫢
  - The map $\varphi : \mathfrak{D} \to \mathfrak{T}$
  - The functional $S[\varphi; C] := \int_{\mathfrak{D}} \mathscr{L}(\varphi, \dot{\varphi}, \ldots; C)$ & BC
- Classical theory: $\varphi_{cl} =$ solution of $\delta_\varphi S[\varphi; C] = 0$
- Quantum theory:

  (all alternative worlds)

  (sum over all alternative histories)
  - The partition functional, $Z[\vartheta; C] := \int \mathbf{D}[\varphi]\, e^{\left(S[\varphi;C] + \int \vartheta \cdot \varphi\right)/i\hbar}$
  - Correlations: $G(\xi_1, \ldots, \xi_n) := \lim_{\vartheta \to 0} \left\{ \frac{1}{Z[\vartheta; C]} \left[\frac{\partial}{\partial \vartheta(\xi_1)} \cdots \frac{\partial}{\partial \vartheta(\xi_n)} Z[\vartheta; C] \right] \right\}$

$$\mathfrak{D} = \mathbb{R}^1_t; \quad \mathfrak{T} = \mathbb{R}^3_{\vec{r}}; \quad \varphi = \vec{r}(t);$$
$$S[\vec{r}; m, k] = \int \mathrm{d}t \left\{ \frac{m}{2} \dot{\vec{r}}^2 - \frac{k}{2} \vec{r}^2 \right\}$$

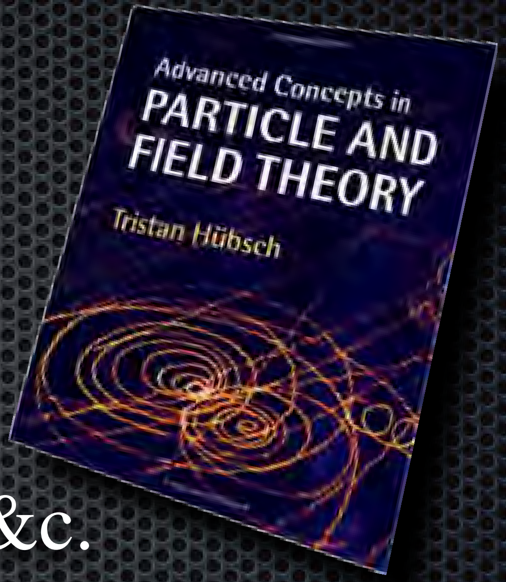
$$\mathfrak{D} = \mathbb{R}^{1,3}_{t,\vec{r}}; \quad \mathfrak{T} = \mathbb{R}^{1,3};$$
$$\varphi = (\, \Phi(\vec{r}, t), \vec{A}(\vec{r}, t) \,)$$

$\ldots$*etc.*

$S[\varphi; C] := \int_{\mathfrak{D}} \mathscr{L}(\varphi, \dot{\varphi}, \ldots; C)$ & BC

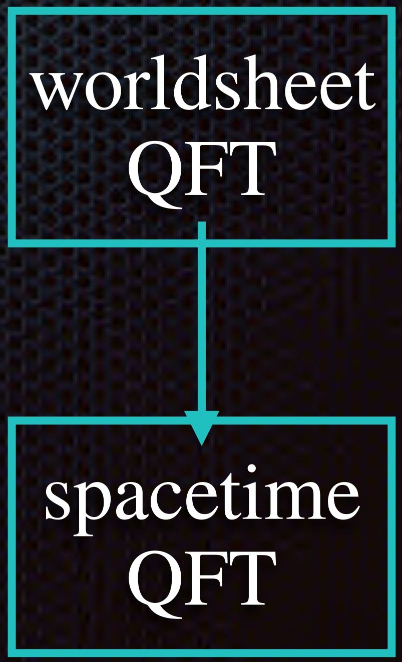
$Z[\vartheta; C] := \int \mathbf{D}[\varphi]\, e^{(S[\varphi;C] + \int \vartheta \cdot \varphi)/i\hbar}$

