(Fundamentalna) Fizika Elementarnih Čestica

Dan 05a: Kalibracioni princip: neabelovsko uopštenje

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Fundamentalna Fizika Elementih Čestica

Program za danas (do pauze):

- **Princip kalibracione (lokalne) symmetrije**
  - Parcijalni izvod i kalibraciono-kovarijantni izvod
  - Opšta transformaciona "pravila"
- **$SU(3)_c$ transformacije**
  - Boja kao 3-dimenzionalan naboj
  - Matrične faze i lokalna simetrija
  - Matrična reprezentacija za $SU(3)$
- **$SU(3)_c$-invarijantni Lagranžijan**
  - Tenzor zakrivljenosti i Bianchi-ev identiteti
  - Jednačine kretanja
  - Očuvanje boje i jednačina kontinuiteta

Princip Kalibracione (Lokalne) Simetrije

Parcijalni i kalibraciono-kovarijantni izvod

- Pre svega:
- Matematički objekat, $\Psi(\vec{r}, t)$, koji predstavlja česticu
- zavisi (je funkcija) od raznih promenljivih:

eksterne
- pozicije u prostor-vremenu, → baza

interne
- faze (kao kompleksna funkcija),
- dodatnih stepeni slobode
  - spin
  - isospin
  - boja
  - …

zavisno od toga kako se "vlakna" "ponašaju"
- continualno
- 1/2…-put diffencijabilno
- glatno
- kompleksno-analitički
- ………

→ "vlakno"

…koji su sve neke "**koordinate**," u širem smislu te reči.

- Kalibracioni prinicip (lokalne simetrije):
  - interne koordinate mogu da slobodno zavise od eksternih.

"vlaknasti svežnjevi/snopovi/…"

Princip Kalibracione (Lokalne) Simetrije

Opšta transformaciona "pravila"

- Izvodi računaju stepen/meru promene
- Stoga, ako $\Psi(\vec{r}, t)$ takodje zavisi od $(\vec{r}, t)$-zavisne faze,
- onda stepen/mera promene $\Psi(\vec{r}, t)$ potiče od
  - varijacije $\Psi(\vec{r}, t)$ eksplicitno, i od
  - varijacije $\Psi(\vec{r}, t)$ implicitno, kroz varijaciju faze.
- Ako se "kalibracija" odnosi na "odmeravanje"/"fiksiranje" te faze,
- …onda "kalibraciono-kovarijantan" izvod
- …mora da sadrži dva dela: $\mathscr{D}_\mu := \partial_\mu + A_\mu(\vec{r}, t)$, **kalibraciono polje**
- gde je $A_\mu(\vec{r}, t)$ kalibracioni potencijal, tj. "koneksija".
- Matematičari: $\mathrm{d}x^\mu \mathscr{D}_\mu := \mathrm{d}x^\mu \partial_\mu + \mathrm{d}x^\mu A_\mu(\vec{r}, t)$ **koneksijska 1-forma**
  - daje definiciju (diferencijalno usmerenog izvoda) koja je nezavisna od izbora (eksternih = prostor-vremenskih) koordinata.

Princip Kalibracione (Lokalne) Simetrije

Opšta transformaciona "pravila"

- Unitarna kalibraciona transformacija (lokalne simetrije) je oblika $U_\varphi := \exp\{\, i\varphi\cdot \mathrm{Q}\,\}, \quad (U_\varphi{}^\dagger = U_\varphi{}^{-1})$
  - φ je niz (realnih) kalibracionih parametara,
  - Q je niz (ermitskih) generatora kalibracionih transformacija.
- Onda,
  - $\Psi(\vec r,t) \to U_\varphi \Psi(\vec r,t)$ & $\Psi(\vec r,t)^\dagger \to (U_\varphi \Psi(\vec r,t))^\dagger = \Psi(\vec r,t)^\dagger\, U_\varphi^\dagger = \Psi(\vec r,t)^\dagger\, U_\varphi^{-1}$;
  - $\mathscr{O}(\vec r,t) \to U_\varphi\, \mathscr{O}(\vec r,t)\, U_\varphi{}^{-1}$ za svaki operator koji deluje na $\Psi(\vec r,t)$;
  - …pa stoga i: $\mathscr{D}(\vec r,t) \to U_\varphi\, \mathscr{D}(\vec r,t)\, U_\varphi{}^{-1}$ (pošto je $\mathscr{D}$ linearan operator).
- Stoga, sa $\mathscr{D}_\mu = \mathbf{1}\,\partial_\mu + \mathscr{A}_\mu$ (gde je $\mathscr{A}_\mu := A_\mu \mathrm{Q}$)
  - $[\partial_\mu + \mathscr{A}_\mu(\vec r,t)] \to U_\varphi\, [\partial_\mu + \mathscr{A}_\mu(\vec r,t)]\, U_\varphi{}^{-1}$ implicira da
  - $\mathscr{A}_\mu(\vec r,t) \to U_\varphi\, \mathscr{A}_\mu(\vec r,t)\, U_\varphi{}^{-1} + U_\varphi\, (\partial_\mu U_\varphi{}^{-1})$

