A Hitchhiker's Guide to Superstring Jump Gates and Other Worlds

T. Hübsch[a]*

[a]Howard University Physics Department,
2355 Sixth Str., NW, Washington DC20059

The general assumptions about the physical spacetime in superstring models are reviewed and reexamined. In particular, it is noted that the original Ansatz has undergone a number of generalizations in the past decade, and here we seek to establish some general facts about the admissible spacetimes in superstring theory. As a byproduct, we find that the generic spacetime offers an amusing and cosmologically novel type of candidate solutions.

1. THE STORY SO FAR

Precisely following in the footsteps of higher-dimensional supergravity models, physically realistic superstring models are sought for by compactifying the extra six dimensions into compact spaces of Planck size [1]. For many of the models that were originally thought not to have such a geometric interpretation, one has been found since. This includes generalizations involving additional vector bundles or sheaves, and possibly also generalizing the category of geometries (as will be seen below). We'll adopt a fairly cavalier point of view on this issue and use hereafter the geometric insight as a general tool, in the belief that perhaps a suitable reformulation of our arguments will always apply.

1.1. Approaches and some results

The first mental picture about superstring theory regards the world-sheet of the string as a (punctured) Riemann surface which is embedded in the (target) spacetime. Here, the spacetime must have 10 dimensions and may be chosen to be of the form $M^{s,1} \times K^{9-s}$, where K^{9-s} is a $9-s$-dimensional compact space, and $M^{s,1}$ is the $s+1$-dimensional Minkowski (flat) spacetime. For $s = 3$, we get the usual Ansatz, and requiring $N = 1$ supersymmetry in the effective field theory on $M^{3,1}$ forces K^6 to be a complex compact manifold of precisely $SU(3)$ holonomy (rather than a subgroup thereof)—Calabi-Yau manifolds [1].

Note however that the other solutions for K^6 ($K3 \times T^2$ and T^6), while producing more supersymmetry ($N = 2$ and $N = 4$, respectively), share one crucial property: all these are Ricci-flat. Furthermore, note that the Minkowski spacetime itself is Ricci-flat.

The second mental picture about superstring theory regards the world-sheet of the string as the spacetime of the underlying 2-dimensional field theory. The degrees of freedom parametrizing the spacetime for the effective field theory to be compared with the real world then comes as a certain subset of the field space in this 2-dimensional field theory. The (very degenerate) ground states of superstring theory with $N = 1$ supersymmetry in the effective 'real world' $M^{3,1}$ were identified with certain (2,0)-superconformal field theories.

From this second point of view, the analogue of Ricci-flatness is an anomaly cancellation requirement, as is perhaps most easily seen in the linear σ-models [2,3]. In fact, Ricci-flatness seems to be a required characteristic in all string theory, although rigorous results to this effect exist only for special cases [4–9].

- We will assume Ricci-flatness to be a general (stringy) required characteristic.

The third mental picture about superstring theory is rather novel and so still very preliminary. It involves the equally sketchy M- and F-theories, about which only particular facts are known and only regarding special compactifications of these. However, it is clear that the

*On leave from the Rudjer Bošković Institute, Zagreb, Croatia. Work supported by the Department of Energy grant DE-FG02-94ER-40854.

effective spacetime in these is (total) 11- and 12-dimensional; while the first one is consistently identified as (a compactification of) the 11-dimensional supergravity, the latter will presumably turn out to be a constraint theory (perhaps topological in some sense) [10,11]. In any case, it is quite clear that this approach will radically contribute to the general understanding of string theory, whereas it already is contributing on the technical side.

1.2. New techniques and some results

While the relationship between the first (geometrical) and the second (2-d field theory) approaches has long been studied, the linear σ-models finally put this on a really firm ground. In fact they show that the two classes of models are not identical, but rather a sort of analytic continuation of each other in the space of 'radial' deformations. Quite importantly then, questions independent of these 'radial' moduli can be answered in whichever model it turns out easier to calculate, and more exactly.

Similarly, 'mirror symmetry' relates models the geometrical interpretation of which is radically different, in a way that certain 'easy' results in one model correspond to 'hard' results in the other, and the other way around. Thereby again, certain questions can be answered in whichever model it turns out easier to calculate, and more exactly. 'Mirror symmetry' was also discovered by correlating models in the first (geometrical) and the second (2-d field theory) approach [12].

More recently, but similarly to 'mirror symmetry', various dualities have been found to relate even very different (super)string theories (or their sectors), including the M- and the F-theory. Typically, these dualities provide information that pertains to stringy non-perturbative sectors of a class of models and so are without precedent in the development of string theory. In addition, studying models with more than the phenomenologically interesting $N = 1$ supersymmetry and/or higher-dimensional un-compactified factor of the spacetime produces similarly unprecedented understanding of at least some non-perturbative effects.