Grand-Gödelian Guide

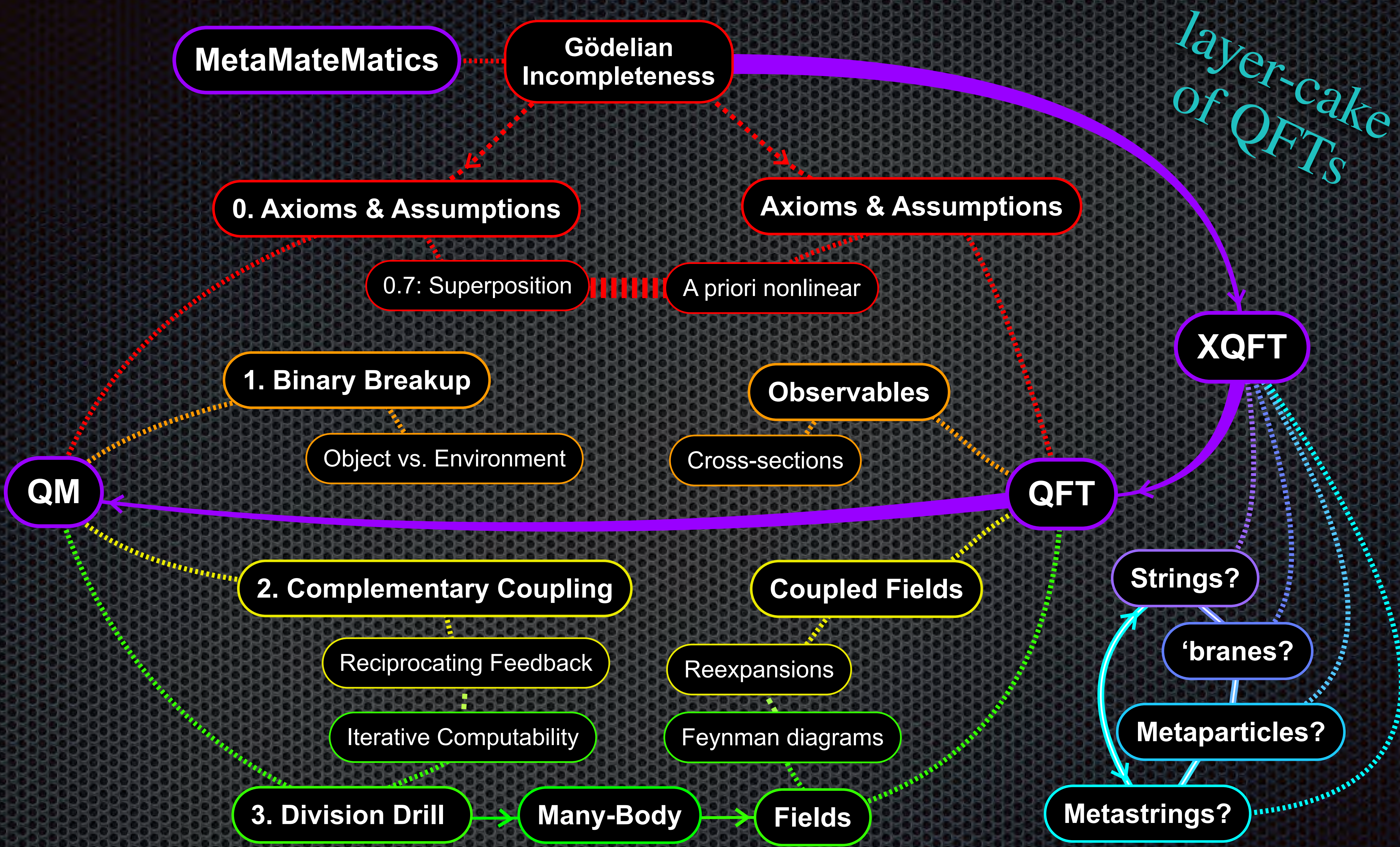


- We *can* turn the tables:
  - Choose: $\mathfrak{D}$; φ as "fields" on $\mathfrak{D}$; $S[\varphi; C]$ by symmetries &c. (1st layer)
  - Compute $\mathfrak{T}$ as the range of so constrained φ
- Induce "layers" of models
  - Define $e^{(S_{\text{eff}}[\phi;\widetilde{C}])/i\hbar} := \int \mathbf{D}[\varphi]\, e^{(S[\varphi+\phi;C])/i\hbar}$ & seek $\delta_Q C := (\widetilde{C} - C) \overset{!}{=} 0$
  - 1979, Friedan: for $S[X^\mu; g_{\mu\nu}] = \int \mathrm{d}^2\xi\, \gamma^{ij}(\xi)\, (\partial_i X^\mu)(\partial_j X^\nu)\, g_{\mu\nu}(X)$
  - $\delta_Q\, g_{\mu\nu}(X) \overset{!}{=} 0 \;\Rightarrow\;$ Einstein field equations!
  - …which makes the Einstein-Hilbert action stationary (2nd layer)
  - Generalized to all "familiar" gauge QFT models

Gödelian extensibility



layer-cake of QFTs



The Biger Picture

Go Forth & Calculate!

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