$$\widetilde{\mathscr{A}_\mu(\vec r,t)} = U_\varphi\, \mathscr{A}_\mu(\vec r,t)\, U_\varphi{}^{-1} \boxed{- iU_\varphi\, (\partial_\mu \varphi)\cdot \mathrm{Q}\, U_\varphi{}^{-1} \qquad \text{nehomogeni član}}$$

elektrodinamika: $\vec A \to \vec A + (\vec\nabla \lambda)$

$SU(3)_c$ Transformacije

Boja kao 3-dimenzioni naboj

- Setimo se:
  - $\Delta^{++} = [uuu]$,
  - $\Delta^{-} = [ddd]$,
  - $\Omega^{-} = [sss]$.

  Spin-$^3/_2$ barioni
  S-stanja; bez orbitalnog ugaonog momenta
  Prostorno simetrična talasna funkcija

- Pa, ili kvarkovi nisu ni bozoni ni fermioni

$$[\,b_i\,,\,b_j^{\dagger}\,] = \delta_{ij}, \qquad [\,b_i\,,\,b_j\,] = 0 = [b_i^{\dagger}, b_j^{\dagger}], \quad \text{bozoni,}$$

$$\{\,f_i\,,\,f_j^{\dagger}\,\} = \delta_{ij}, \qquad \{\,f_i\,,\,f_j\,\} = 0 = \{f_i^{\dagger}, f_j^{\dagger}\}, \quad \text{fermoni,}$$

- već <u>parafermioni</u> (O.W. Greenberg, 1964)

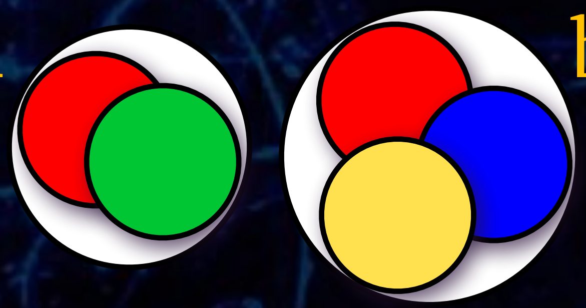
$$\left.\begin{array}{l} \{\,\tilde{f}_{i,\alpha}\,,\,\tilde{f}^{\dagger}_{j,\alpha}\,\} = \delta_{ij}, \qquad \{\,\tilde{f}_{i,\alpha}\,,\,\tilde{f}_{j,\alpha}\,\} = 0 = \{\,\tilde{f}^{\dagger}_{i,\alpha}\,,\,\tilde{f}^{\dagger}_{j,\alpha}\,\}, \\ [\,\tilde{f}_{i,\alpha}\,,\,\tilde{f}^{\dagger}_{j,\beta}\,] = \delta_{ij}, \qquad [\,\tilde{f}_{i,\alpha}\,,\,\tilde{f}_{j,\beta}\,] = 0 = [\,\tilde{f}^{\dagger}_{i,\alpha}\,,\,\tilde{f}^{\dagger}_{j,\alpha}\,], \quad \alpha \neq \beta, \end{array}\right\} \text{parafermioni,}$$