Furthermore, the linear σ-model is maximally adapted to a reinterpretation in terms of toric geometry. A generic such construction involves typically many different types of models, geometric- and non-geometric-looking, but the straightforward links to toric geometry (and some 'mirror symmetry' considerations) provide a rigorous geometric interpretation to all of them [17]. This process however necessarily generalizes the category of possible K^6 into objects called 'stratified pseudomanifolds'—which are n-dimensional manifolds with m-dimensional ($m < n-1$) 'glued' onto them.

- The inclusion of stratified pseudomanifolds as candidate K^6 provides a geometrical interpretation to the largest class of models known to date.

1.3. Stringy cosmic strings

Clearly, this assumption of simple factorization (or, equivalently, that the compact 'internal' space K^{9-s} is constant over $M^{s,1}$) is by far not the generic case.

The analysis of a simple model [13] where K^6 has a T^2 factor, showed that models where this T^2 varies over 2 dimensions of $M^{3,1}$ are perfectly consistent solutions (assuming that the total spacetime is smooth). Ref. [14] then showed that it is possible—and in fact easy—to make *all* Ricci-flat spaces K^6 variable in spacetime. For technical reasons, this required to Wick-rotate the Minkowski $M^{3,1}$ into an Euclidean space X^4 and assume that it is in fact a complex 2-space. The Ricci-flat K^6 is then fibred over X^4, in such a way that this non-compact space is also Ricci-flat and (for technical reasons) smooth.

The novel feature of these spacetimes is that they exhibit cosmic strings in the $M^{3,1}$ spacetime. Unlike their Grand-Unified Theory relatives, these cosmic strings have a finite mass per unit length and the genus and the number of self-intersections of this cosmic world-sheet are easily calculable topological invariants of the whole 10-dimensional spacetime. Moreover, at the core of the cosmic string the 'internal' K^6 singularizes in precisely the fashion proven to connect topologically distinct K^6's [15].

At least for compactifications of Type II strings (with $N = 2$ supersymmetry, which enables the mechanism), the physical mechanism behind the conifold transitions [15] is also understood [16]. In this context at least, the physics of stringy cosmic string spacetimes [14] is perfectly well understood—including the previously puzzling core of the cosmic strings.

• We assume that a generalization (still a mystery) of this mechanism will similarly apply in cases of $N = 1$ supersymmetry.

The bottom line in these models is that the total, 10-dimensional spacetime (upon euclideanization) becomes a non-compact Ricci-flat complex 5-fold, in agreement with our assumption in § 1.1. These spaces are all fibrations of K^6 over (the euclideanized version of) $M^{3,1}$, where it is still perfectly consistent to think of K^6 as small—of Planck size—while the non-compact $M^{3,1}$ is naturally (infinitely) big.

2. GENERIC STRINGY SPACETIMES

Clearly, such a undemocratic distribution of size poses a tough question: why should have 3+1 spacetime dimensions undergo an inflationary expansion, while other 6 remained curled up and small?

In the generic case, of course, all 10 dimensions have expanded more or less alike, and we end up with a 10-dimensional spacetime of possibly complicated topology, but all of which is big. Whether or not such models can even pretend to have a reasonable physical interpretation, *these are the generic models*, and it behooves us to familiarize ourselves with them, if we are to chart out all possible stringy vacua. Note that there are two types of such models:

1. The 10-dimensional spacetime is compact, but much larger than the horizon distance, and so appears flat—at least in four dimensions (see below).

2. The 10-dimensional spacetime is at least partially non-compact, akin to the stringy cosmic string spacetimes.

The discussion below pertains to both, the modifications being technical [18].

2.1. Surprise!

For the purposes of this note, we are looking for at least one generic feature of Ricci-flat 10-spaces.

In the context of stratified pseudomanifolds (and certainly including ordinary manifolds) and the accompanying intersection homology theory [19], there is the notion of a 'small map'. It provides what is known as a 'small resolution'. In complex 5-folds, the mildest (and so most typical) kinds of singular points have a small resolution—which replaces the singular point with a complex 2-fold, i.e., a real 4-space. The crucial property of these 4-spaces is that they are isolated, i.e., rigid. This is in exact analogy to the fact that in 3-dimensional Ricci-flat manifolds there exist so-called $(-1,-1)$-curves, which have no deformation; they resemble ridges or creases in the manifold.

Given such isolated 4-spaces, a whole subset of the cohomology of the 10-space will be supported only on one such 4-space. That is, there are forms that vanish elsewhere, and are perfectly well-behaved precisely on this 4-space. This is perhaps easiest seen in a residue-type formalism, as it was used recently to provide an explicit representation of all (physically interesting) cohomology on Calabi-Yau 3-folds [20].