- …ili…

$SU(3)_c$ Transformacije

Boja kao 3-dimenzioni naboj

- … ili: Kvarkovi *jesu* fermioni,
- …ali imaju dodatni stepen slobode.
  - Januar 1965: Boris V. Struminsky, Dubna (Moskva, Russia)
  - …tada sa N. Bogolyubov-im + Albert Tavchelidze-om
  - Maj 1965, A. Tavchelidze: ICTP, Trieste (Italy)
  - Decembar 1965, Moo-Young Han + Yoichiro Nambu
    - celobrojno naelektrisani, obojeni kvarkovi + 8 (boja-antiboja) gluoni
  - Konačna verzija (sa razlomački naelektrisanim kvarkovima): 1974, William Bardeen, Harald Fritzsch & Murray Gell-Mann
  - Kvark: $\Psi_n^{\alpha A}(\vec{r}, t)$, gde:
    - $n = u, d, s, c, b, t$ označava "ukus"
    - α = crveno, žuto, plavo označava "boju"
    - $A = 1, 2, 3, 4$ osnačava komponentu Dirac-ovog spinora
- P.S.: Greenberg naknadno dokazuje ekvivalentnost…



$SU(3)_c$ Transformacije

Matrične faze i lokalna simetrija

$U_\varphi := \exp\{ i\varphi\cdot\mathrm{Q} \}$

- Bez pisanja Dirac-ovih komponenti,

$$\boldsymbol{\Psi}_n(\mathrm{x}) = \hat{\mathrm{e}}_\alpha \Psi_n^\alpha(\mathrm{x}) = \hat{\mathrm{e}}_\mathrm{r}\Psi_n^\mathrm{r}(\mathrm{x}) + \hat{\mathrm{e}}_\mathrm{y}\Psi_n^\mathrm{y}(\mathrm{x}) + \hat{\mathrm{e}}_\mathrm{b}\Psi_n^\mathrm{b}(\mathrm{x}) = \begin{bmatrix} \Psi_n^\mathrm{r}(\mathrm{x}) \\ \Psi_n^\mathrm{y}(\mathrm{x}) \\ \Psi_n^\mathrm{b}(\mathrm{x}) \end{bmatrix},$$

- …gde $n = u, d, s, c, b, t$ naznačava “ukus.”
- Pisanjem boja u matričnom formatu,
  - faza talasne funkcije kvarka postaje 3×3 matrica,
  - kao i operator unitarne transformacije faze, U_φ

$$\boldsymbol{\Psi}_n(\mathrm{x}) \to e^{i(\varphi/\hbar)\cdot(g_c \mathrm{Q})}\,\boldsymbol{\Psi}_n(\mathrm{x}), \qquad \varphi\cdot\mathrm{Q} := \varphi^a\,\mathrm{Q}_a$$

- gde su Q_a 3×3 matrice
  - ermitske, da bi U_φ bile unitarne,
  - bez traga, da bi U_φ bile unimodularne.

Dijagonalna transformacija, sa $\mathrm{Q}_0 = \mathbf{1}$,
deluje podjednako na sve boje,
kao elektromagnetizam…
…samo što $g_c \neq g_e$

$SU(3)_c$ Transformacije

Matrične faze i lokalna simetrija

Reprezentacija generatora transformacije (J_i) zavisi od toga na šta deluju

- Kalibracione (lokalno simetrijske) transformacije

$$[i\hbar \not{\mathscr{D}} - mc]\boldsymbol{\Psi}_n(\mathrm{x}) = 0 \to [i\hbar \not{\mathscr{D}}' - mc]\boldsymbol{\Psi}'_n(\mathrm{x}) = 0$$

$$\not{\mathscr{D}} := \gamma^\mu \mathscr{D}_\mu, \quad \mathscr{D}_\mu \to \mathscr{D}'_\mu := U_\varphi \mathscr{D}_\mu U_\varphi^{-1}$$

Dirac-ove matrice

$$U_\varphi = e^{i(\varphi^a/\hbar)(g_c \mathrm{Q}_a)}$$

- Multi-komponentnost: $\not{\mathscr{D}}\boldsymbol{\Psi}_n = \gamma^\mu \mathscr{D}_\mu \boldsymbol{\Psi}_n$

"boja" komponenta Dirac-ovog spinora

$$(\not{\mathscr{D}}_\mu \boldsymbol{\Psi}_n)^\alpha = \gamma^\mu \mathscr{D}_\mu{}^\alpha{}_\beta \boldsymbol{\Psi}_n^\beta \qquad (\not{\mathscr{D}}_\mu \boldsymbol{\Psi}_n)^{\alpha A} = [\gamma^\mu]^A{}_B \mathscr{D}_\mu{}^\alpha{}_\beta \boldsymbol{\Psi}_n^{\beta B}$$

- …koji se obično ne pišu eksplicitno.
- U opštem slučaju: $\not{\mathscr{D}} = \gamma^\mu \left[\mathbf{1}\,\partial_\mu + \frac{ig_c}{\hbar c} A^a_\mu \mathrm{Q}_a\right]$
- gde forma operatora Q_a zavisi od objekta na koji deluje
  - Q_a je 3×3 matrica ako je $\boldsymbol{\Psi}$ 3-komponentna matrica-kolona

$SU(3)_c$ Transformacije

Matrična reprezentacija SU(3)

$$U_\varphi = e^{i(\varphi^a/\hbar)(g_c Q_a)}$$

$$\not{\mathscr{D}} = \gamma^\mu \big[\mathbf{1}\,\partial_\mu + \underbrace{\tfrac{ig_c}{\hbar c} A^a_\mu Q_a}_{\mathscr{A}_\mu}\big]$$

- Iz "opšteg" formalizma sledi da je:

$$\mathscr{A}'_\mu = U_\varphi \mathscr{A}_\mu U_\varphi^{-1} + U_\varphi(\partial_\mu U_\varphi^{-1}) = \frac{ig_c}{\hbar c}\big[A^a_\mu - c(\partial_\mu \varphi^a)\big] U_\varphi Q_a U_\varphi^{-1}$$

  - pa $A'^a_\mu Q_a = \big[A^a_\mu - c(\partial_\mu\varphi^a)\big]\, U_\varphi Q_a U_\varphi^{-1}$.
  - Q_a su "generatori": $[Q_a, Q_b] = i f_{ab}{}^c Q_c$, $f_{ab}{}^c$="strukturna const."
  - Q_a su 3×3 matrice koje deluju na (kvark) 3-vektor boje.

- Ali, kako Q_a deluje na osam 4-vektorskih potencijala A^a_μ?

$$A'^a_\mu Q_a = \big[A^a_\mu - c(\partial_\mu\varphi^a)\big]\big(1 + \tfrac{ig_c}{\hbar}\varphi^b Q_b + \ldots\big) Q_a \big(1 - \tfrac{ig_c}{\hbar}\varphi^b Q_b + \ldots\big)$$

$$= A^a_\mu Q_a + \frac{ig_c}{\hbar} A^a_\mu \varphi^b Q_b Q_a - \frac{ig_c}{\hbar} A^a_\mu \varphi^c Q_a Q_c - c(\partial_\mu \varphi^a) Q_a + \ldots$$

$$\delta A^a_\mu Q_a = -c(\partial_\mu\varphi^a) Q_a - \frac{ig_c}{\hbar} A^b_\mu \big([Q_b, Q_c] = (i f_{bc}{}^a) Q_a\big)\varphi^c$$

$$\delta A^a_\mu = -c\Big[\delta^a_c \partial_\mu + \frac{g_c}{\hbar c} A^b_\mu \Big(\boxed{[\widetilde{Q}_b]_c{}^a := (f_{bc}{}^a)}\Big)\Big]\varphi^c = -c(\mathscr{D}_\mu \varphi)^a$$

$SU(3)_c$-invarijantni Lagranžijan

Tenzor zakrivljenosti i Bianchi-ev identitet

- Abelovsko/neabelovske razlike: $\mathscr{D}'_\mu = U_\varphi \mathscr{D}_\mu U_\varphi^{-1}$ implicira
  - $A'_\mu = A_\mu - (\partial_\mu \varphi)$ za elektrodinamiku
  - ali $A'^{\,a}_\mu = A^a_\mu - c(\mathscr{D}_\mu \varphi)^a = A^a_\mu - c(\partial_\mu \varphi^a) - \frac{g_c}{\hbar} f_{bc}{}^a A^b_\mu \varphi^c$