Given such a 4-space, the massless fields corresponding to this local cohomology would exist only on this 4-space, not elsewhere. In a favorable such 4-space, there could also exists a 'diagonal' cohomology basis for the whole 10-space in which the cohomology elements local to the 4-space would have no Yukawa couplings with those supported elsewhere in the 10-space. This provides a novel candidate cosmology, one in which the other dimensions are inaccessible not because they are curled up and small, but because we cannot exist there.

• The matter and radiation made up of these massless fields local to the 4-space would be topologically constrained to the 4-space and would not detect the rest of the 10-space or matter and fields therein.

The foregoing assumed all the time that $N = 1$ supersymmetry is maintained in the $M^{3,1}$ spacetime of the effective field theory. This of course puts constraints on the type of the normal bundle of the exceptional 4-space inside the 10-space. Further constraints are provided by the desired particle content. Assuming that this can all be satisfied, there remain two remarks to make at this preliminary level.

The 10-space metric produces standard 10-dimensional gravity, the Coulomb force of which decays as $\sim 1/r^8$. (If background configurations for other gauge fields are required as in the usual Ansatz, there are similar corresponding very short range Coulomb fields to be detected.) The question remains, whether it is possible to find a consistent $N = 1$ supersymmetric background field configuration where there is also a 'little' metric: a component that vanishes outside the 4-space, but therein produces the usual 4-dimensional gravity.

The other remark has to do with supersymmetry breaking, is needless to say highly speculative at this stage and is included for the Reader's amusement.

The topological constraints mentioned above stem from holomorphy of the euclideanized Ansatz, which in turn stems from supersymmetry. Therefore, when supersymmetry is broken at a scale $\sim M_S$, the previous rigorous constraint becomes a type of a a finite potential well—and therefore can be overcome by sufficiently energetic fields, possibly enveloping and object—this being a gedanken-prototype of a "warp bubble". Now the normal bundle of the 4-space inside the 10-space has a peculiar topology likened often to a helicoidal stairwell where the vertical shaft would correspond to the 4-space. If then a 4-space object can be sent, however near, outside the 4-space and returned at a different hyper-angle, it will reappear at a different point in the 4-space. This then would be a gedanken-prototype of a "hyperspace jump-gate".

REFERENCES

1. P. Candelas, G. Horowitz, A. Strominger and E.Witten: Nucl. Phys. B258 (1985) 46.
2. E. Witten, Nucl. Phys. B403 (1993) 159.
3. J. Distler and S. Kachru, Phys. Lett. 336B (1994), 368-375.
4. I.B. Frenkel, H. Garland and G.J. Zuckerman, Proc. Natl. Acad. Sci. USA 83 (1986) 8442.
5. M.J. Bowick and S.G. Rajeev: Phys. Lett. 58 (1987) 535, *ibid* 58(E) (1987) 1158, Nucl. Phys. B293 (1987) 348.
6. L. Alvarez-Gaumé, C. Gomez and C. Reina: 190B (1987) 55.
7. K. Pilch and N.P. Warner: Class. Quantum Grav. 4 (1987) 1183.
8. P. Oh and P. Ramond: Phys. Lett. 195B (1987) 130.
9. D. Harari, D.K. Hong, P. Ramond and V.G.J. Rogers: Nucl. Phys. B294 (1987) 556.
10. T. Hubsch, Math. Rev. (1996).
11. S.J. Gates, Jr. and P. van Nieuwenhuizen, private communication.
12. B.R. Greene and M. R. Plesser, Nucl. Phys. B338 (1990) 15.
13. B. Greene, A. Shapere, C. Vafa and S.-T. Yau, Nucl. Phys. B337 (1990) 1.
14. P. Green and T. Hübsch, Int. J. Mod. Phys. A9 (1994) 3203.
15. P.S. Green and T. Hübsch, Phys. Rev. Lett. 61 (1988) 1163, Commun. Math. Phys. 119 (1989) 431.
16. A. Strominger, Nucl. Phys. B451 (1995) 109-120.
17. P.S. Aspinwall, B.R. Greene and D.R. Morrison, Nucl. Phys. B416 (1994) 414, IAS preprint IASSNS-HEP-93/43, Nucl. Phys. B420 (1994) 184, IAS preprint IASSNS-HEP-93/81.
18. S.-T. Yau, private communication.
19. F. Kirwan, An Introduction to Intersection Homology Theory, (Longman Scientific & Technical, Essex, 1988).
20. P. Berglund and T. Hübsch, Int. J. Mod. Phys. A10 (1995) 3381–3430.