nelinearno

elektrodinamika: $\mathscr{D}\varphi = [\partial + A\mathrm{Q}]\varphi = \partial\varphi$

jer $A_\mu \to A_\mu + \partial_\mu \varphi$

- Takodje, za elektrodinamiku je
  - $F'_{\mu\nu} = F_{\mu\nu}$, tako da su $\vec{E}$ i $\vec{B}$ su kalibraciono invarijantni!
  - $F'_{\mu\nu} = \big[\partial_\mu A'_\nu - \partial_\nu A'_\mu\big] = \Big[\big(\partial_\mu A_\nu - \partial_\mu(\partial_\nu \varphi)\big) - \big(\partial_\nu A_\mu - \partial_\nu(\partial_\mu \varphi)\big)\Big] = \big[\partial_\mu A_\nu - \partial_\nu A_\mu\big]$
- Ali, u neabelovskom (matričnom) slučaju:
  - $\big[\partial_\mu A'^{\,a}_\nu - \partial_\nu A'^{\,a}_\mu\big] \neq \big[\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\big]$
  - $\big[\partial_\mu A'^{\,a}_\nu - \partial_\nu A'^{\,a}_\mu\big] \neq U_\varphi \big[\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\big] U_\varphi^{-1}$
  - …čak ni samo za infinitezimalne kalibracione transformacije $U_\varphi \approx \mathbf{1} + \frac{ig_c}{\hbar} \varphi^a \mathrm{Q}_a$

sve zbog $[\mathrm{Q}_a, \mathrm{Q}_b] = i f_{ab}{}^c \mathrm{Q}_c$

$SU(3)_c$-invarijantni Lagranžijan

Tenzor zakrivljenosti i Bianchi-ev identitet

- U elektrodinamici:
  - $[\mathscr{D}_\mu, \mathscr{D}_\nu]\, f(\mathrm{x}) = \big[\partial_\mu + \frac{iq}{\hbar c} A_\mu, \partial_\nu + \frac{iq}{\hbar c} A_\nu\big]\, f(\mathrm{x}) = \frac{iq}{\hbar c}\big[\big(\partial_\mu A_\nu - \partial_\nu A_\mu\big) = F_{\mu\nu}\big]\, f(\mathrm{x})$
  - Ovo mora da bude komutator, da rezultat u uglastoj zagradi ne bi bio diferencijalni operator, već "obična" funkcija.
  - A komutator upravo računa razliku u… pa, …u komutaciji.
- U opštem: $[\mathscr{D}, \mathscr{D}] = (\text{torzija}) \cdot \mathscr{D} + (\text{zakrivljenost})$
- Stoga računamo:
  - $\mathbb{F}_{\mu\nu} := \frac{\hbar c}{i g_c} [\mathscr{D}_\mu, \mathscr{D}_\nu] = \frac{\hbar c}{i g_c} [\partial_\mu + \frac{i g_c}{\hbar c} A^b_\mu \mathsf{Q}_b, \partial_\nu + \frac{i g_c}{\hbar c} A^c_\nu \mathsf{Q}_c]$
  - $= \big(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\big)\mathsf{Q}_a + \frac{\hbar c}{i g_c}\big(\frac{i g_c}{\hbar c}\big)^2 A^b_\mu A^c_\nu \big([\mathsf{Q}_b, \mathsf{Q}_c] = i f_{bc}{}^a \mathsf{Q}_a\big)$
  - pa je $\mathbb{F}_{\mu\nu} := F^a_{\mu\nu}\, \mathsf{Q}_a$ i $F^a_{\mu\nu} := (\partial_\mu A_\nu - \partial_\nu A_\mu) - \frac{g_c}{\hbar c} f_{bc}{}^a A^b_\mu A^c_\nu$

$SU(3)_c$-invarijantni Lagranžijan

Tenzor zakrivljenosti i Bianchi-ev identitet

- Po definiciji, imamo da je
  - $\mathbb{F}_{\mu\nu} \to \mathbb{F}'_{\mu\nu} := \frac{\hbar c}{ig_c}[\mathscr{D}'_\mu, \mathscr{D}'_\nu] = \frac{\hbar c}{ig_c}[U_\varphi \mathscr{D}_\mu U_\varphi^{-1}, U_\varphi \mathscr{D}_\nu U_\varphi^{-1}]$
  - $= \frac{\hbar c}{ig_c} U_\varphi [\mathscr{D}_\mu, \mathscr{D}'_\nu] U_\varphi^{-1}$
  - $= U_\varphi \mathbb{F}_{\mu\nu} U_\varphi^{-1}$

Nije invarijantno nego kovarijantno!

- Nezavisno,
  - $\mathscr{D}_\mu \mathbb{F}_{\nu\rho} f(\mathrm{x}) = [\mathscr{D}_\mu, \mathbb{F}_{\nu\rho}] f(\mathrm{x}) = \frac{\hbar c}{ig_c}\big[\mathscr{D}_\mu, [\mathscr{D}_\nu, \mathscr{D}_\rho]\big] f(\mathrm{x})$
  - gde važi $\big[A,[B,C]\big] + \big[B,[C,A]\big] + \big[C,[A,B]\big] = 0$
  - pa onda $\varepsilon^{\mu\nu\rho\sigma}\mathscr{D}_\mu \mathbb{F}_{\nu\rho} f(\mathrm{x}) = \frac{\hbar c}{ig_c}\varepsilon^{\mu\nu\rho\sigma}\big[\mathscr{D}_\mu, [\mathscr{D}_\nu, \mathscr{D}_\rho]\big] f(\mathrm{x}) = 0$
- Za sve $SU(n)$ grupe: $\mathrm{Tr}[\mathbb{F}_{\mu\nu}] = F^a_{\mu\nu}\,\mathrm{Tr}[\mathrm{Q}_a] = 0$
  - Važi za sve *poluproste* Lie-jeve grupe (= bez abelovskih faktora).

$SU(3)_c$-invarijantni Lagranžijan

Jednačine kretanja

- Pošto se matrična zakrivljenost transformiše transformacijom sličnosti, $\mathbb{F}'_{\mu\nu} = U_\varphi \, \mathbb{F}_{\mu\nu} \, U_\varphi^{-1}$
  - ...na šta je funkcija traga invarijantna, $\mathrm{Tr}[UXU^{-1}] = \mathrm{Tr}[X]$,
  - $\mathrm{Tr}[\mathbb{F}_{\mu\nu}\,\mathbb{F}^{\mu\nu}] \to \mathrm{Tr}[\mathbb{F}'_{\mu\nu}\,\mathbb{F}'^{\mu\nu}] = \mathrm{Tr}[U_\varphi \mathbb{F}_{\mu\nu} U_\varphi^{-1}\, U_\varphi \mathbb{F}^{\mu\nu} U_\varphi^{-1}]$
  - $= \mathrm{Tr}[\mathbb{F}_{\mu\nu}\,\mathbb{F}^{\mu\nu} U_\varphi^{-1}\, U_\varphi]$
  - $= \mathrm{Tr}[\mathbb{F}_{\mu\nu}\,\mathbb{F}^{\mu\nu}]$
- pa biramo:
  - $\mathscr{L}_{\mathrm{QCD}} = \sum_n \mathrm{Tr}\Big[\overline{\boldsymbol{\Psi}}_n[i\hbar c\, \not{\mathscr{D}} - m_n c^2]\boldsymbol{\Psi}_n\Big] - \frac{1}{4{g_c}^2}\mathrm{Tr}[\mathbb{F}_{\mu\nu}\mathbb{F}^{\mu\nu}]$
  - $= \sum_n \mathrm{Tr}\Big[\overline{\boldsymbol{\Psi}}_{\alpha n}[i\boldsymbol{\gamma}^\mu(\hbar c\,\delta^\alpha_\beta\partial_\mu + i g_c A^a_\mu \tfrac{1}{2}[\boldsymbol{\lambda}_a]^\alpha{}_\beta) - m_n c^2\delta^\alpha_\beta]\boldsymbol{\Psi}^\beta_n\Big] - \frac{1}{4{g_c}^2} F^a_{\mu\nu}\, F_a^{\mu\nu}$
  - gde trag ostaje još samo po komponentama Dirac-ovih spinora i matrice, $\mathrm{Tr}[\overline{\boldsymbol{\Psi}}_{\alpha n}\ldots\boldsymbol{\gamma}^\mu\ldots\boldsymbol{\Psi}^\beta_n] = \overline{\Psi}_{\alpha n A}\ldots[\gamma^\mu]^A_B\ldots\Psi^{\beta B}_n$

$SU(3)_c$-invarijantni Lagranžijan

Jednačine kretanja

$$\mathscr{L}_{\rm QCD} = \sum_n {\rm Tr}\big[\overline{\boldsymbol{\Psi}}_n[i\hbar c\,\not{\mathscr{D}} - m_n c^2]\boldsymbol{\Psi}_n\big] - \frac{1}{4 g_c{}^2}{\rm Tr}[\mathbb{F}_{\mu\nu}\mathbb{F}^{\mu\nu}]$$

- Varijacija po A^a_μ daje:
  - $\mathscr{D}_\mu F^{a\,\mu\nu} = g_c \sum_n \overline{\Psi}_{n\alpha A}\,[\gamma^\nu]^A_B\,\frac{1}{2}[\lambda^a]^\alpha{}_\beta\,\Psi^{\beta B}_n$
  - $J^{a\,\mu}_{(q)} = g_c \sum_n \overline{\Psi}_{n\alpha A}\,[\gamma^\nu]^A_B\,\frac{1}{2}[\lambda^a]^\alpha{}_\beta\,\Psi^{\beta B}_n$ — kvarkovska struja boje
- U elektrodinamici je
  - $(\mathscr{D}_\mu F^{\mu\nu} = \partial_\mu F^{\mu\nu}) = g_e \overline{\Psi}_A\,[\gamma^\nu]^A_B\,\Psi^B =: J^\nu_e$ — struja naelektrisanja
  - Pa je $\partial_\nu J^\nu_e = \frac{4\pi\epsilon_0 c}{4\pi}\partial_\mu\partial_\nu F^{\mu\nu} \equiv 0$ pošto je $F_{\mu\nu} \equiv -F_{\nu\mu}$
- Isto ne sledi za $\mathscr{D}_\mu F^{a\,\mu\nu} \neq \partial_\mu F^{a\,\mu\nu}$ za ne-abelovske sile
- Umesto toga: $\mathscr{D}_\mu F^{a\,\mu\nu} = J^{a\,\nu}_{(q)}$,
  - pa je $\mathscr{D}_\nu J^{a\,\nu}_{(q)} = \mathscr{D}_\nu \mathscr{D}_\mu F^{a\,\mu\nu} = -\frac{1}{2}[\mathscr{D}_\mu, \mathscr{D}_\nu]\,F^{a\,\mu\nu} = -\frac{1}{2} f_{bc}{}^a\,F^b_{\mu\nu}\,F^{c\,\mu\nu} \equiv 0$
  - pošto je $f_{bc}{}^a = -f_{cb}{}^a$

$SU(3)_c$-invarijantni Lagranžijan

Očuvanje boje i jednačina kontinuiteta

- Neabelovski kovarijantni izvod nije jednačina kontinuiteta:
  - $0 = \mathscr{D}_\mu J^{a\,\mu}_{(q)} = \partial_\mu J^{a\,\mu}_{(q)} - \frac{g_c}{\hbar c} f_{bc}{}^a A^b_\mu J^{c\,\mu}_{(q)}$
  - daje $\frac{\mathrm{d}}{\mathrm{d}t}\int_V \mathrm{d}^3\vec{r}\;\partial_\mu J^{a\,0}_{(q)} = -\oint_{\partial V}\mathrm{d}^2\vec{r}\,\partial_\mu \vec{J}^{\,a}_{(q)} - \frac{g_c}{\hbar c} f_{bc}{}^a \int_V \mathrm{d}^3\vec{r}\,(A^b_\mu J^{c\,\mu}_{(q)})$
- …pošto dodatni član na desnoj strani ne iščezava u opštem.
- Medjutim, $J^{a\,\nu}_{(q)} = \mathscr{D}_\mu F^{a\,\mu\nu} = \partial_\mu F^{a\,\mu\nu} - \frac{g_c}{\hbar c} f_{bc}{}^a A^b_\mu F^{c\,\mu\nu}$
  - pa $\partial_\mu F^{a\,\mu\nu} = J^{a\,\nu}_{(c)} := J^{a\,\nu}_{(q)} + \frac{g_c}{\hbar c} f_{bc}{}^a A^b_\mu F^{c\,\mu\nu}$
  - daje $\partial_\nu J^{a\,\nu}_{(c)} = \partial_\nu \partial_\mu F^{a\,\mu\nu} \equiv 0$

stacionarna struja boje
$J^{a\,\nu}_{(c)} := J^{a\,\nu}_{(q)} + \frac{g_c}{\hbar c} f_{bc}{}^a A^b_\mu F^{c\,\mu\nu}$

- …pa i kvarkovi i gluoni doprinose očuvanom naboju boje:
  - $Q^a_{(c)} := \int \mathrm{d}^3\vec{r}\; J^{a\,0}_{(c)} := g_c \int \mathrm{d}^3\vec{r}\,\Big(\sum_n \mathrm{Tr}[\overline{\boldsymbol{\Psi}}_n \gamma^0 \boldsymbol{\lambda}^a \boldsymbol{\Psi}_n] + \frac{1}{\hbar c} f_{bc}{}^a A^b_\mu F^{c\,\mu 0}\Big)$
  - gde je trag i po bojama kvarkova i po Dirac-ovim komponentama

$SU(3)_c$-invarijantni Lagranžijan

Očuvanje boje i jednačina kontinuiteta

- Ovo menja analogon Gauss-Ampère-ovih zakona.
- Pogledajmo $\nu = 0$ slučaj jednačine $\mathscr{D}_\mu F^{a\,\mu\nu} = J^{a\,\nu}_{(q)}$:
  - $\partial_\mu F^{a\,\mu 0} - \frac{g_c}{\hbar c} f_{bc}{}^a A^b_\mu F^{c\,\mu 0} = J^{a\,0}_{(q)}$
- …i definišemo: $\vec{E}^a := \hat{e}_i F^{a\,i0}, \quad \vec{A}^a := -\hat{e}^i A^a_i, \quad \rho^a_{(q)} := J^{a\,0}_{(q)}$
- Onda je $\boxed{\vec{\nabla} \cdot \vec{E}^a = \rho^a_{(q)} - \frac{g_c}{\hbar c} f_{bc}{}^a \vec{A}^b \cdot \vec{E}^c}$ Gauss-ov zakon za boju
- Valja primetiti:
  - Nemoguće je napisati neabelovske analogone Maxwell-ovih jednačina bez korišćenja kalibracionih potencijala
  - I kvarkovi i gluoni služe kao “izvori” za polje sile boje
  - Analogon Maxwell-ovih jednačina su nelinearne po poljima

$SU(3)_c$-invarijantni Lagranžijan

Očuvanje boje i jednačina kontinuiteta

- Da sumiramo:
  - Maxwell-ove jednačine $\mathscr{D}_\mu \mathbb{F}^{\mu\nu} = \mathbb{J}^\nu_{(q)}$ i $\varepsilon^{\mu\nu\rho\sigma}(\mathscr{D}_\mu \mathbb{F}_{\nu\rho}) = 0$
  - kvarkovska struja: $\mathbb{J}^\nu_{(q)} := g_c \sum_n \overline{\Psi}_{n\alpha A} [\gamma^\mu]^A_B \frac{1}{2} [\lambda^a]^\alpha{}_\beta \Psi^{\beta B}_n \mathsf{Q}_a$
  - Kompletna struja: $\mathbb{J}^\nu_{(c)} := \mathbb{J}^\nu_{(q)} + \frac{ig_c}{\hbar c} [\mathbb{A}_\mu, \mathbb{F}^{\mu\nu}]$ $\quad \partial_\mu \mathbb{F}^{\mu\nu} = \mathbb{J}^\nu_{(c)}$
  - Jednačina kontinuiteta: $\partial_\nu \mathbb{J}^\nu_{(c)} = 0$
  - Očuvani naboj boje: $\frac{\mathrm{d}}{\mathrm{d}t} \int_V \mathrm{d}^3\vec{r}\ \mathbb{J}^0_{(c)} = -\oint_{\partial V} \mathrm{d}^2\vec{r} \cdot \vec{J}_{(c)}$

Kafica?

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